# Canyon Crest Academy

# Integrated Math 3
# Module 4 Honors
# &
# 3.3, 3.5
# Trigonometric Functions

Adapted from

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## 3.3H Warm Up
***Special Rights – A Solidify Understanding Task***



In previous courses you have studied the Pythagorean Theorem and right triangle trigonometry. Both of these mathematical tools are useful when trying to find missing sides of a right triangle.

1. What do you need to know about a right triangle in order to use the Pythagorean Theorem?

2. What do you need to know about a right triangle in order to use right triangle trigonometry?

While using the Pythagorean Theorem is fairly straight forward (you only have to keep track of the legs and hypotenuse of the triangle), right triangle trigonometry generally requires a calculator to look up values of different trigonometry ratios. There are some right triangles, however, for which knowing a side length and an angle measure is enough to calculate the value of the other sides without using trigonometry. These are known as ***special right triangles*** because their side lengths can be found by relating them to another geometric figure for which we know a great deal about its sides.

**One type of special right triangle is a $45° - 45° - 90°$ triangle.**

3. Draw a $45° - 45° - 90°$ triangle and assign a specific value to one of its sides. (For example, let one of the legs measure 5 cm, or choose to let the hypotenuse measure 8 inches. You will want to try both approaches to perfect your strategy.) Now that you have assigned a measurement to one of the sides of your triangle, find a way to calculate the exact measures of the other two sides. As part of your strategy, you may want to relate this triangle to another geometric figure that may be easier to think about.

4. Generalize your strategy for a $45° - 45° - 90°$ triangle by letting one side of the triangle measure *x*. Show how the exact measures of the other two sides can be represented in terms of *x*. Make sure to consider cases where *x* is the length of a leg, as well as the case where *x* is the length of the hypotenuse.

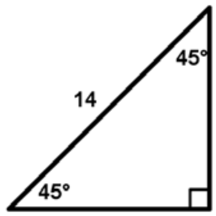
**<u>Another type of special right triangle is a $30° - 60° - 90°$ triangle</u>.**

5. Draw a $30° - 60° - 90°$ triangle and assign a specific value to one of its sides. Find a way to calculate the exact measures of the other two sides. As part of your strategy, you may want to relate this triangle to another geometric figure that may be easier to think about.

6. Generalize your strategy for $30° - 60° - 90°$ triangles by letting one side of the triangle measure *x.* Show how the exact measures of the other two sides can be represented in terms of *x.* Make sure to consider cases where *x* is the length of a leg, as well as the case where *x* is the length of the hypotenuse.

Find the missing sides of each special right triangle using the $45° - 45° - 90°$ or $30° - 60° - 90°$ triangle rules. Leave answers with simplified radicals, where necessary.

7.

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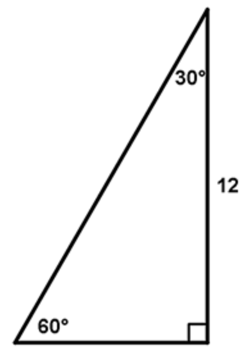


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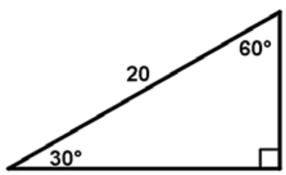


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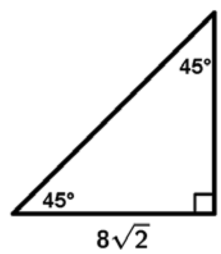
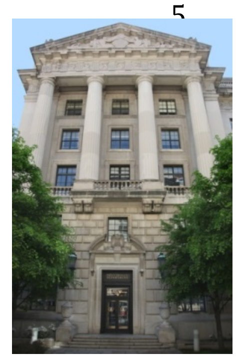


# 3.3H More Than Right
## *A Develop Understanding Task*



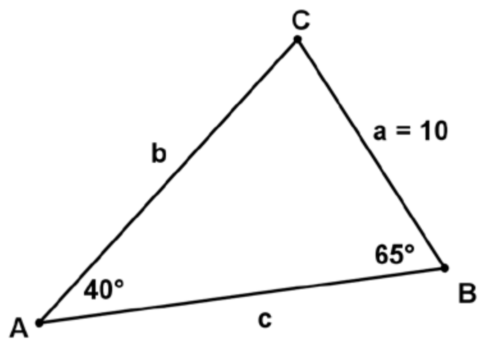


We can use right triangle trigonometry and the Pythagorean Theorem to solve for missing sides and angles in a ***right triangle***. What about ***other*** triangles? How might we find unknown sides and angles in acute or obtuse triangles if we only know a few pieces of information about them?

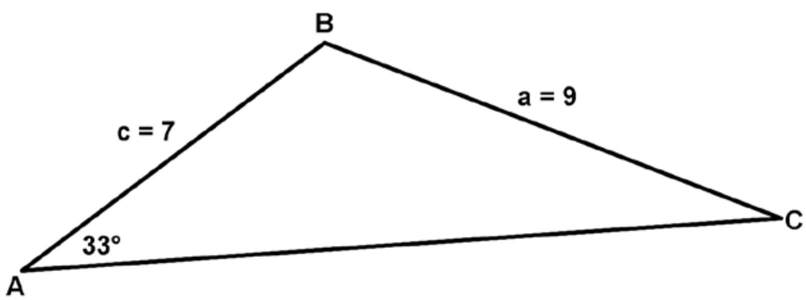
In the previous homework and in today's warm up, we found it might be helpful to create right triangles by drawing an altitude in a non-right triangle. We can then apply trigonometry or the Pythagorean Theorem to the smaller right triangles, which may help us learn something about the sides and angles in the larger triangle.

See if you can devise a strategy for finding the missing sides and angles of each of these triangles.

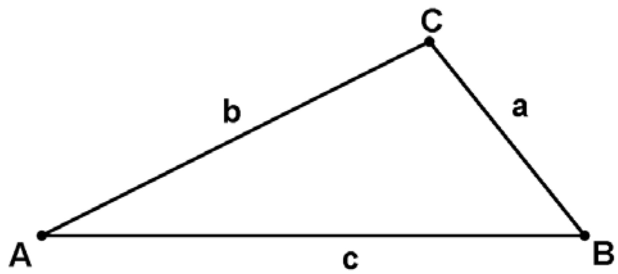
1.




2.



3. See if you can generalize the work you have done on questions 1 and 2 by finding relationships between sides and angles in the following diagram. Unlike the previous two questions, this triangle contains an ***obtuse angle*** at *C*. Find as many relationships as you can between sides *a, b,* and *c* and the related angles *A, B,* and *C.*




**LAW OF SINES**

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

**LAW OF COSINES**

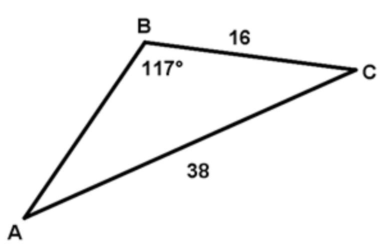
$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$b^2 = a^2 + c^2 - 2ac\cos B$$

$$c^2 = a^2 + b^2 - 2ab\cos C$$

Use the Law of Sines and/or the Law of Cosines to find the missing side lengths and angle measures in each triangle below. Round your answers to the nearest tenth.

4.

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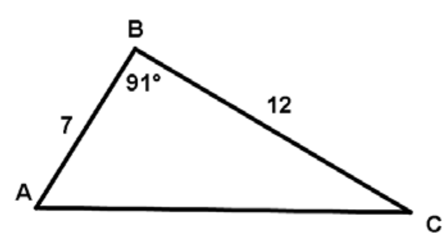

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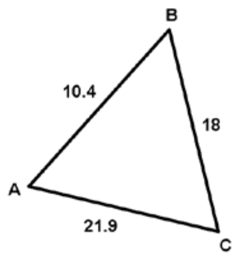
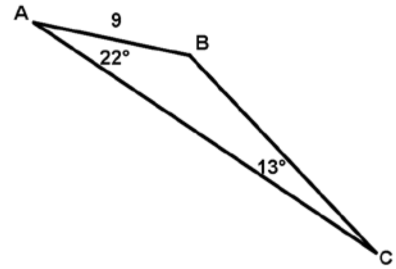
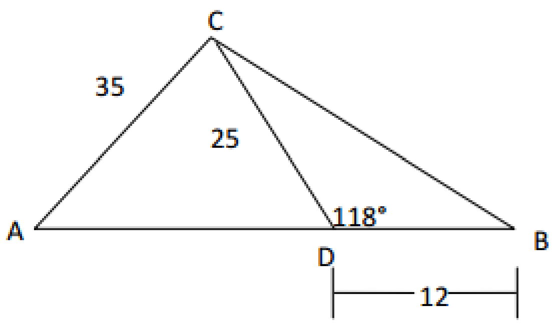


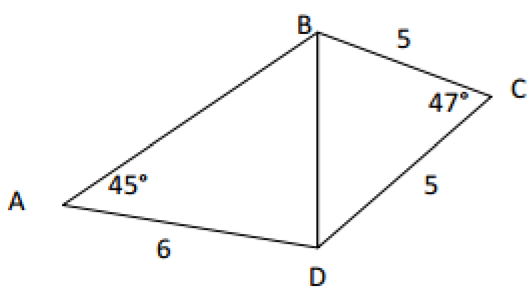

7.




8. Find angle A.



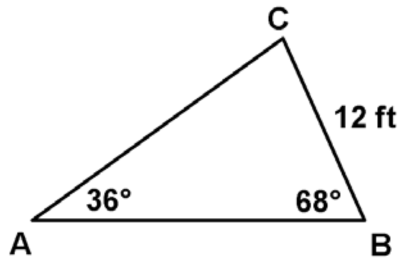

9. Find angle ABD.



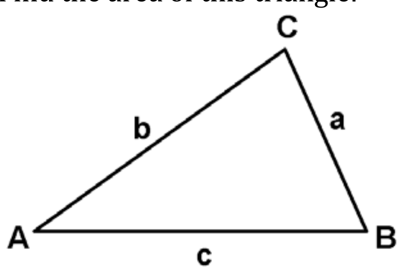
# 3.5H Triangle Areas by Trig
## *A Practice Understanding Task*


Find the area of the following two triangles using the strategies and procedures you have developed in the past few tasks. For example, draw an altitude as an auxiliary line, use right triangle trigonometry, use the Pythagorean Theorem, or use the Law of Sines or the Law of Cosines to find needed information.

1. Find the area of this triangle.




2. Find the area of this triangle.



Jumal and Jabari are helping Jumal's father with a construction project. He needs to build a triangular frame as a component of the project, but he has not been given all the information he needs to cut and assemble the pieces of the frame. He is even having a hard time envisioning the shape of the triangle from the information he has been given.

Here is the information about the triangle that Jumal's father has been given:

- Side $a = 10.00$ meters
- Side $b = 15.00$ meters
- $\angle A = 40.0°$

Jumal's father has asked Jumal and Jabari to help him find the measure of the other two angles and the missing side of this triangle.

3. Carry out each student's described strategy. Then draw a diagram showing the shape and dimensions of the triangle that Jumal's father should construct. (Note: To provide as accurate information as possible, Jumal and Jarbari decide to round all calculated sides to the nearest centimeter, that is, to the nearest hundredth of a meter, and all angle measures to the nearest tenth of a degree.

   <u>Jumal's Approach</u>

   - Find the measure of $\angle B$ using the Law of Sines

   - Find the measure of the third $\angle C$

   - Find the measure of side $c$ using the Law of Sines

   - Draw the triangle

Jabari's Approach

- Solve for $c$ using the Law of Cosines: $a^2 = b^2 + c^2 - 2bc \cos A$

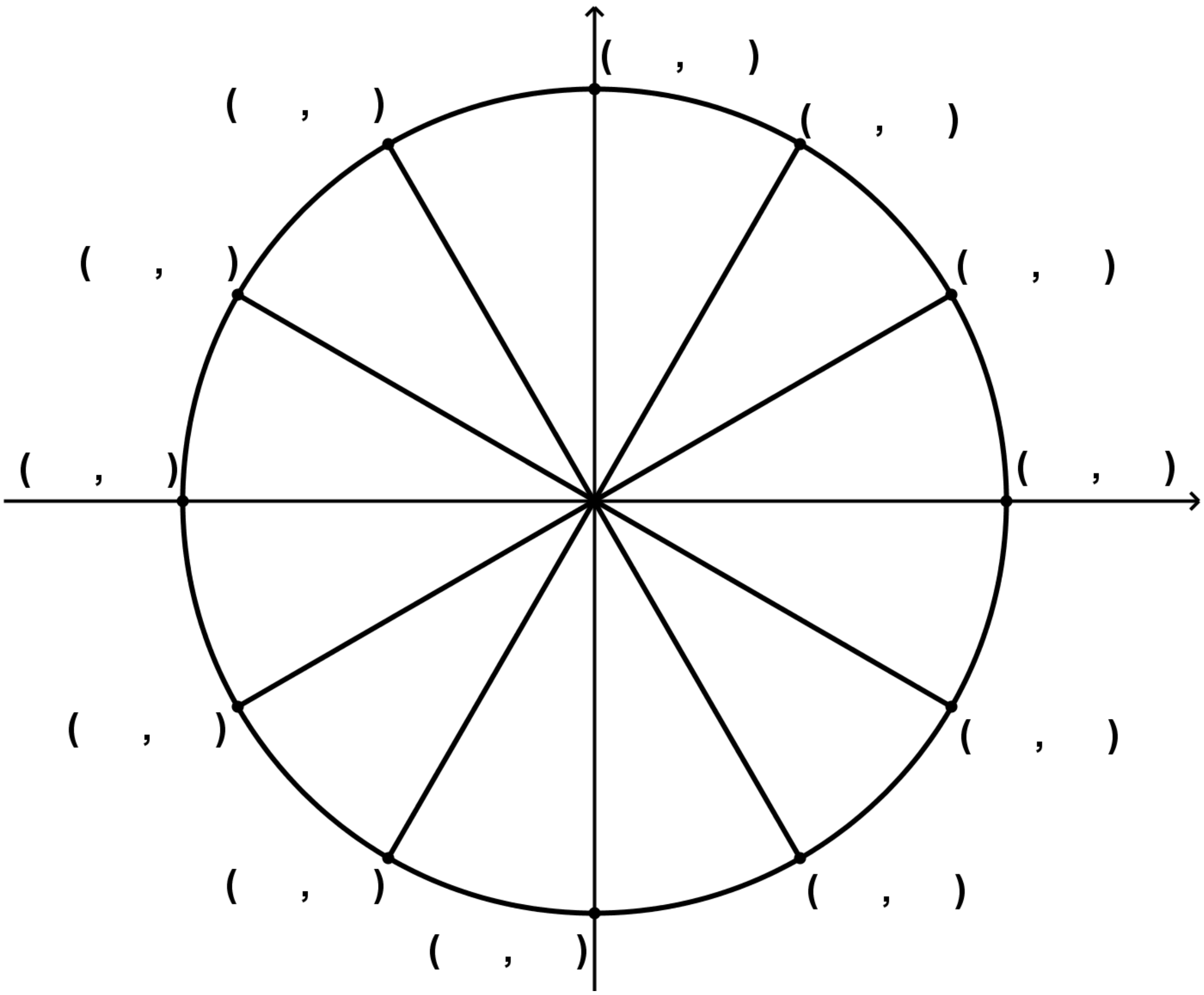
- Jabari is surprised that this approach leads to a quadratic equation, which he solves with the quadratic formula. He is even more surprised when he finds **two reasonable solutions** for the length of side *c.*

Draw both possible triangles and find the two missing angles of each using the Law of Sines.

## 4.1H Warm Up
***Trigonometric Functions – finding exact values***

**Explain the relationship between degrees and radians**

**Complete the unit circle below using your understanding of degrees, radians, and special right triangles**

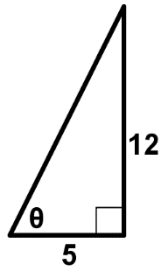


**Review Trigonometric Ratios**
Find the trigonometric ratios for the given triangles. Simplify all answers completely.

$\sin\theta =$

$\cos\theta =$

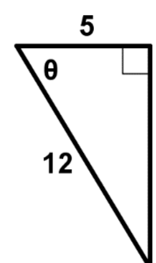
$\tan\theta =$




$\csc\theta =$

$\sec\theta =$

$\cot\theta =$



An angle is in **standard position** if its vertex is located at the origin and one ray is on the positive *x*-axis. The ray on the *x*-axis is called the **initial side** and the other ray is called the **terminal side.**

An angle, in standard position, that is rotated counter clockwise has a **positive measurement**. An angle, in standard position, that is rotated clockwise has a **negative measurement**.

If $\theta$ is an angle in standard position, its **reference angle** is the acute angle formed by the terminal side of $\theta$ and the *x*-axis.

Two angles, in standard position, are **coterminal** if they share a terminal side.

**For each angle measurement below, (A) sketch the angle in standard position, (B) identify the measure of the reference angle, and (C) identify the measure of an angle that is coterminal with the given angle.**

| | |
|---|---|
| 5. $110°$<br>(A) Sketch:<br><br><br><br>(B) Reference Angle:<br><br>(C) Coterminal Angle: | 6. $345°$<br>(A) Sketch:<br><br><br><br>(B) Reference Angle:<br><br>(C) Coterminal Angle: |
| 7. $76°$<br>(A) Sketch:<br><br><br><br>(B) Reference Angle:<br><br>(C) Coterminal Angle: | 8. $-192°$<br>(A) Sketch:<br><br><br><br>(B) Reference Angle:<br><br>(C) Coterminal Angle: |

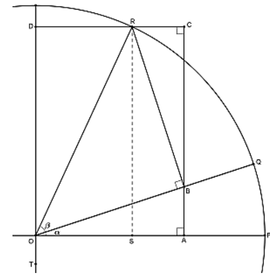
**Find the exact values of the following.**

| | | |
|---|---|---|
| 1. $\cos(210)$ | 2. $\sin(330)$ | 3. $\tan(-750)$ |
| 4. $\sin\left(\frac{\pi}{3}\right) =$ | 5. $\cos\left(\frac{3\pi}{2}\right) =$ | 6. $\tan(\pi) =$ |
| 7. $\tan\left(\frac{\pi}{4}\right) =$ | 8. $\sin\left(\frac{5\pi}{4}\right) =$ | 9. $\cos\left(\frac{3\pi}{4}\right) =$ |
| 10. $\sin\left(\frac{\pi}{4}\right) =$ | 11. $\cos\left(-\frac{\pi}{4}\right) =$ | 12. $\tan\left(\frac{\pi}{2}\right) =$ |
| 13. $\cos\frac{\pi}{6} =$ | 14. $\tan -\frac{11\pi}{6} =$ | 15. $\sin\frac{7\pi}{3} =$ |
| 16. $\sin\left(\frac{\pi}{6}\right) =$ | 17. $\tan\left(\frac{5\pi}{6}\right) =$ | 18. $\cos\left(\frac{13\pi}{6}\right) =$ |

**Find the exact values of the following.**

| | |
|---|---|
| 19. Given $\sin\theta = -\frac{4}{5}$ and $sec\theta > 0$; find<br>$\sin\theta =$ $\csc\theta =$<br>$\cos\theta =$ $\sec\theta =$<br>$\tan\theta =$ $\cot\theta =$ | 20. Given $\tan\theta = \frac{3}{5}$ and $csc\theta < 0$; find<br>$\sin\theta =$ $\csc\theta =$<br>$\cos\theta =$ $\sec\theta =$<br>$\tan\theta =$ $\cot\theta =$ |

# 4.1H Understanding Identities
## *A Practice Understanding Task*

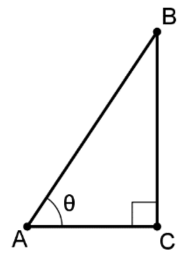


## Part I: Fundamental Trigonometric Identities

Right triangles and the unit circle provide images that can be used to derive, explain, and justify a variety of trigonometric identities.

For example, how might the right triangle diagram below help you justify why the following identity is true for all angles $\theta$ between 0° and 90°?

$$\sin\theta = \cos(90° - \theta)$$

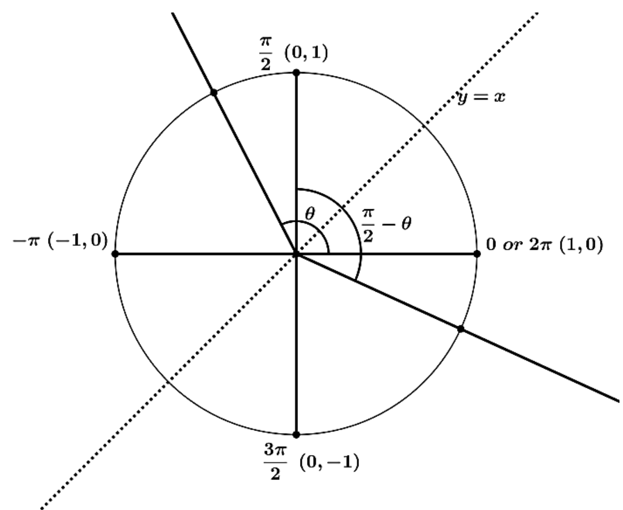



Since we have extended our definition of the sine to include angles of rotation, rather than just the acute angles in a right triangle, we might wonder if this identity is true for all angles $\theta$, not just those that measure between 0° and 90°?

A version of this identity that uses radian rather than degree measure would look like this:

$$\sin\theta = \cos\left(\frac{\pi}{2} - \theta\right)$$

How might you use this unit circle diagram to justify why this identity is true for all angles $\theta$?



Here are some additional trigonometric identities. Use either a right triangle diagram or a unit circle diagram to justify why each is true.

1. $\sin(-\theta) = -\sin\theta$

2. $\cos(-\theta) = \cos\theta$

3. $\sin^2\theta + \cos^2\theta = 1$ [Note: This is the preferred notation for $(\sin\theta)^2 + (\cos\theta)^2 = 1$]

4. $\frac{\sin\theta}{\cos\theta} = \tan\theta$

5. $\tan(-\theta) = -\tan\theta$

# Part II: More Trigonometric Identities

Luna and Happ are in math class trying to solve sin(105°), they decide that there is no way they can solve this problem without using a calculator. Shultis joins their group and realizes 105° could be broken up into two angles that they know how to find the exact value of using a sum.

Which two angles add up to 105° so sin(105°) can be found without using a calculator?

Shultis gives Luna and Happ the formula sheet they forgot to pick up on the way into class.

| | | | |
|---|---|---|---|
| **Half-Angle Formulas** | $\sin\frac{u}{2} = \pm\sqrt{\frac{1-\cos u}{2}}$ | $\cos\frac{u}{2} = \pm\sqrt{\frac{1+\cos u}{2}}$ | $\tan\frac{u}{2} = \frac{1-\cos u}{\sin u}$<br>$= \frac{\sin u}{1+\cos u}$ |
| **Double Angle Formulas** | $\sin 2u = 2\sin u\cos u$<br>$\cos 2u = \cos^2 u - \sin^2 u = 2\cos^2 u - 1 = 1 - 2\sin^2 u$<br>$\tan 2u = \frac{2\tan u}{1-\tan^2 u}$ | | |
| **Sum and Difference Formulas** | $\sin(u+v) = \sin u\cos v + \cos u\sin v$<br>$\sin(u-v) = \sin u\cos v - \cos u\sin v$<br>$\cos(u+v) = \cos u\cos v - \sin u\sin v$<br>$\cos(u-v) = \cos u\cos v + \sin u\sin v$ | $\tan(u+v) = \frac{\tan u + \tan v}{1-\tan u\tan v}$<br>$\tan(u-v) = \frac{\tan u - \tan v}{1+\tan u\tan v}$ | |

Using the formula $\sin(u+v) = \sin u\cos v + \cos u\sin v$, Shultis solves the problem:

$$\sin(60+45) = \sin 60\cos 45 + \cos 60\sin 45$$

$$\sin(60+45) = \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) + \left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$$

$$\sin(60+45) = \left(\frac{\sqrt{6}}{4}\right) + \left(\frac{\sqrt{2}}{4}\right) = \left(\frac{\sqrt{6}+\sqrt{2}}{2}\right)$$

However, using the calculator Happ and Luna got 0.9659258…

Did they get the same answer?

**Use the formulas above to solve the following problems.**

1. Use the angle sum or difference of angles formulas to answer the following questions.

   a. Find $\sin(75°)$

   b. Find $\cos(195°)$

The group continues with the classwork and comes across the questions $\sin(165)$ . Luna uses the same strategy as before

Luna's work:

$$\sin(120 + 45) = \sin 120 \cos 45 + \cos 120 \sin 45$$

Happ says, I agree but we could also use the half angle formula since $\frac{330}{2}$=165

Happ's work:

$$sin\,\frac{330}{2} = \pm\sqrt{\frac{1-\cos(330)}{2}}$$

$$sin\,\frac{330}{2} = \pm\sqrt{\frac{1-\frac{\sqrt{3}}{2}}{2}} =$$

2. If $\sin A = -\frac{3}{5}$ with $\pi < A < \frac{3\pi}{2}$ and $\cos B = \frac{12}{13}$ with $0 < B < \frac{\pi}{2}$, find…

   a. $\cos(A + B)$

   b. $\cos(2A)$

   c. $\sin(A - B)$

   d. $\sin(2A)$

**4.2H Warm-Up**
***Finding trig expressions***

1. Find cosθ if $sin\theta = \frac{2}{3}$ and $0 \le \theta \le \frac{\pi}{2}$

2. Find tanθ if $sin\theta = \frac{3}{7}$ and $\frac{\pi}{2} \le \theta \le \pi$

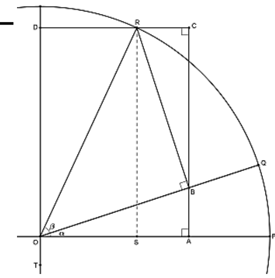
3. Find $csc\theta$ if $\cos\theta = \frac{-\sqrt{3}}{2}$ and $\pi \le \theta \le \frac{3\pi}{2}$

4. Find $sec\theta$ if $sin\theta = \frac{1}{3}$ and $\frac{3\pi}{2} \le \theta \le 2\pi$

5. If $\tan\theta = -\frac{4}{5}$, $270° < \theta < 360°$, find all the remaining functions of θ.

6. Find the values of the six trig. functions of θ, if θ is an angle in standard position with the point (-5, -12) on its terminal ray.

# 4.2H More Identities
## *A Practice Understanding Task*



**Simplify the following:**

1. $cos\,(270° - x) =$

2. $sin\,(x + \frac{\pi}{2}) =$

**Verify the following trigonometric identities.**

3. $\cot x + \tan x = \frac{1}{\sin x \cos x}$

4. $\frac{\sin^4 x - \cos^4 x}{\sin^2 x - \cos^2 x} = 1$

5. $\frac{\tan^2 x}{\tan^2 x + 1} = \sin^2 x$

6. $\frac{\sin^2 x + 4 \sin x + 3}{\cos^2 x} = \frac{3 + \sin x}{1 - \sin x}$

7. $\sin 2x = \frac{2\tan x}{1+\tan^2 x}$

8. $1 + \sin 2x = (\sin x + \cos x)^2$

9. $\sin 2x = (\tan x)(1 + \cos 2x)$

10. $\cos 2x = \frac{1-\tan^2 x}{1+\tan^2 x}$

## 4.3H Warm-Up

Using sum, difference, double and half angle Identities

**Find each of the following numbers:**

If $\sin A = \frac{12}{13}$, $0 < A < \frac{\pi}{2}$ and $\cos B = -\frac{8}{17}$, $\pi < B < \frac{3}{2}\pi$

1. sin (A + B)

2. cos (A – B)

3. tan (A + B )

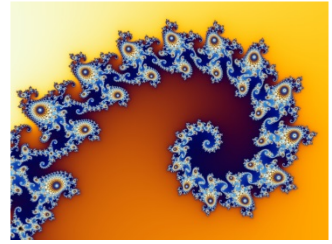
4. Simplify:

   **(a)** $\sin(67\frac{1}{2}{}^\circ)$

   **(b)** $\cos\left(\frac{\pi}{8}\right)$

   **(c)** $\sin\left(\frac{5\pi}{8}\right)$

   **(d)** $\cos(202\frac{1}{2}{}^\circ)$



## 4.3H How Many Solutions Can There Be?
***A Practice Understanding Task***

Throughout this module, you will investigate various periodic phenomena: finding the height of a rider on a Ferris Wheel, describing the high and low tide, and examining the body temperature of a newly discovered animal, just to name a few.

All of these situations involve the use of trigonometric equations to find particular solutions. Many times, these contexts offer an infinite amount of solutions. These solutions were typically evenly spaced. Many trigonometric equations have an infinite amount of solutions and some have no solutions. Furthermore, you have explored trigonometric identities, which have solutions for *any* given angle value.

Below are some basic trigonometric equations for you to solve on the interval $[0, 2\pi)$.

1. $\sin x = \frac{\sqrt{3}}{2}$
2. $\cos x = \frac{\sqrt{2}}{2}$
3. $\csc x = -2$
4. $\tan x = -1$
5. $\cos x = 1$
6. $\sin x\,(1 + \cos x) = 0$

Because of the restricted domain given for the equations above, there were only a **finite number** of solutions for each problem. If there was no restriction on the domain, there would likely be an **infinite amount** of solutions.

7. Consider the equation: $\sin x = \frac{\sqrt{2}}{2}$

   a. What are the solutions in the domain $[0, 2\pi)$?

   b. One of the solutions from part a is in the first quadrant on the unit circle. Find four more solutions that are also in the first quadrant if there is no domain restriction. (Hint: think of **coterminal** angles)

   c. In what quadrant are the other solutions located? List four of these solutions.

   d. Generalize all solutions to the equation $\sin x = \frac{\sqrt{2}}{2}$.

Find all solutions to the following trigonometric equations. Be resourceful in your methods – you may want to consider factoring and using a trigonometric identity along the way.

8. $7\cos x + 9 = -2\cos x$

9. $(\tan x - 1)(\cos x + 1) = 0$

10. $2\sin^2 x - \sin x - 1 = 0$

11. $\sec^2 x - 2 = 0$

12. $\cos 2x = \frac{\sqrt{3}}{2}$

13. $3\sin^2 x + 7\sin x + 4 = 0$

14. $\sin 2x = \sin x$

15. $\cos 2x + \cos x + 1 = 0$

16. $\sin\left(x + \frac{\pi}{4}\right) + \sin\left(x - \frac{\pi}{4}\right) = 1$

17. $\sin 2x \cos x - \cos 2x \sin x = \frac{\sqrt{2}}{2}$

# 4.4H Warm Up

*Solving Trig equations*

Solve the following equations and list the solutions in the interval $(0, 2\pi]$

1. $3cot^2x - 1 = 0$

2. $2sin^2x + 5sin\,x = 3$

Solve the following equations and list the general solutions.

3. $2tan^2\frac{x}{2} - tan\frac{x}{2} - 6 = 0$

4. $sec3x\ sin3x - 3sin3x = 0$

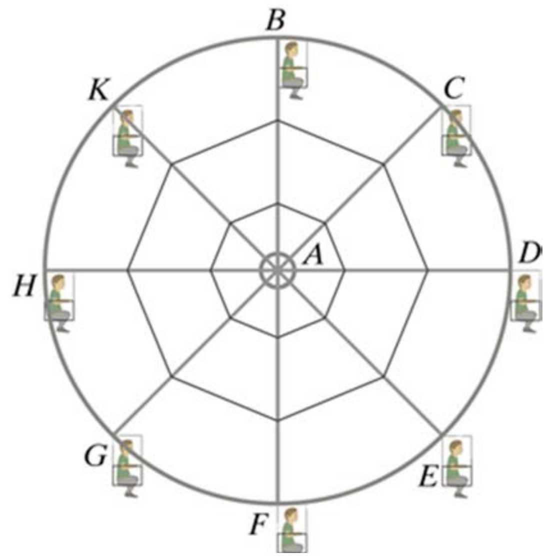
# 4.4H "Sine" Language
## *A Solidify Understanding Task*



## Part 1
Carlos and Clarita are participating in an afterschool enrichment class where they have to build a model replica of a Ferris wheel.

They are required to use the following measurements:

- The Ferris wheel has a radius of 1 meter.
- The center of the Ferris wheel is at ground level




Carlos has also been carefully timing the rotation of the wheel by placing a doll in the cart in position D and has observed the following additional fact:

- The Ferris wheel makes one complete rotation counterclockwise every 360 seconds

1. Using this new information, how many degrees does the Ferris wheel rotate per second (angular speed)?

2. How high will the doll be 10 seconds after passing position A on the diagram?

3. Calculate the height of the doll at each of the following times *t*, where *t* represents the number of seconds since the doll passed position A on the diagram. Keep track of any patterns you notice in the ways you calculate the height. As you calculate each height, record the time and height on the diagram as the coordinates (time, height).

| **Elapsed time, *t*, since passing position A** | **Calculations** | **Height of the doll** |
|---|---|---|
| 10 sec | | |
| 20 sec | | |
| 30 sec | | |
| 45 sec | | |
| 60 sec | | |
| 90 sec | | |

4. Examine your calculations for finding the height of the doll during the first 90 seconds after passing position A. During this time, the angle of rotation of the doll is somewhere between 0° and 90°. Write a general formula for finding the height of the doll during this time interval.

5. Calculate the height of the doll at each of the following times $t$, where $t$ represents the number of seconds since the doll passed position A on the diagram. Keep track of any patterns you notice in the ways you calculate the height. As you calculate each height, record the time and height on the diagram as the coordinates (time, height).

| | | |
|---|---|---|
| 120 sec | | |
| 150 sec | | |
| 180 sec | | |
| 225 sec | | |
| 270 sec | | |
| 315 sec | | |
| 330 sec | | |
| 360 sec | | |
| 400 sec | | |
| 420 sec | | |
| 630 sec | | |
| 660 sec | | |
| 720 sec | | |

6. How might you find the height of the doll in other "quadrants" of the Ferris wheel, when the angle of rotation is greater than 90°?

## Part 2

Carlos and Clarita are making notes of what they have observed about this new way of defining $\sin\theta$.

Carlos: "For some angles the calculator gives me positive values for $\sin\theta$, and for other angles it gives me negative values."

1. Without using your calculator, list at least five angles of rotation for which the value of $\sin\theta$ produced by the calculator should be **positive**.

2. Without using your calculator, list at least five angles of rotation for which the value of $\sin\theta$ produced by the calculator should be **negative**.

Clarita: "Yeah, and sometimes we can't even draw a triangle at certain positions on the Ferris wheel, but the calculator still gives us values for the sine at those angles of rotation."

3. List possible angles of rotation that Clarita is talking about - positions for which you can't draw a triangle to use as a reference. Then, without using your calculator, give the value of the sine that the calculator should provide at those positions.

Carlos: "And, because of the symmetry of the circle, some angles of rotation should have the same values for the sine."

4. Without using your calculator, list at least five **pairs** of angles that should have the same sine value.

Clarita: "Right! And if we go around the circle more than once, the calculator still gives us values for the sine of the angle of rotation, and multiple angles have the same value of the sine."

Carlos: "Can you have a sine value for an angle less than zero?"

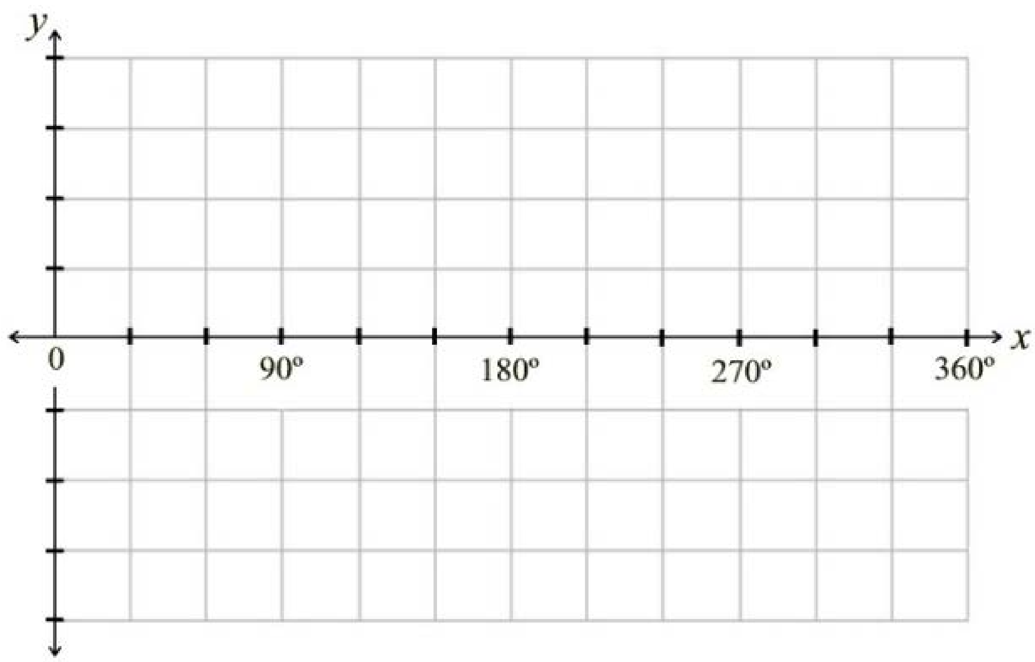
5. List some angles that satisfy Clarita's statement. Explain why her statement is true.

6. a. For which angles of rotation are the values of sine positive?

   b. For which angles of rotation are the values of sine negative?

7. Explain how you find the angle of rotation in **quadrant II, III, and IV** when the reference angle has a measurement of $\theta$.

8. Based on the data you calculated, as well as any additional insights you might have about riding on Ferris wheels, sketch a graph of the height of the doll on this Ferris wheel as a function of the elapsed time since the rider passed the position farthest to the right on the Ferris wheel. We can consider this position as the rider's starting position at time $t = 0$. Be sure to include the starting position.




9. Write the equation of the graph you sketched in question 8.

10. Carlos realizes that a Ferris wheel at ground level does not make sense.

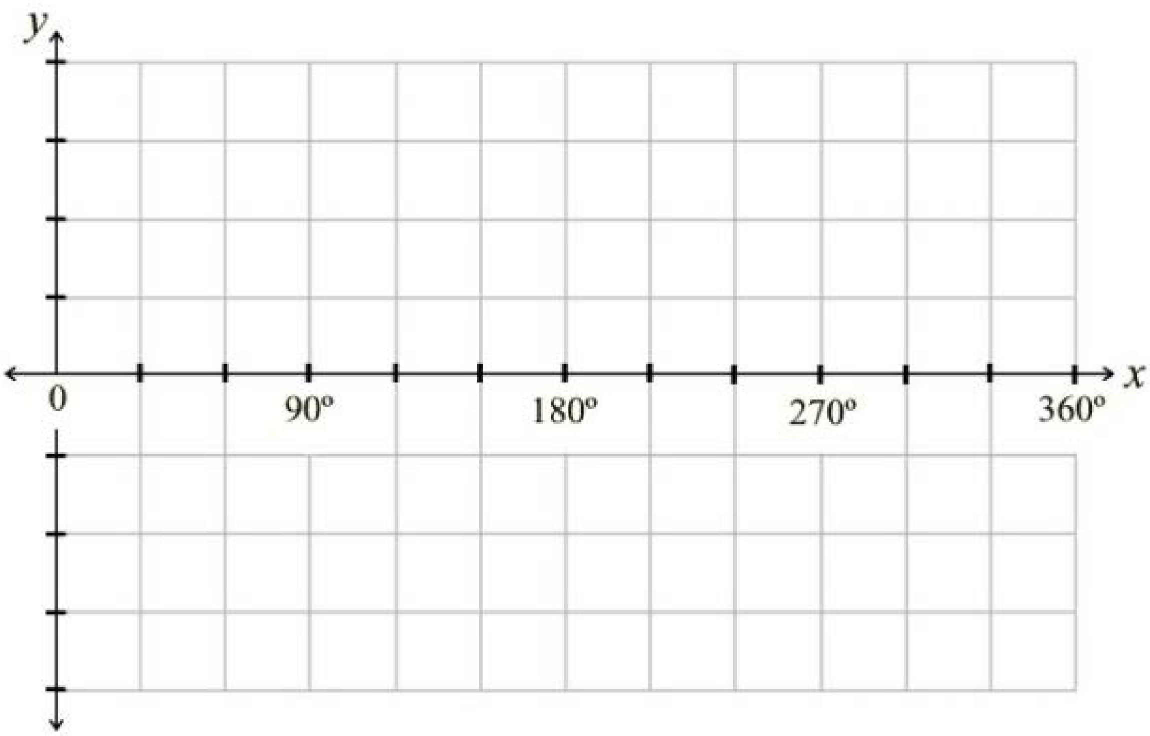
a. How would the graph and equation change if he built the Ferris wheel 2 meters above ground?

New Equation:

b. How would the graph and equation change if he built the Ferris wheel 1 meter above the ground with a radius of 3?

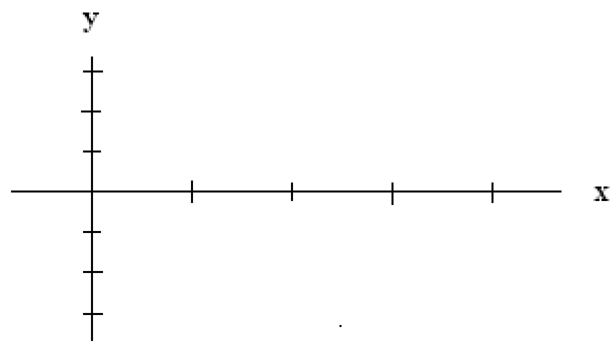
New Equation:

Graph both equations from part a and b on the graph below.

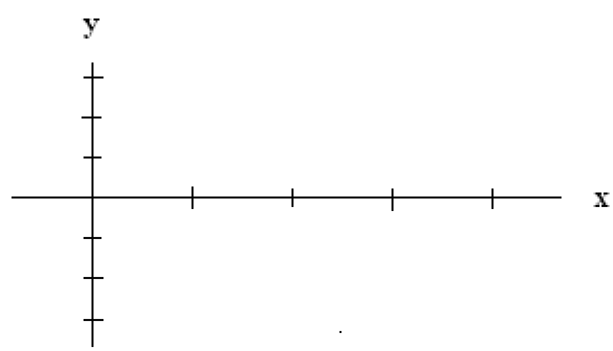


## Practice

$y = 5sinx - 1$ Amplitude________ Midline ___________




$y = -3\ sinx + 2$ Amplitude________ Midline ___________



# 4.5H Warm-Up

## More Ferris Wheels

*A Solidify Understanding Task*



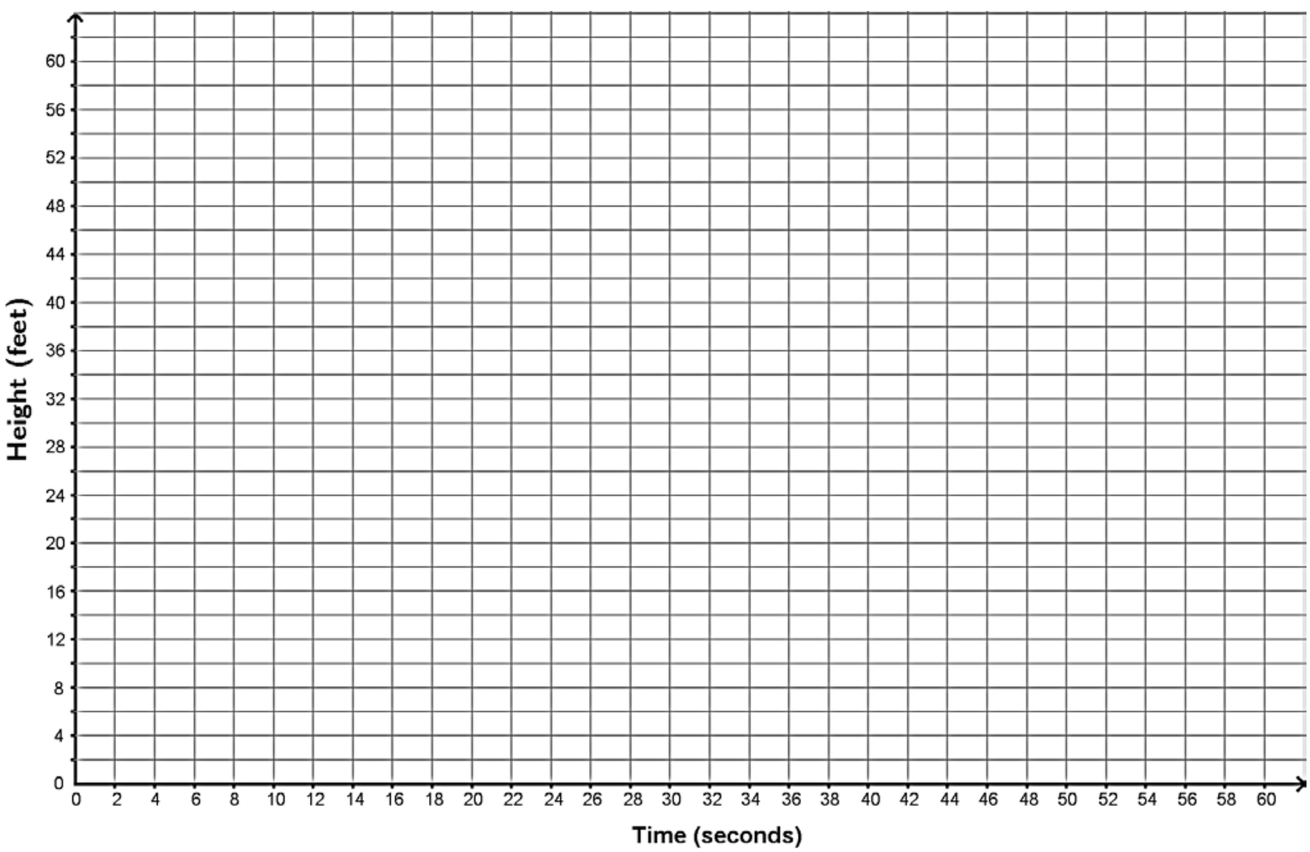
In a previous task, *"Sine" Language*, you calculated the height of a doll on a model Ferris wheel at different times $t$, where $t$ represented the elapsed time after the rider passed the position farthest to the right on the Ferris wheel.

Use the following facts for a new Ferris wheel :

- The Ferris wheel has a radius of 25 feet.
- The center of the Ferris wheel is 30 feet above the ground.
- The Ferris wheel makes one complete rotation counterclockwise every 24 seconds.

1. Using what you learned in the previous task, as well as any additional insights you might have about riding on Ferris wheels, sketch a graph of the height of a rider on this Ferris wheel as a function of the elapsed time since the rider passed the position farthest to the right on the Ferris wheel. We can consider this position as the rider's starting position at time $t = 0$. Be sure to include the starting position.

| t | 3 | 4 | 6 | 8 | 9 | 12 | 16 | 20 | 24 |
|---|---|---|---|---|---|---|---|---|---|
| f(t) | | | | | | | | | |




2. Write the equation of the graph you sketched in question 1.

3. We began this task by considering the graph of the height of a rider on a Ferris wheel with a radius of 25 feet and center 30 feet off the ground, which makes one revolution counterclockwise every 24 seconds. How would your **graph** change if:

   a. the radius of the wheel was larger? or smaller?

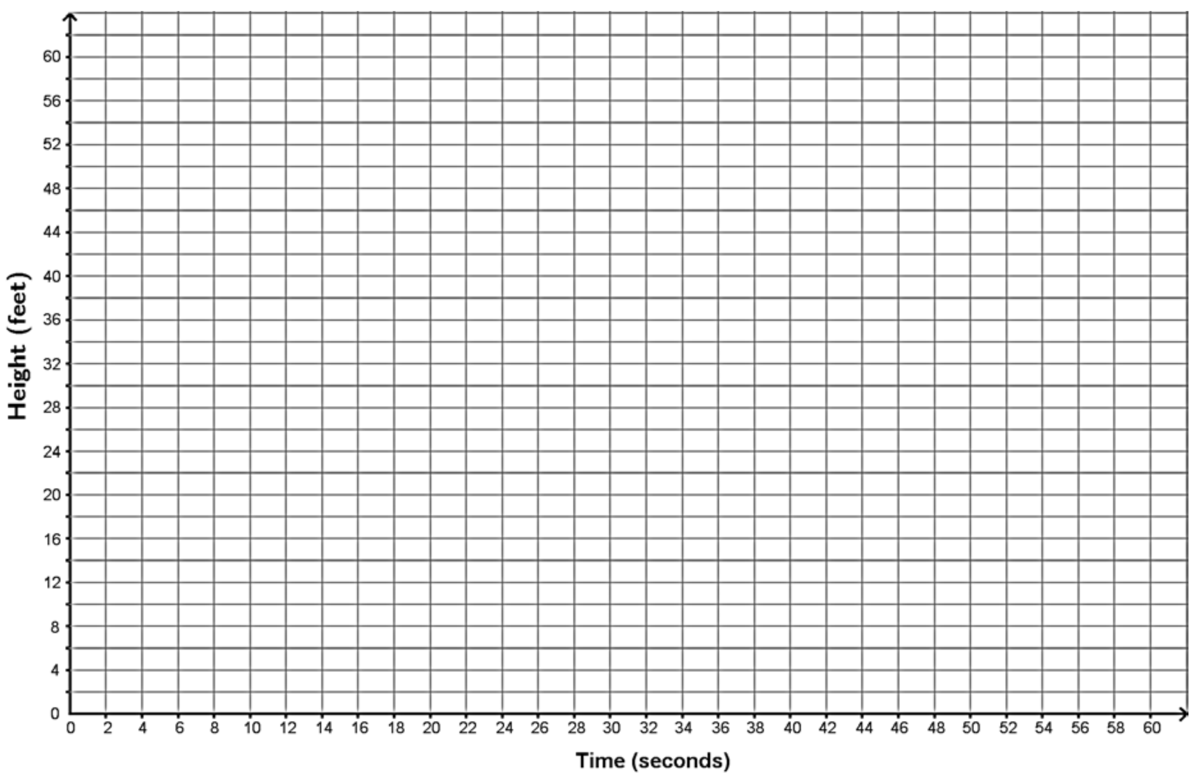
   b. the height of the center of the wheel was greater? or smaller?

   c. the wheel rotates faster? or slower?

4. Of course, Ferris wheels do not all have this same radius, center height, or time of rotation. Describe a different Ferris wheel by changing at least one of the facts listed above.

   Description of my Ferris wheel:

5. Sketch a graph of the height of a rider on your Ferris wheel from question 4 as a function of the elapsed time since the rider passed the position farthest to the right on the Ferris wheel.




6. Write the equation of the graph you sketched in question 5.

7. How does the **equation** of the rider's height change if:

   a. the radius of the wheel is larger? or smaller?

   b. the height of the center of the wheel is greater? or smaller?

   c. the wheel rotates faster? or slower?

8. Write the equation of the height of a rider on each of the following Ferris wheels $t$ seconds after the rider passes the farthest right position.

   a. The radius of the wheel is 30 feet, the center of the wheel is 45 feet above the ground, and the angular speed of the wheel is 15 degrees per second counterclockwise.

   b. The radius of the wheel is 50 feet, the center of the wheel is at ground level (you spend half of your time below ground), and the wheel makes one revolution ***clockwise*** every 15 seconds.

9. Explain how each part of the functions below changes the original $y = sinx$ graph.

   a. $y = 10\sin(2x) + 6$

   b. $y = -5\sin\left(\frac{2}{3}x\right) - 3$

# 4.5H Moving Shadows
## *A Practice Understanding Task*



In spite of his nervousness, Carlos enjoys his first ride on the Ferris wheel. He does, however, spend much of his time with his eyes fixed on the ground below him. After a while, he becomes fascinated with the fact that, since the sun is directly overhead, his shadow moves back and forth across the ground beneath him as he rides around on the Ferris wheel.

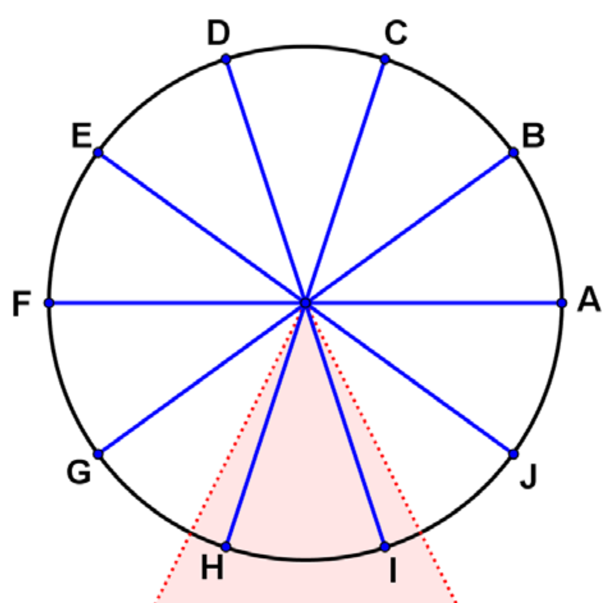
Recall the following facts for the model replica Ferris wheel Carlos is building:

- The Ferris wheel has a radius of 1 meter.
- The center of the Ferris wheel is 1 meter above the ground.
- The Ferris wheel makes one complete rotation counterclockwise every 360 seconds

To describe the location of the dolls' shadow as it moves back and forth on the ground beneath, we could measure the shadow's **horizontal distance** (in feet) to the right or left of the point directly beneath the center of the model Ferris wheel, with locations to the right of the center having positive value and locations to the left of the center having negative values. For instance, in this system, the doll's shadow's location will have a value of 1 meter when he is at the position farthest to the right of the center on the Ferris wheel, and a value of $-1$ when it is at a position farthest to the left of the center.

1. In this new measurement system, if the doll's shadow is at 0 feet, where is the doll sitting on the Ferris wheel?

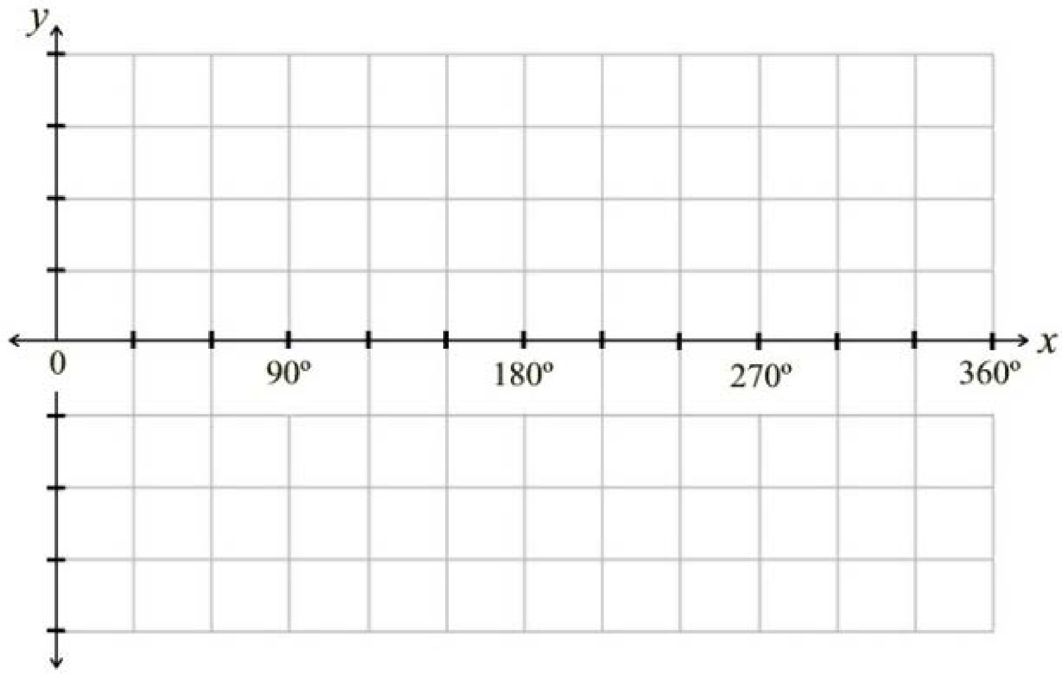
2. In our previous work with the Ferris wheel, $t$ represents the number of seconds since the doll passed the position farthest to the right on the Ferris wheel. How long will it take the doll to be at the position described in question 1?



3. Calculate the location of the doll's shadow from center at the times *t* given in the following table, where *t* represents the number of seconds since Carlos passed the position farthest to the right on the Ferris wheel. **Keep track of any patterns you notice in the ways you calculate the location of the shadow**. As you calculate each location, plot the location of the shadow from center on the graph on the following page.

| **Elapsed time since passing position A** | **Location of the shadow from center** |
|---|---|
| 30 sec | |
| 60 sec | |
| 90 sec | |
| 135 sec | |
| 150 sec | |
| 180 sec | |
| 210 sec | |
| 225 sec | |
| 240 sec | |
| 300 sec | |
| 315 sec | |
| 330 sec | |
| 360 sec | |
| 390 sec | |
| 410 sec | |
| 450 sec | |
| 480 sec | |
| 495 sec | |
| 510 sec | |

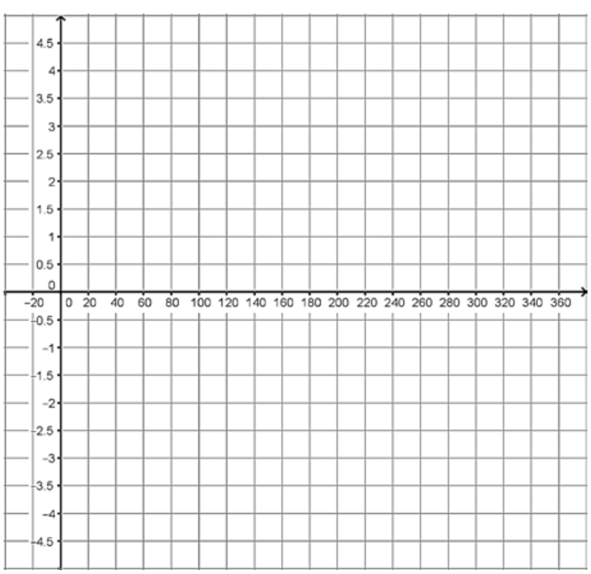
4. Sketch a graph of the horizontal location of the doll's shadow from center as a function of time *t*, where *t* represents the elapsed time after the doll passes position A, the farthest right position on the Ferris wheel.



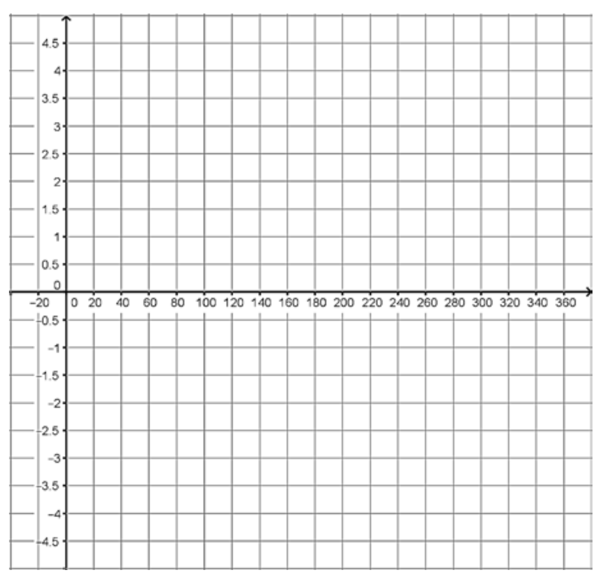

5. Write a general equation for finding the location of the shadow at any instant in time.

**Graph the following functions.**

6. $y = 2\cos x$



7. $y = -3\cos x + 2$



# 4.6H Warm-up

*Degrees to Radians*

**During the spring runoff of melting snow, the stream of water powering this waterwheel causes it to make one complete revolution counterclockwise every 3 seconds.**

Write an equation to represent the height of a particular paddle of the waterwheel above or below the water level at any time, *t,* after the paddle emerges from the water.

18. Write your equation so the height of the paddle will be graphed correctly on a calculator set in *degree* mode.

19. Revise your equation so the height of the paddle will be graphed correctly on a calculator set in *radian* mode.

**During the summer months, the stream of water powering this waterwheel becomes a "lazy river" causing the wheel to make one complete revolution counterclockwise every 12 seconds.**

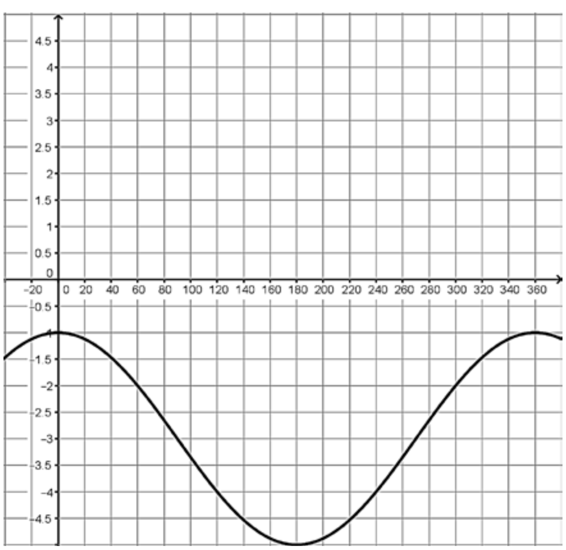
Write an equation to represent the height of a particular paddle of the waterwheel above or below the water level at any time, *t,* after the paddle emerges from the water.

20. Write your equation so the height of the paddle will be graphed correctly on a calculator set in *degree* mode.

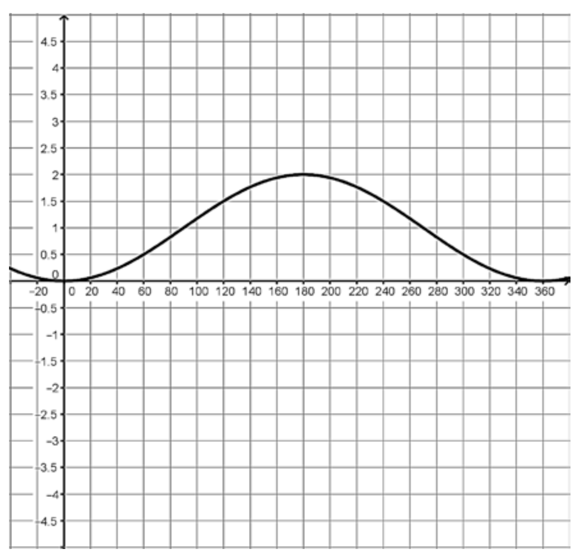
21. Revise your equation so the height of the paddle will be graphed correctly on a calculator set in *radian* mode.

**Given the graphs below, write at least one function that can be used to model the graph.**

8.



9.



# 4.6H High Noon and Sunset Shadows
## *A Develop Understanding Task*

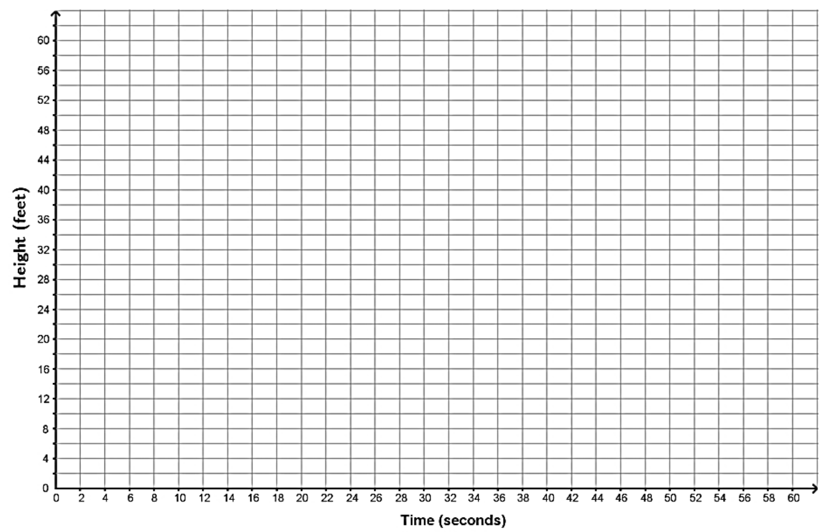


In this task, we revisit the Ferris wheel that caused Carlos so much anxiety. Recall the following facts from previous tasks:

- The Ferris wheel has a radius of 25 feet.
- The center of the Ferris wheel is 30 feet above the ground.
- The Ferris wheel makes one complete rotation counterclockwise every 20 seconds.

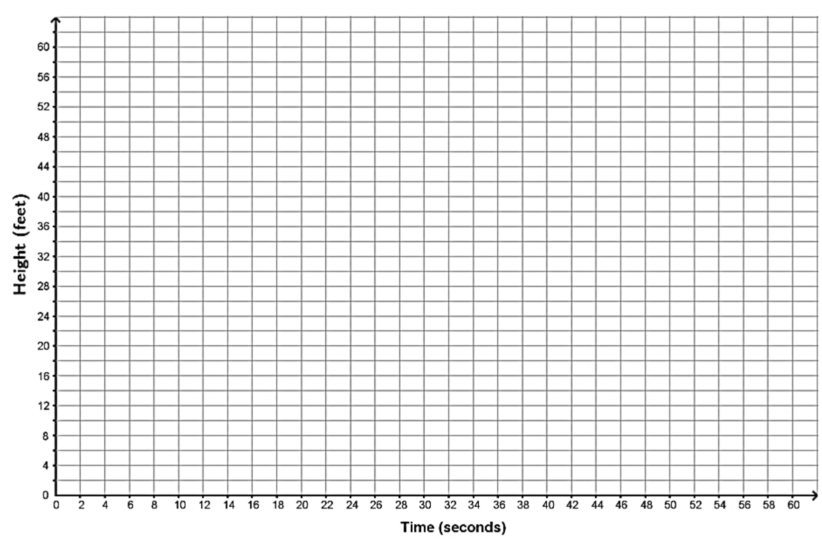
The Ferris wheel is located next to a high-rise office complex. At sunset, a rider casts a shadow on the exterior wall of the high-rise building. As the Ferris wheel turns, you can watch the shadow of the rider rise and fall along the surface of the building. In fact, you know an equation that would describe the height of this "sunset shadow."

1. Write the equation of this "sunset shadow."




At noon, when the sun is directly overhead, a rider casts a shadow that moves left and right along the ground as the Ferris wheel turns. In fact, you know an equation that would describe the motion of this "high noon shadow."

2. Write the equation of this "high noon shadow."



3. Based on your previous work, you probably wrote these equations in terms of the angle of rotation being measured in degrees. Revise your equations so the angle of rotation is measured in **radians**.

   a. The “sunset shadow” equation in terms of radians:

   b. The “high noon shadow” equation in terms of radians:

4. In the equations you wrote in question 3, where on the Ferris wheel was the rider located at time $t = 0$? We will refer to the position as the rider’s ***initial position*** on the wheel.

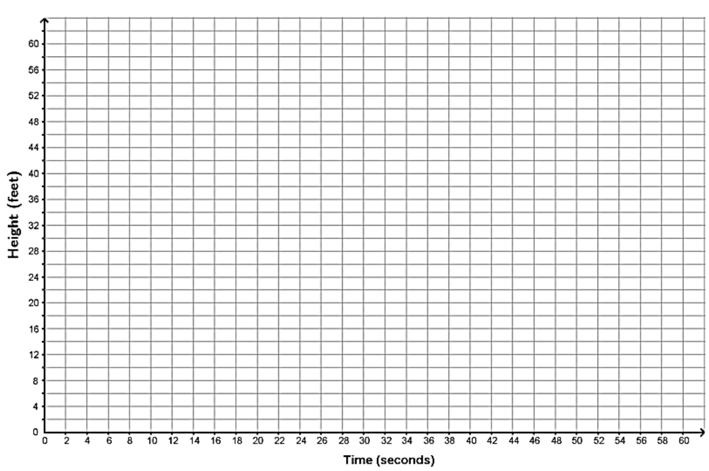
   a. Initial position for the “sunset” shadow equation:

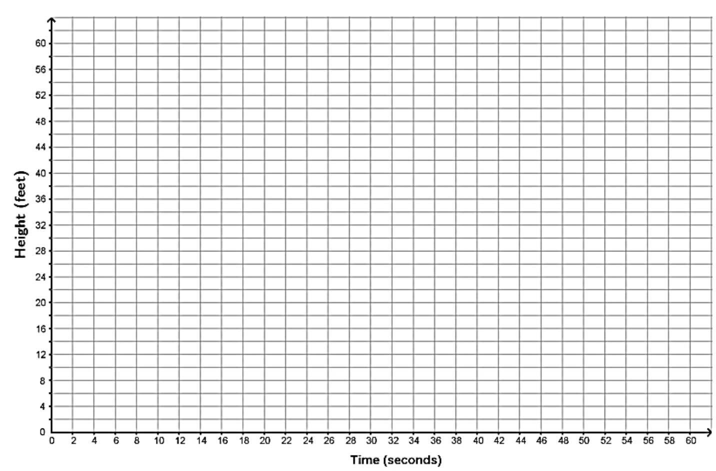
   b. Initial position for the “high noon” shadow equation:

5. Revise your equations from question 3 so that the rider’s initial position at $t = 0$ is **at the top of the wheel**. Sketch a graph of the new functions.

   a. The “sunset shadow” equation, initial position at the top of the wheel:

   b. The “high noon shadow” equation, initial position at the top of the wheel:

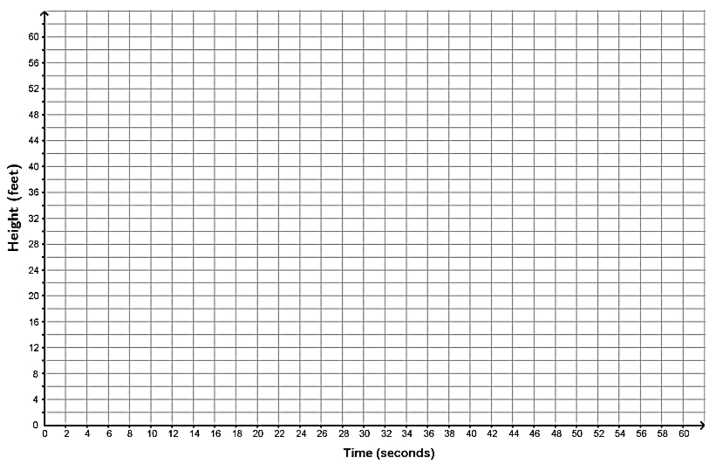


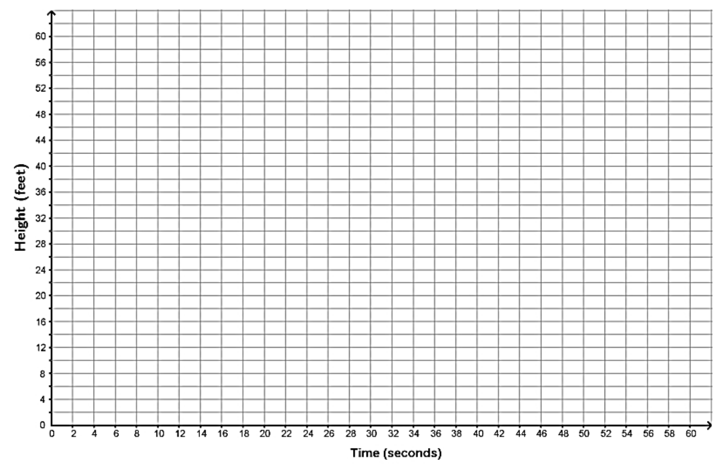



6. Revise your equations from question 3 so that the rider's initial position at $t = 0$ is **at the bottom of the wheel**.

   a. The "sunset shadow" equation, initial position at the bottom of the wheel:

   b. The "high noon shadow" equation, initial position at the bottom of the wheel:

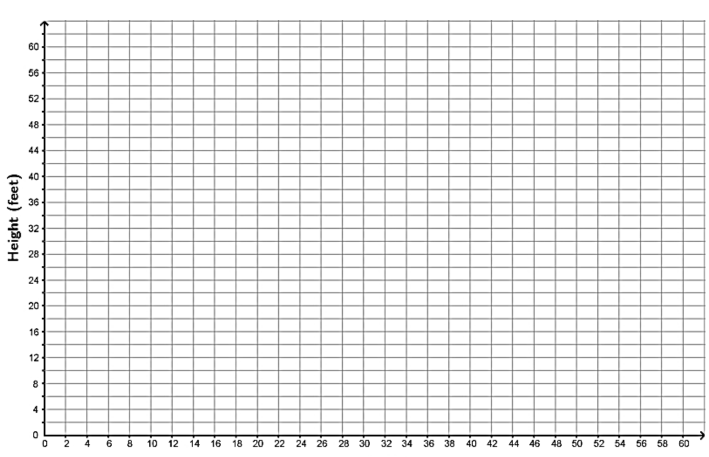


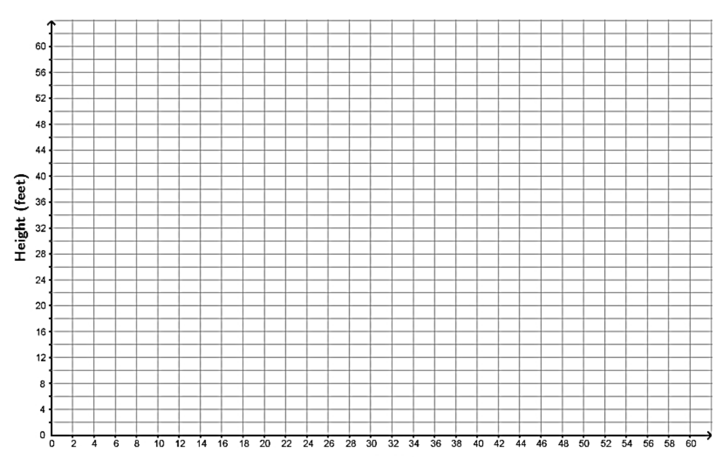




7. Revise your equations from question 3 so that the rider's initial position at $t = 0$ is **at the point farthest to the left of the wheel.**

   a. The "sunset shadow" equation, initial position at the point farthest to the left of the wheel:

   b. The "high noon shadow" equation, initial position at the point farthest to the left of the wheel:

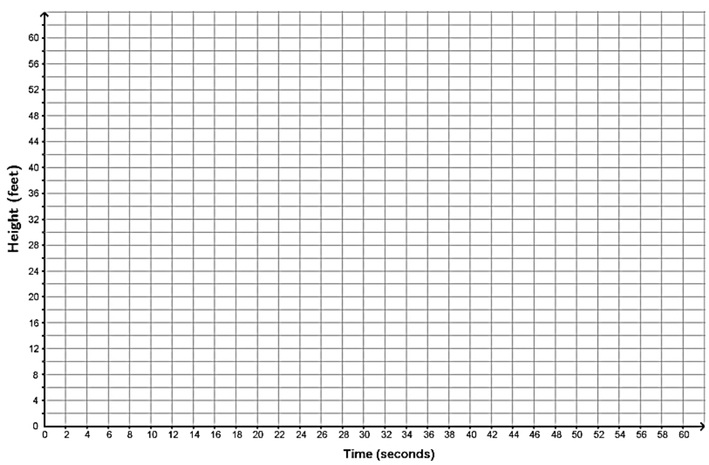


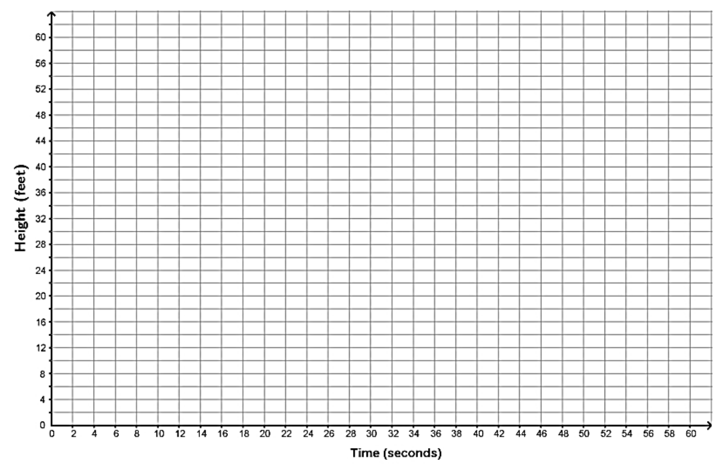



8. Revise your equations from question 3 so that the rider's initial position at $t = 0$ is **halfway between the farthest point to the right on the wheel and the top of the wheel.**

   a. The "sunset shadow" equation, initial position halfway between the farthest point to the right on the wheel and the top of the wheel:

   b. The "high noon shadow" equation, initial position halfway between the farthest point to the right on the wheel and the top of the wheel:

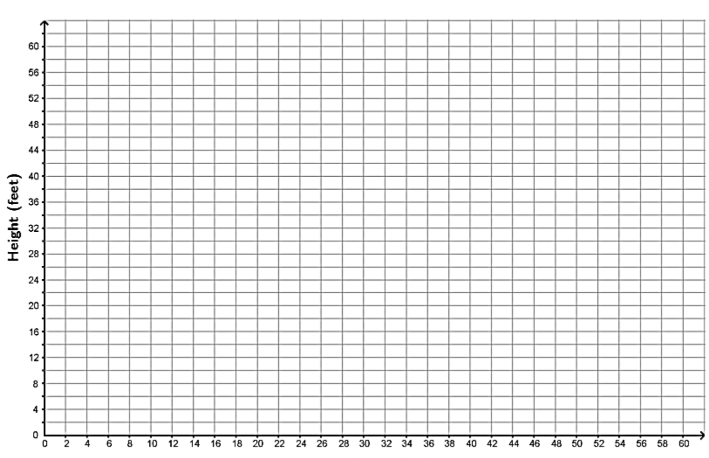


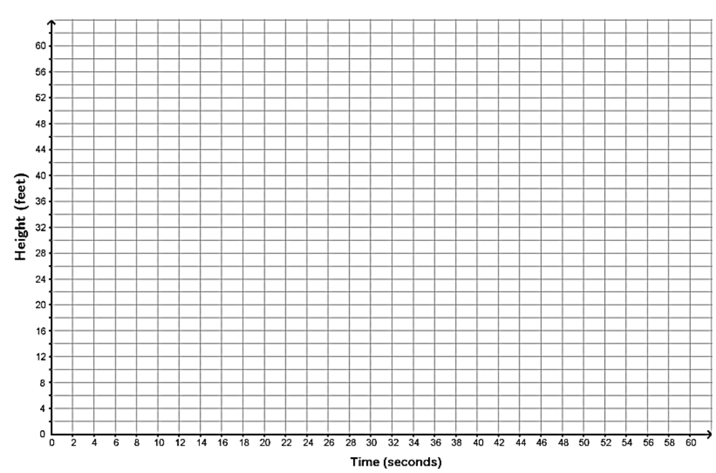




9. Revise your equations from question 3 so that the wheel **rotates twice as fast**.

   a. The "sunset shadow" equation for the wheel rotating twice as fast:

   b. The "high noon shadow" equation for the wheel rotating twice as fast:

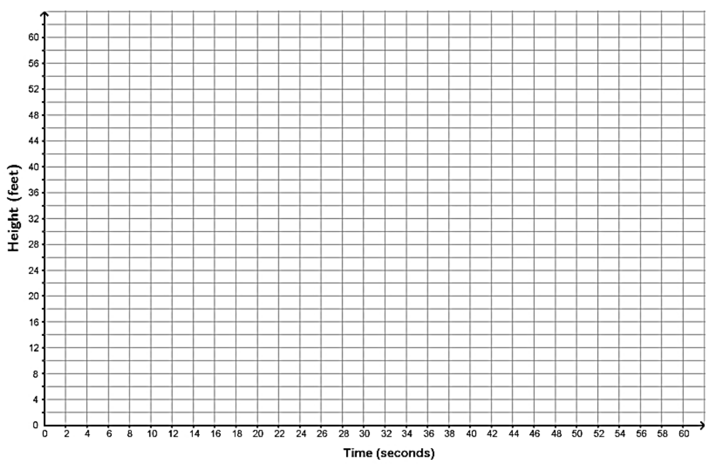


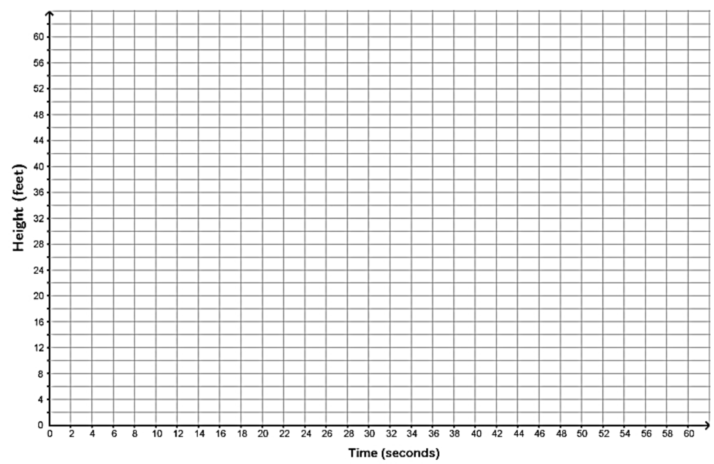



10. Revise your equations from question 3 so that the **radius of the wheel is twice as large and the center of the wheel is twice as high.**

    a. The “sunset shadow” equation for a radius twice as large and the center twice as high:

    b. The “high noon shadow” equation for a radius twice as large and the center twice as high:




11. Below is the standard form for a sine and cosine trigonometric function. Explain how to find the following features on a graph using standard form.

$$\boldsymbol{y = A\sin(Bx - C) + D} \qquad \boldsymbol{y = A\cos(Bx - C) + D}$$

    a. y-intercept:

    b. Midline(Vertical Shift):

    c. Amplitude:

    d. Period:

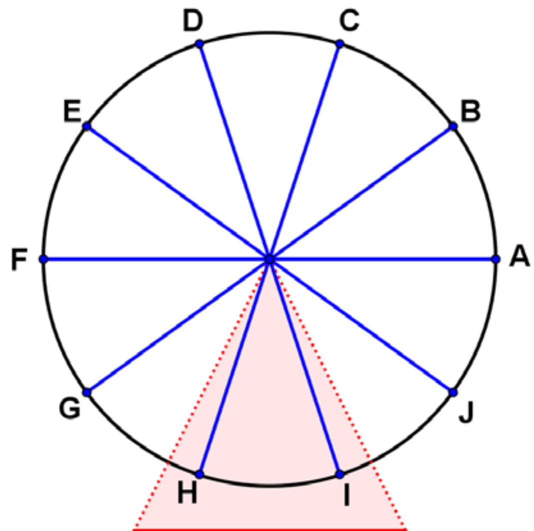
    e. Phase (Horizontal) Shift:



## 4.7H Warm Up

***Getting on the Right Wavelength - A Practice Understanding Task***

Below is a new Ferris wheel that has a radius of 40 feet, whose center is 50 feet from the ground, and makes one revolution counterclockwise every 18 seconds.




1. Write the equation of the height from the ground of the rider at any time $t$, if at $t = 0$ the rider is at **position A**. Use radians to measure the angles of rotation.

2. At what time(s) is the rider 70 feet above the ground? Show the details of how you answered this question.

3. Use your answer from question 2 to write an equation to show when the rider is 70 feet above the ground if the Ferris wheel goes around forever.

4. If you used a sine function in question 1, revise your equation to model the same motion with a cosine function. If you used a cosine function in question 1, revise your equation to model the motion with a sine function.

5. Write the equation of the height of the rider at any time $t$, if at $t = 0$, the rider is at **position D**. Use radians to measure the angles of rotation.

6. For the equation you wrote in question 4, at what time(s) is the rider 80 feet above the ground? Show or explain the details of how you answered this question.

7. Use your answer from question 6 to write an equation to show when the rider is 80 feet above the ground if the Ferris wheel goes around forever.

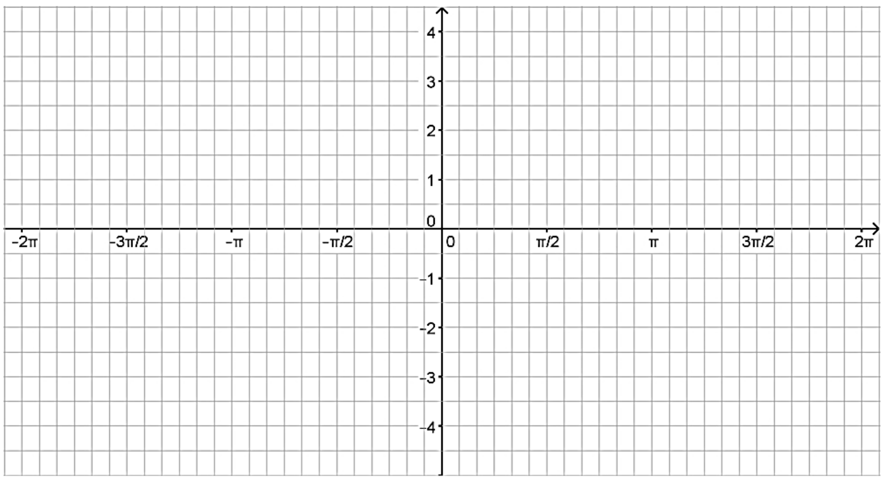
# 4.7H *Graphing Sine and Cosine*

*A Practice Understanding Task*



Graph the following functions using radians and identify the listed features.

1. $y = -\sin 2x + 3$




Domain:

Range:

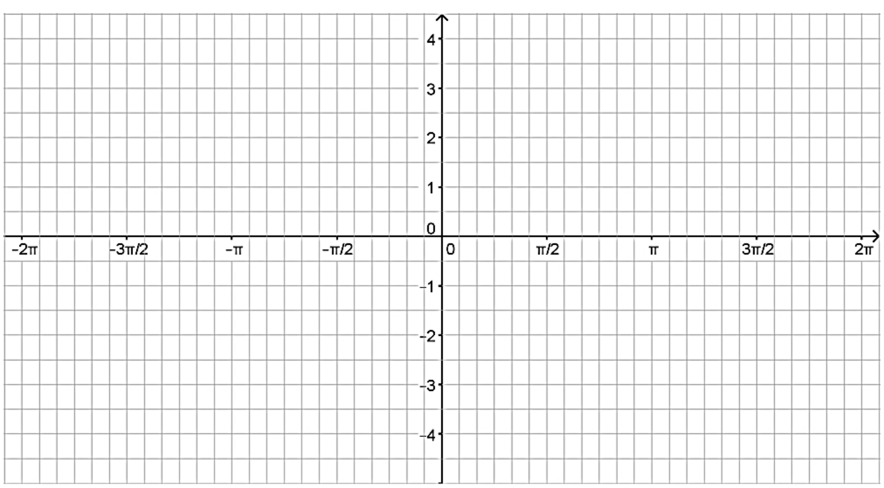
Amplitude:

Period:

Phase Shift:

Midline:

2. $y = -3\sin 2x + 1$




Domain:

Range:

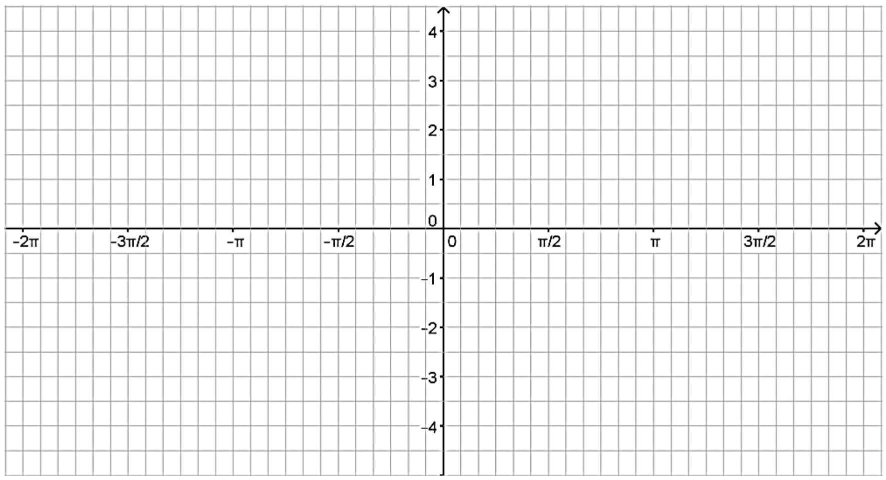
Amplitude:

Period:

Phase Shift:

Midline:

3. $y = 3\cos 3x - 2$




Domain:

Range:

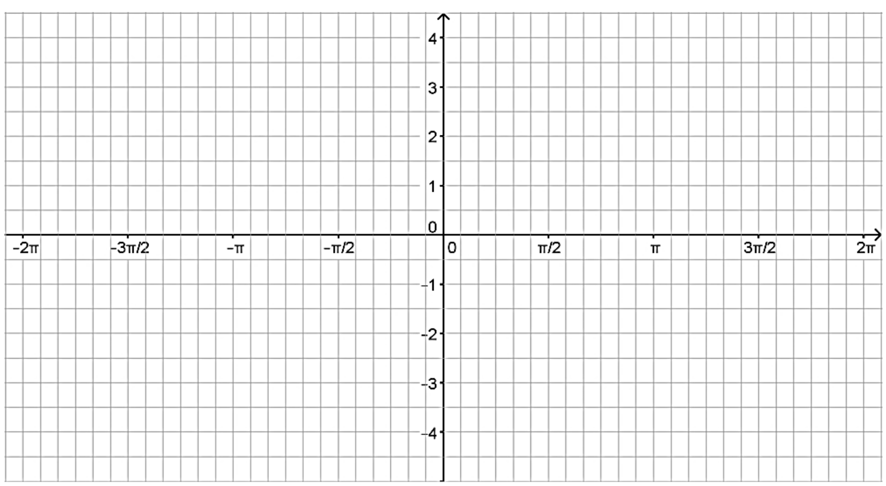
Amplitude:

Period:

Phase Shift:

Midline:

4. $y = \frac{3}{2}\cos\left(\frac{x}{2} - \pi\right) + 1$




Domain:

Range:

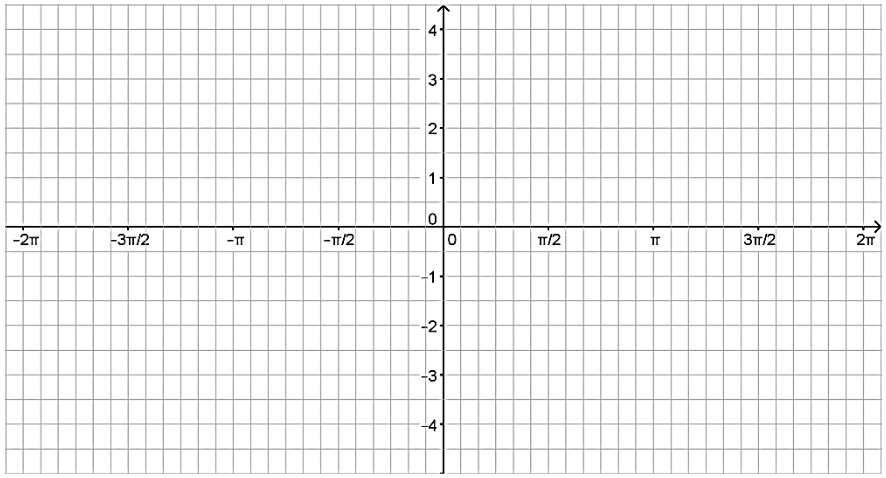
Amplitude:

Period:

Phase Shift:

Midline:

5. $y = \frac{1}{2}\cos\left(\theta + \frac{\pi}{6}\right)$




Domain:

Range:

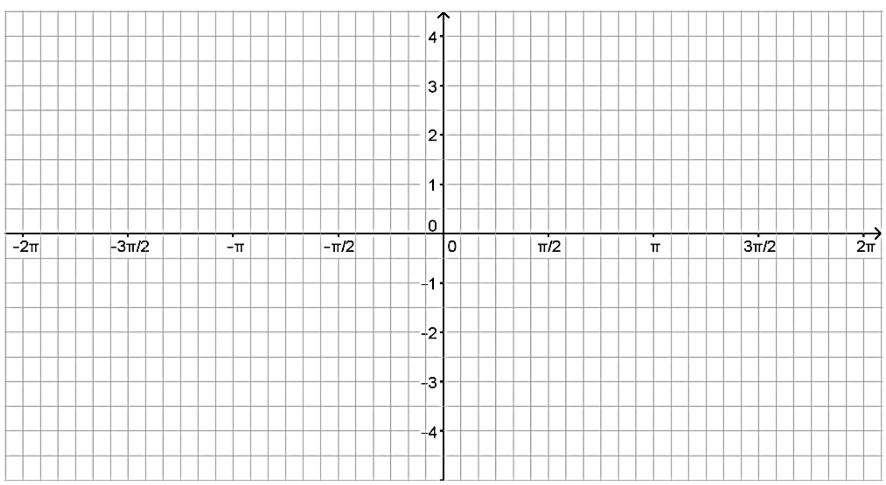
Amplitude:

Period:

Phase Shift:

Midline:

6. $y = \cos\left(2\left(x - \frac{\pi}{3}\right)\right) - 1$




Domain:

Range:

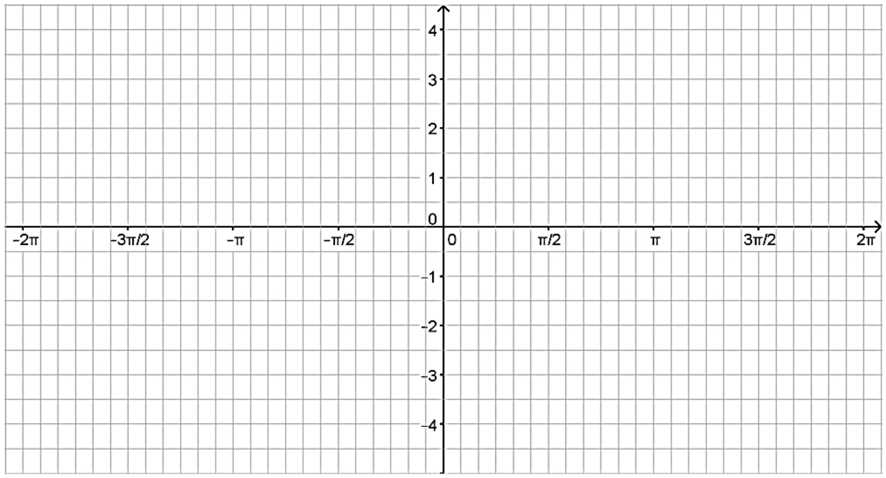
Amplitude:

Period:

Phase Shift:

Midline:

7. $y = \sin\left(\frac{2}{3}\left(\theta - \frac{\pi}{2}\right)\right)$




Domain:

Range:

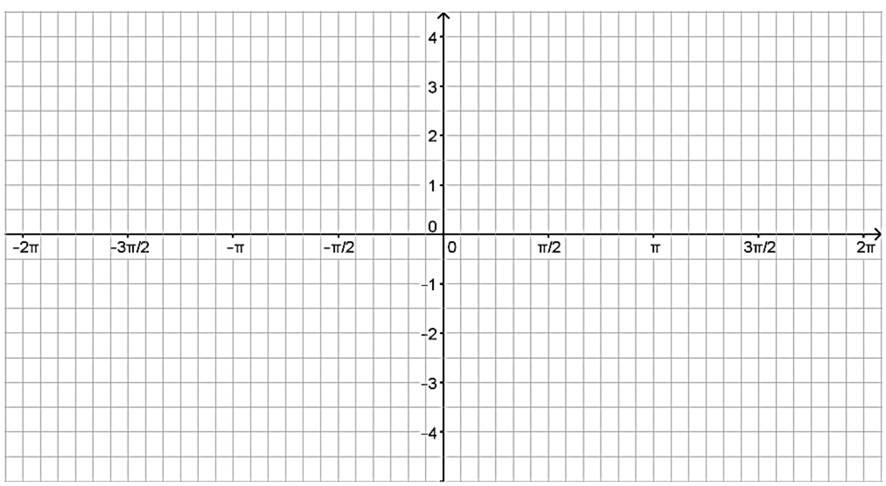
Amplitude:

Period:

Phase Shift:

Midline:

8. $y = \frac{1}{2}\sin\left(2x + \frac{\pi}{2}\right) - 1$




Domain:

Range:

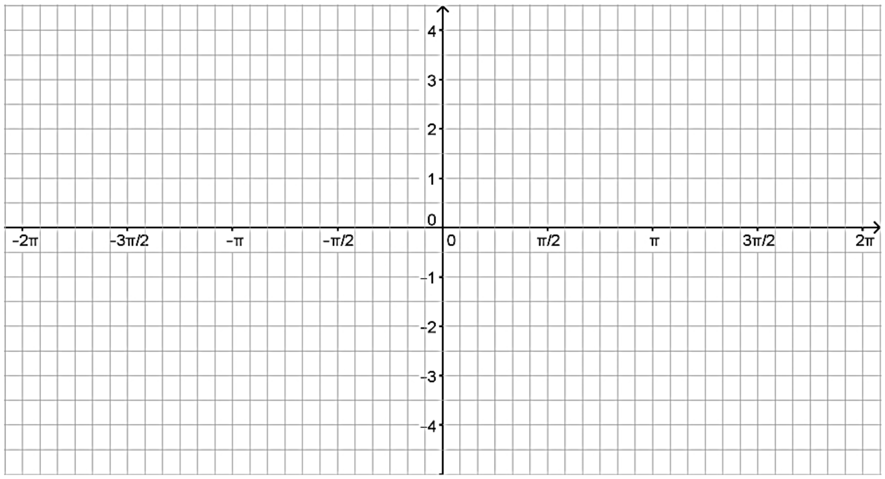
Amplitude:

Period:

Phase Shift:

Midline:

9. $y = -\cos\left(\frac{1}{2}\left(x - \frac{\pi}{2}\right)\right) - 2$




Domain:

Range:

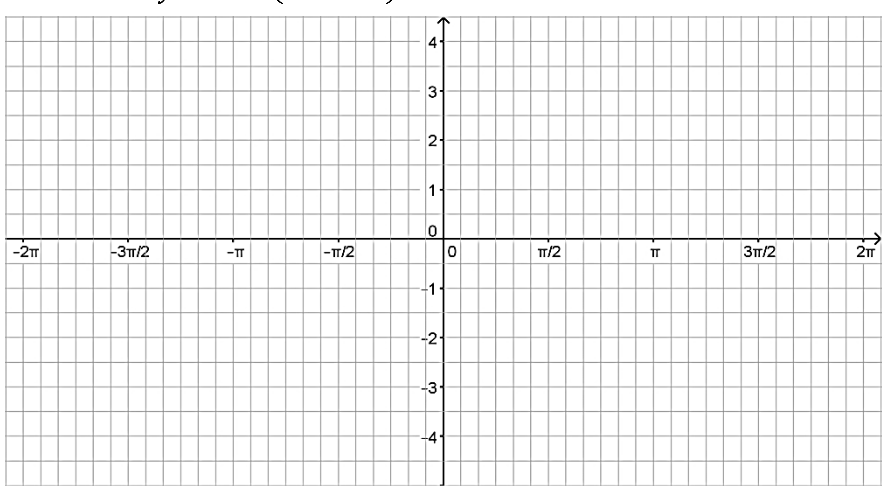
Amplitude:

Period:

Phase Shift:

Midline:

10. $y = 3\sin(2x - 2\pi) - 2$




Domain:

Range:

Amplitude:

Period:

Phase Shift:

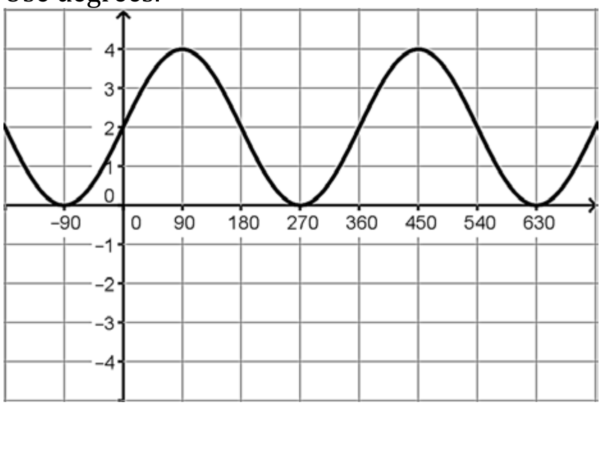
Midline:

## 4.8H Warm Up

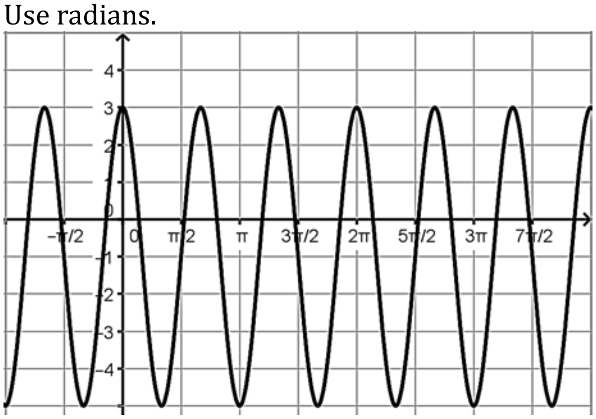
*Transformations of Sine and Cosine & Solving trigonometric equations*

Write a sine and a cosine equation for each of the graphs below.

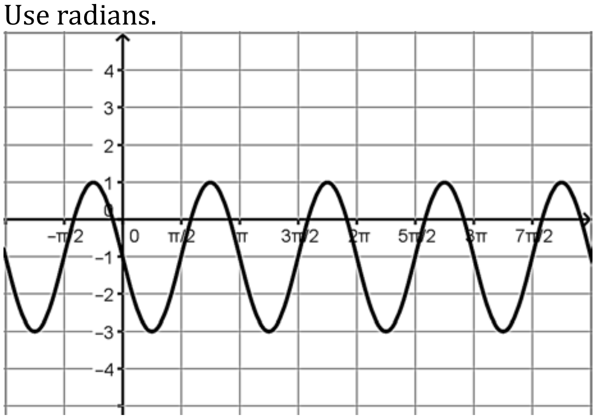
1. Use degrees.




2. Use radians.




3. Use radians.



Solve each of the equations below and write the general solution.

4. $2\sin 3x \sec 3x = \sec 3x$

5. $\sin 2x + 2\cos x = 0$

6. $tan^2 x + 3\sec x + 3 = 0$

7. $4sin^4 x + 3sin^2 x - 1 = 0$



# 4.8H High Tide

*A Solidify Understanding Task*

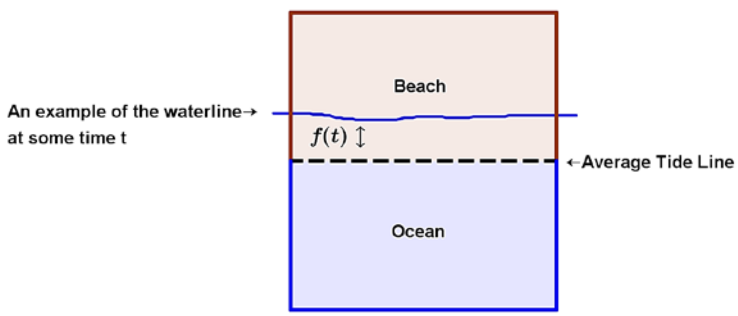
Perhaps you have built an elaborate sand castle at the beach only to have it get swept away by the in-coming tide.

Frustrated by having your sandcastle knocked down by the tide, you decide to pay attention to the tides so that you can keep track of how much time you have to build and admire your sand castle.

You have a friend who is in calculus who will be going to the beach with you. You give your friend some data from the almanac about high tides along the ocean, as well as a contour map of the beach you intend to visit, and ask her to come up with an equation for the water level on the beach on the day of your trip. According to your friend's analysis, the water level on the beach will fit this equation:

$$f(t) = 20\sin\left(\frac{\pi}{6}t\right)$$

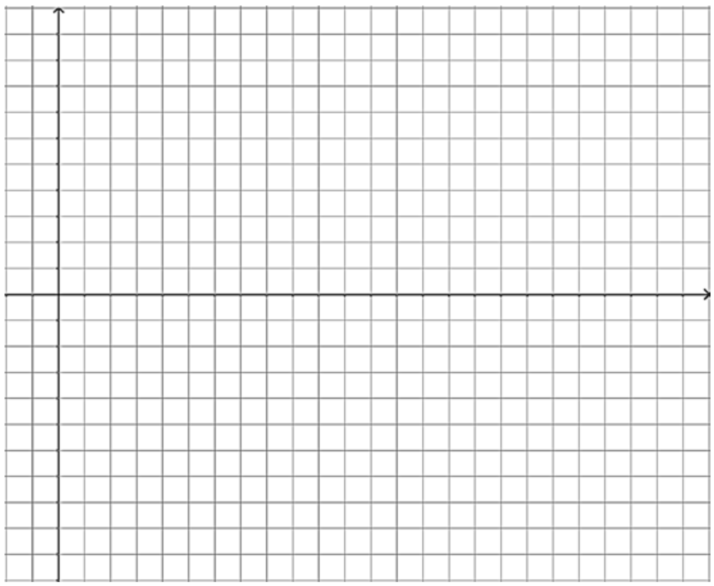
In this equation, $f(t)$ represents how far the tide is "in" or "out" from the average tide line. This distance is measured in feet and $t$ represents the elapsed time, in hours, since midnight.




1. What is the highest up the beach (compared to its average position) that the tide will be during the day? This is called ***high tide***. What is the lowest that the tide will be during the day? This is called ***low tide***.

2. Suppose you plan to build your castle right on the average tide line just as the water has moved below that line. How much time will you have to build your castle before the incoming tide destroys your work?

3. Use the given function and your answers to questions 1 and 2 to sketch the graph of two complete tide cycles. Label and scale the axes.



4. Suppose you want to build your castle 10 feet below the average tide line to take advantage of the damp sand. What is the maximum amount of time you will have to make your castle? How can you convince your friend that your answer is correct? Use your graph to estimate this amount of time and then use algebra & the inverse sine function to find the exact answer.

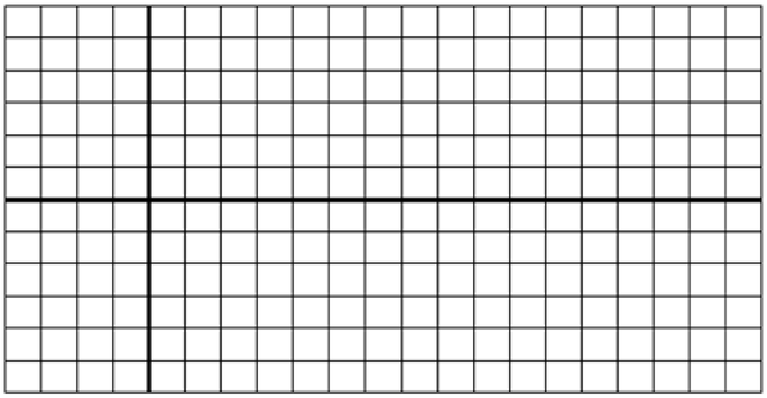
5. Suppose you want to build your castle 15 feet above the average tide line to give you more time to admire your work. What is the maximum amount of time you will have to make your castle? How can you convince your friend that your answer is correct? Use your graph to estimate this amount of time and then use algebra & the inverse sine function to find the exact answer.

6. Suppose you decide you only need two hours to build and admire your castle. What is the lowest point on the beach where you can build the castle?

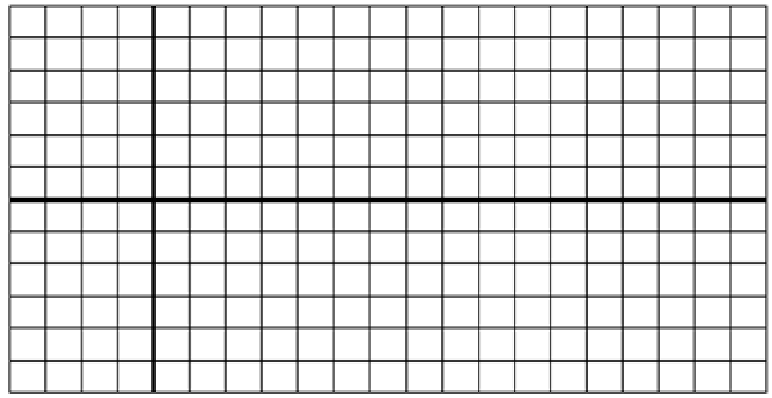
**4.9H Warm Up**

Graphing Trig Functions where the Period is rational

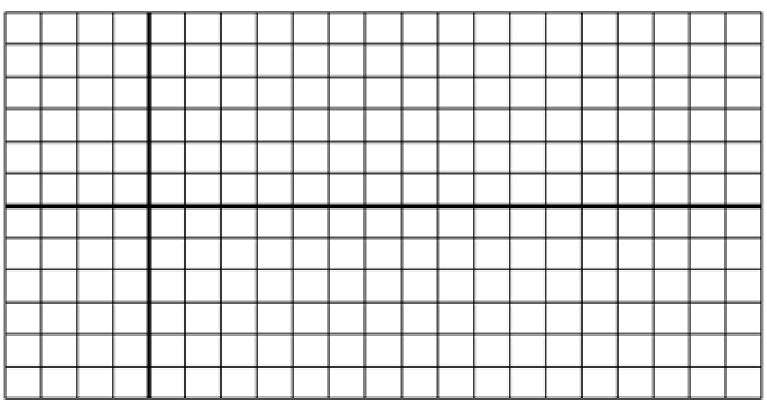
1. $y = 2\sin\left(\frac{1}{3}x + \frac{2\pi}{3}\right)$



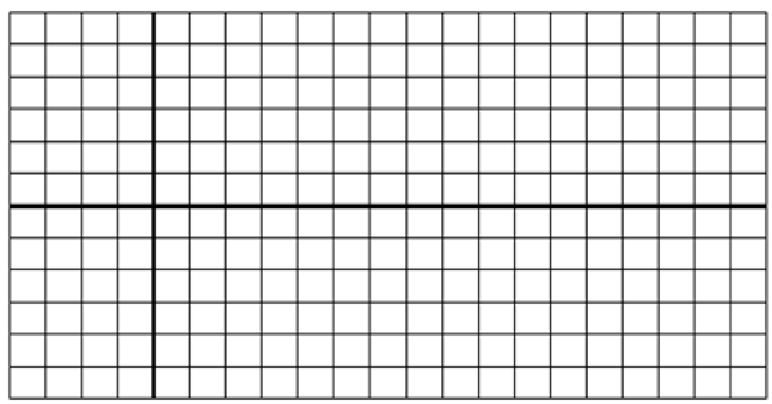
2. $y = 2\sin\left(\frac{\pi}{3}x + \frac{2\pi}{3}\right)$



3. $y = 2\cos(2x + \pi)$



4. $y = 2\cos(2\pi x + \pi)$



5. Why does the period become rational instead of irrational*(in terms of pi)* on problems 2 and 4?

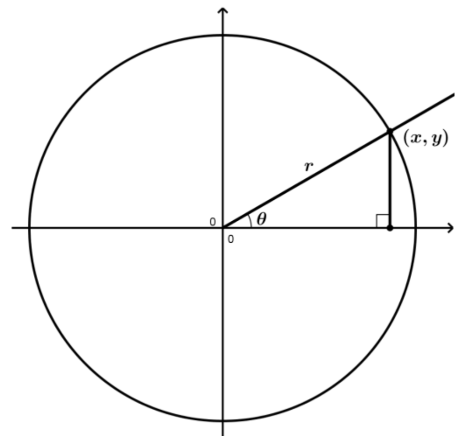
## 4.9H Off on a Tangent
### *A Develop and Solidify Understanding Task*



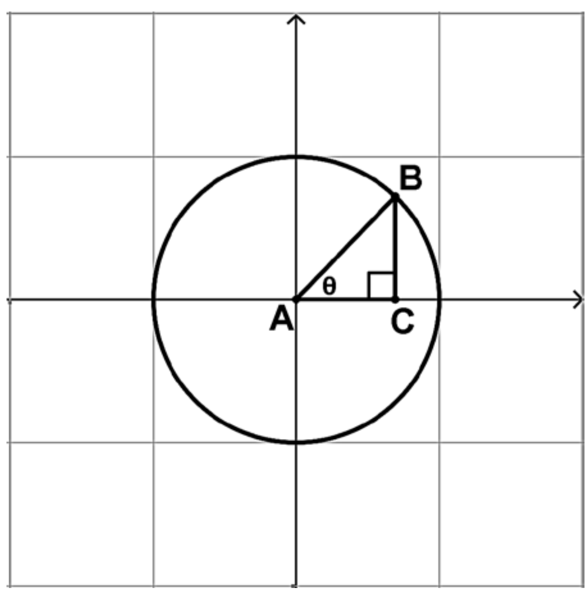
Recall that the right triangle definition of the tangent ratio is:

$$\tan\theta = \frac{\text{length of side opposite angle } \theta}{\text{length of side adjacent to angle } \theta}$$

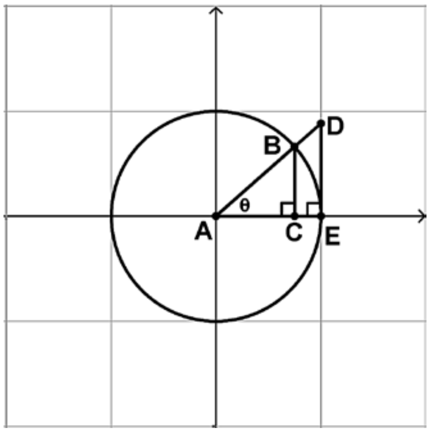
1. Revise the right triangle definition of tangent to find the tangent of **any angle of rotation** drawn in standard position on the unit circle. Explain why your definition is reasonable.




2. We have observed that on the unit circle the value of sine and cosine can be represented with the length of a line segment. Indicate on the following diagram which segment's length represents the value of $\sin\theta$ and which represents the value of $\cos\theta$ for the given angle $\theta$.



3. There is also a line segment that can be defined on the unit circle so that its length represents the value of $\tan\theta$. Consider the length of $\overline{DE}$ in the unit circle diagram below. Note that $\Delta ADE$ and $\Delta ABC$ are similar right triangles. Write a convincing argument explaining why the length of $\overline{DE}$ is equivalent to the value of $\tan\theta$ for the given angle, $\theta$.




4. Extend your thinking about $y = \tan\theta$ by considering the length of $\overline{DE}$ as $\theta$ rotates through positive angles from 0 radians to $2\pi$ radians. Using your unit circle diagrams from the task, *Water Wheels and the Unit Circle*, give exact values for the following trigonometric expressions:

| | | |
|---|---|---|
| a. $\tan\frac{\pi}{6} =$ | b. $\tan\frac{5\pi}{6} =$ | c. $\tan\frac{7\pi}{6} =$ |
| d. $\tan\frac{\pi}{4} =$ | e. $\tan\frac{3\pi}{4} =$ | f. $\tan\frac{11\pi}{6} =$ |
| g. $\tan\frac{\pi}{2} =$ | h. $\tan\pi =$ | i. $\tan\frac{7\pi}{3} =$ |
| j. $\tan -\frac{\pi}{3} =$ | k. $\tan\frac{3\pi}{2} =$ | l. $\tan -\frac{\pi}{4} =$ |

5. On the coordinate axes below, sketch the graph of $y = \tan\theta$ by considering the length of $\overline{DE}$ as $\theta$ rotates through angles from 0 radians to $2\pi$ radians. Explain any interesting features you notice in your graph.

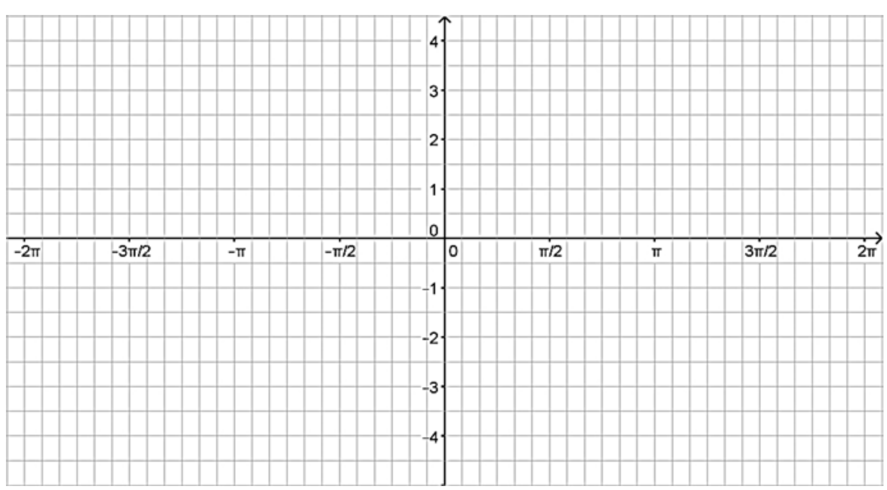
x-intercepts:

Period:

Asymptotes:

Domain:

Range:



Based on these definitions and your work in this module, determine how to classify each of the following trigonometric functions.

- A function $f(x)$ is classified as an **odd function** if $f(-\theta) = -f(\theta)$.
- A function $f(x)$ is classified as an **even function** if $f(-\theta) = f(\theta)$.

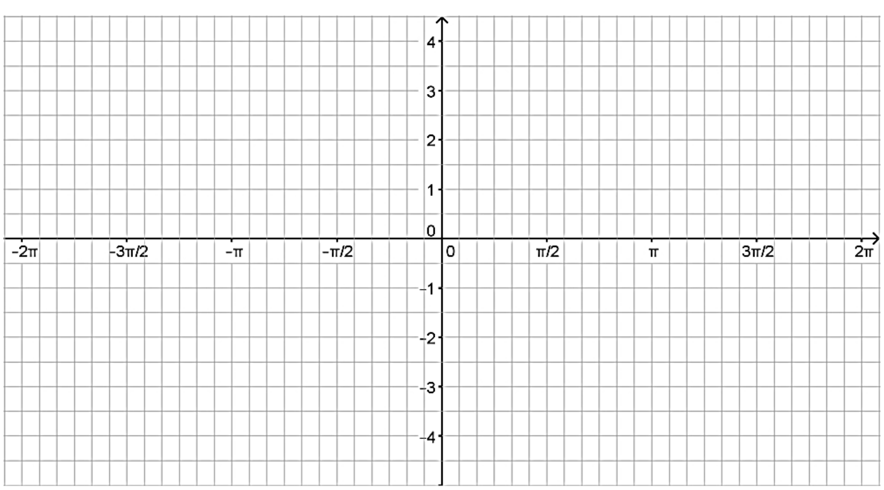
a. The function $y = \sin x$ would be classified as an [**odd function, even function, neither an even or odd function**]. Give evidence for your response.

b. The function $y = \cos x$ would be classified as an [**odd function, even function, neither an even or odd function**]. Give evidence for your response.

c. The function $y = \tan x$ would be classified as an [**odd function, even function, neither an even or odd function**]. Give evidence for your response.

7. Graph each tangent function. Be sure to identify the locations of the asymptotes.

a. $y = \tan 2\theta$




Period:

Phase Shift:

Vertical Shift:

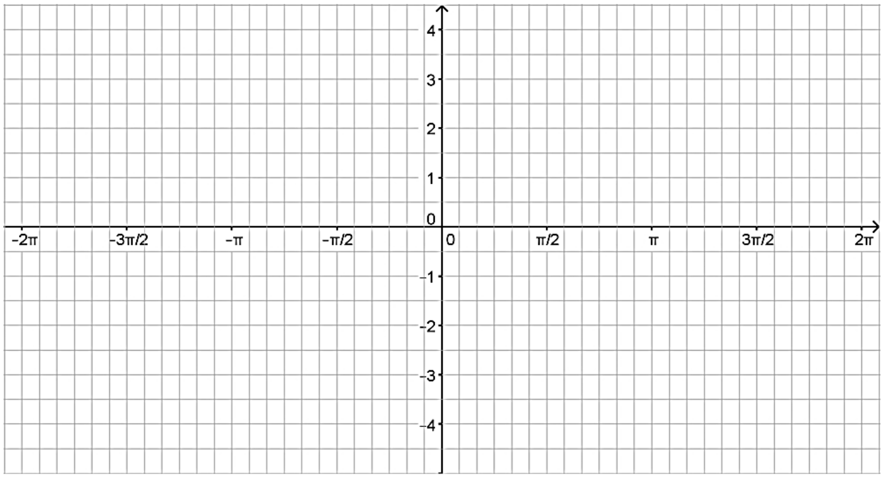
x-intercepts:

Asymptotes:

Domain:

Range:

b. $y = 2\tan\theta - 3$




Period:

Phase Shift:

Vertical Shift:

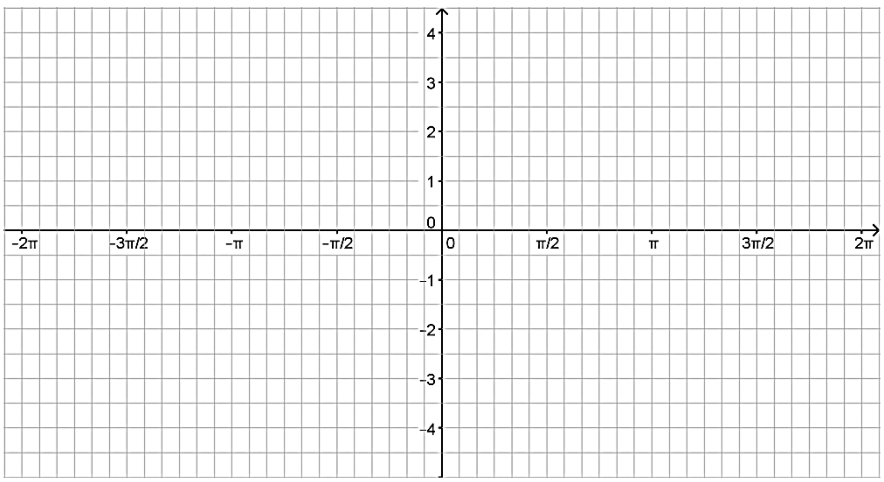
x-intercepts:

Asymptotes:

Domain:

Range:

c. $y = 1 + \tan\frac{1}{2}\theta$




Period:

Phase Shift:

Vertical Shift:

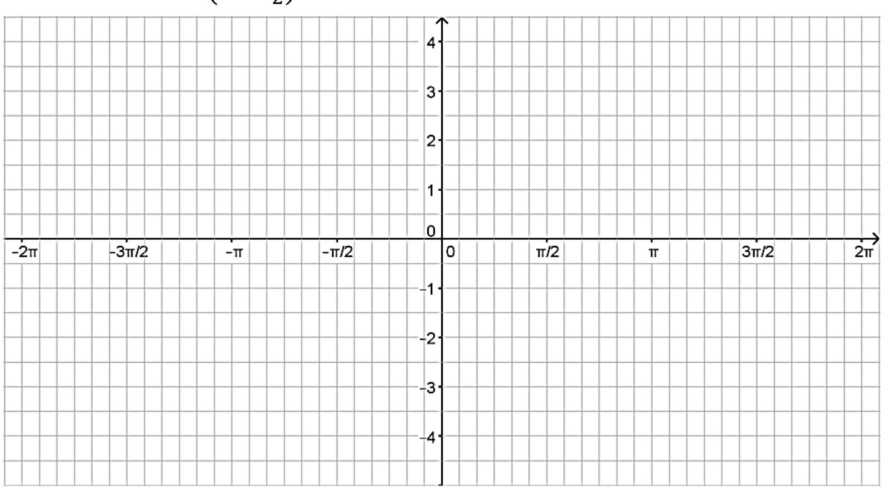
x-intercepts:

Asymptotes:

Domain:

Range:

d. $y = -\tan\left(\theta - \frac{\pi}{2}\right) + 2$




Period:

Phase Shift:

Vertical Shift:

x-intercepts:

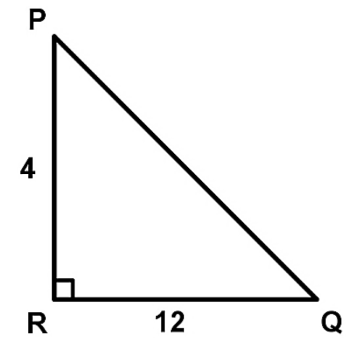
Asymptotes:

Domain:

Range:

# 4.10H Warm Up
## *Reciprocal Trigonometric Values*

Use the triangle below to identify the following trigonometric ratios. Be sure to give exact values.




1. $\sin Q =$

2. $\csc Q =$

3. $\cos Q =$

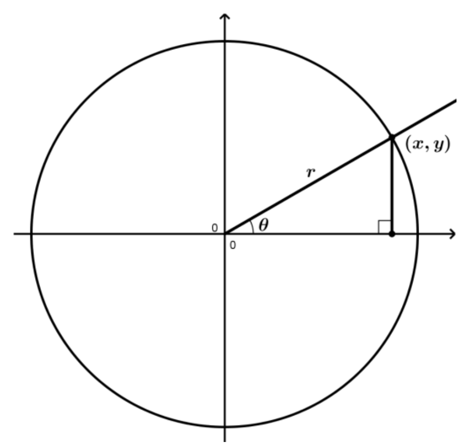
4. $\sec Q =$

5. $\tan Q =$

6. $\cot Q =$

Recall how we extended the definitions of sine, cosine, and tangent for all angles of rotation:

$\boldsymbol{\sin\theta = \frac{y}{r}}$ $\boldsymbol{\cos\theta = \frac{x}{r}}$ $\boldsymbol{\tan\theta = \frac{y}{x}}$




7. How might the definitions of cosecant, secant, and cotangent be extended to all angles of rotation?

   $\csc\theta =$

   $\sec\theta =$

   $\cot\theta =$

# 4.10H Reciprocating the Graphs
## *A Develop and Solidify Understanding Task*

Carlos and Clarita were graphing trigonometric functions on their math homework and were wondering what the graphs of the reciprocal functions would look like.

Carlos stated: "Since $\sin\theta$ and $\cos\theta$ both have maximum and minimum values of 1 and $-1$, I think these will be the maximum and minimum values for $\csc\theta$ and $\sec\theta$ as well."

Clarita disagreed: "No way! Think about the values between the zeros and the maximum or between the zeros and the minimum. What happens if you take their reciprocals? The numbers become really large."

1. Who do you agree with? Explain.

2. Complete the table of values for angles in quadrant I for $y = \sin\theta$ and $y = \csc\theta$. Graph these two functions in different colors on the coordinate plane. Use symmetry to complete the sketch of each function for angles on a domain of $[-2\pi, 2\pi]$. Reminder: $\csc\theta = \frac{1}{\sin\theta}$.

| $\theta$ | $\sin\theta$ | $\csc\theta$ |
|---|---|---|
| 0 | | |
| $\frac{\pi}{6}$ | | |
| $\frac{\pi}{4}$ | | |
| $\frac{\pi}{3}$ | | |
| $\frac{\pi}{2}$ | | |
| | | |
| | | |
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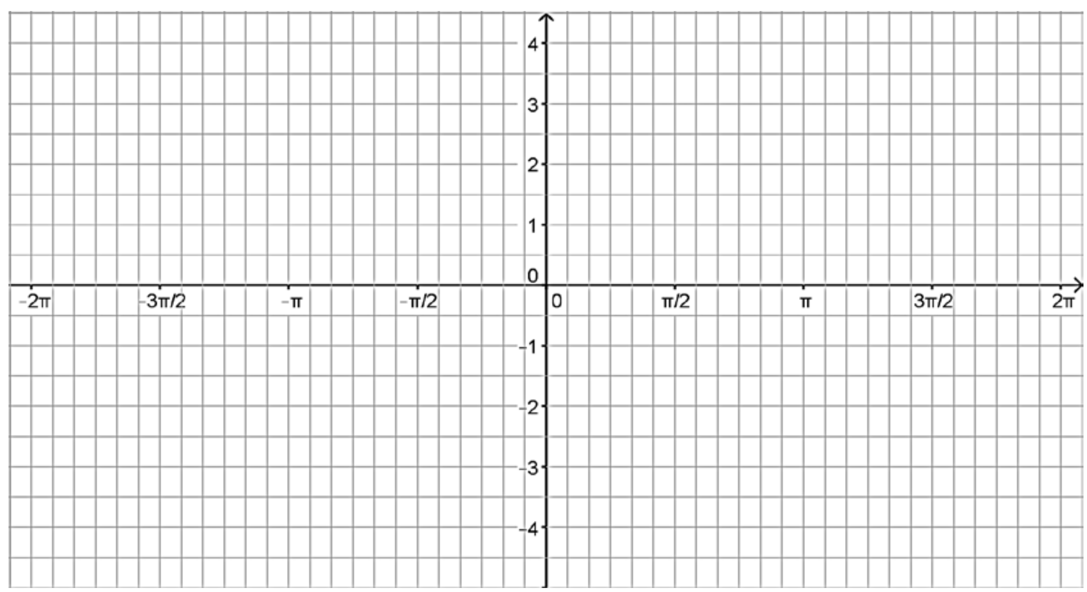
| | | |
| | | |




3. What happens to the graph of $y = \csc\theta$ when the graph of $y = \sin\theta$ crosses the $x$-axis?

4. Explain why $y = \csc\theta$ has the shape that appears in your graph in question 2.

5. Complete the table of values for angles in quadrant I for $y = \cos\theta$ and $y = \sec\theta$. Graph these two functions in different colors on the coordinate plane. Use symmetry to complete the sketch of each function for angles on a domain of $[-2\pi, 2\pi]$. Reminder: $\sec\theta = \frac{1}{\cos\theta}$.

| $\theta$ | $\cos\theta$ | $\sec\theta$ |
|---|---|---|
| 0 | | |
| $\frac{\pi}{6}$ | | |
| $\frac{\pi}{4}$ | | |
| $\frac{\pi}{3}$ | | |
| $\frac{\pi}{2}$ | | |
| | | |
| | | |
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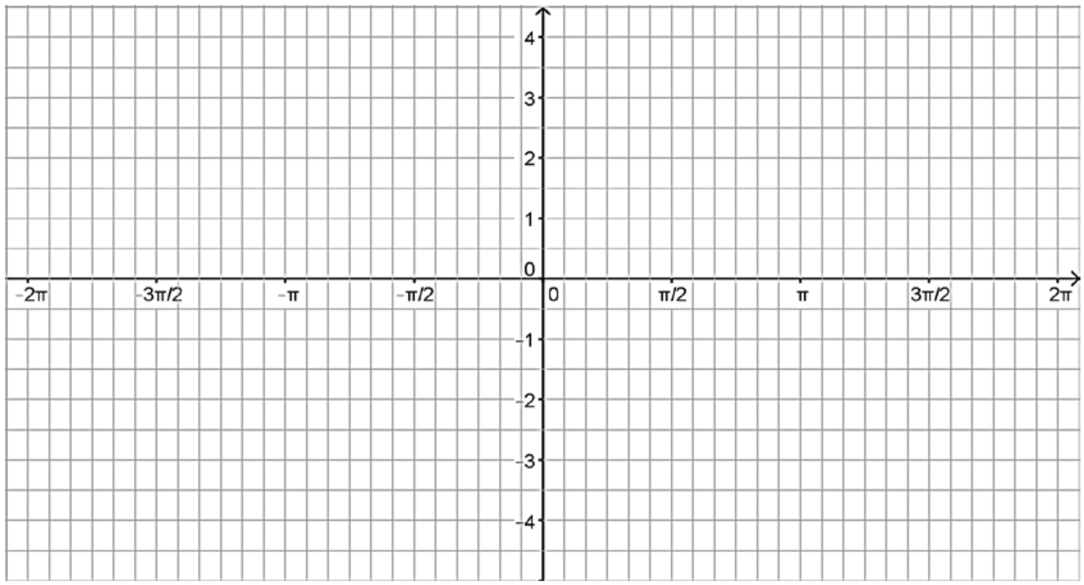


6. What happens to the graph of $y = \sec\theta$ when the graph of $y = \cos\theta$ crosses the $x$-axis?

7. Explain why $y = \sec\theta$ has the shape that appears in your graph in question 5.

Clarita says: "The graphs of $y = \csc\theta$ and $y = \sec\theta$ look so strange. What would happen if we take the reciprocals of the values of $\tan\theta$?"

Carlos adds: "That graph will look crazy because $y = \tan\theta$ has asymptotes. What is the reciprocal of an asymptote?"

8. Answer Carlos' question: What is the reciprocal of an asymptote? Hint: think about what type of ratio creates an asymptote.

9. Complete the table of values for angles in quadrant I for $y = \tan\theta$ and $y = \cot\theta$. Graph these two functions in different colors on the coordinate plane. Use symmetry to complete the sketch of each function for angles on a domain of $[-\pi, \pi]$. Reminder: $\cot\theta = \frac{1}{\tan\theta}$.

| $\theta$ | $\tan\theta$ | $\cot\theta$ |
|---|---|---|
| $0$ | | |
| $\frac{\pi}{6}$ | | |
| $\frac{\pi}{4}$ | | |
| $\frac{\pi}{3}$ | | |
| $\frac{\pi}{2}$ | | |
| | | |
| | | |
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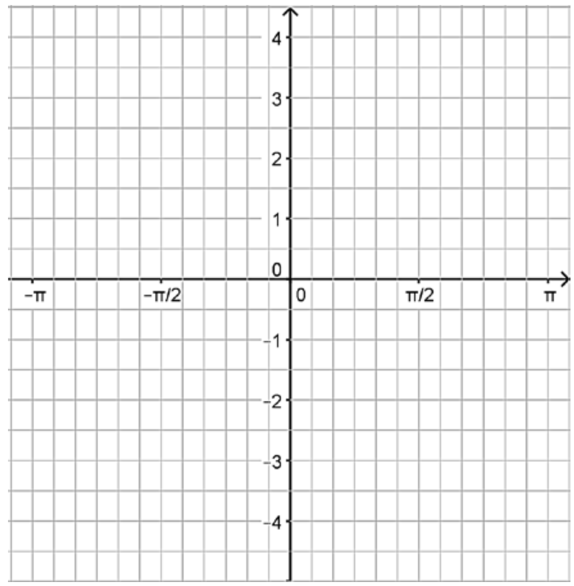


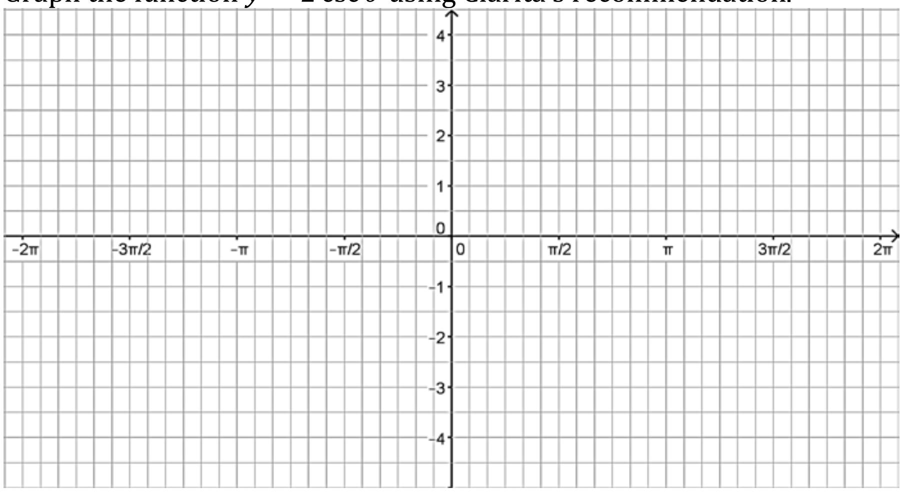

10. What happens to the graph of $y = \cot\theta$ when the graph of $y = \tan\theta$ crosses the $x$-axis?

11. What happens to the graph of $y = \cot\theta$ when the graph of $y = \tan\theta$ has an asymptote?

Carlos: “Now that we have seen the basic graphs of $y = \csc\theta$, $y = \sec\theta$, & $y = \cot\theta$, what would happen if we change the numbers in the functions?”

Clarita responds: “Okay. Let’s try graphing $y = 2\csc\theta$ by first graphing $y = 2\sin\theta$.”

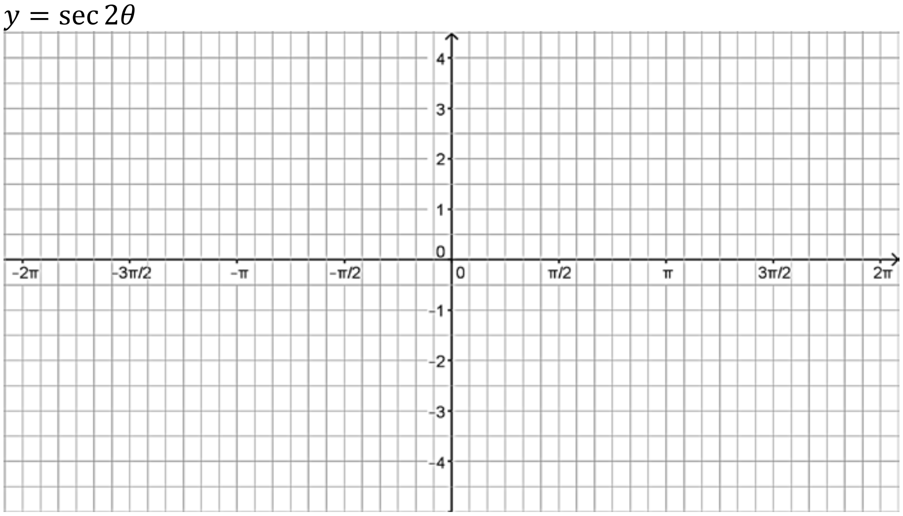
12. Graph the function $y = 2\csc\theta$ using Clarita’s recommendation.



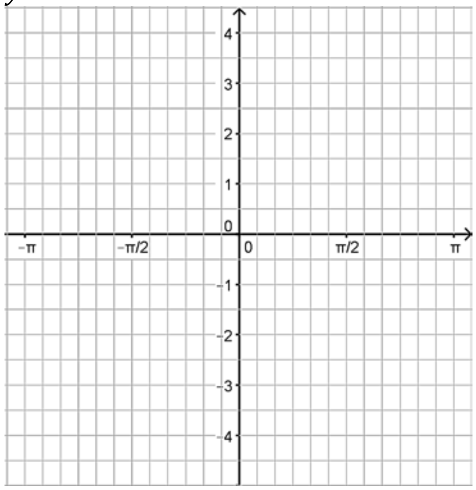

Carlos says: “I’m starting to understand. Let’s try a few more.”

Graph the following functions by first graphing the associated sine, cosine, or tangent function.

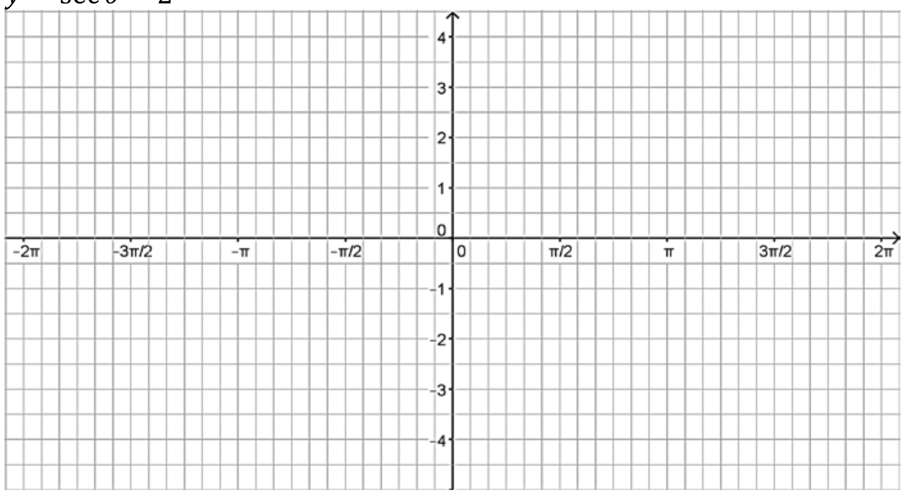
13. $y = \sec 2\theta$



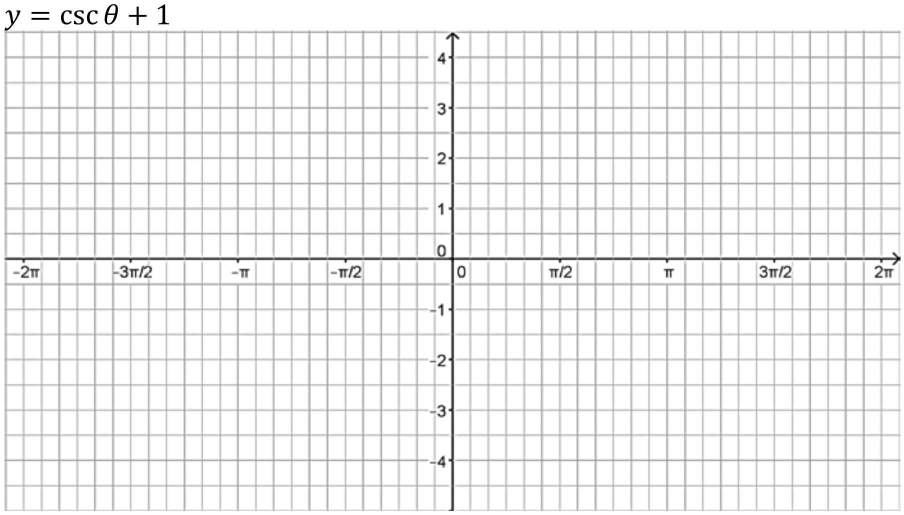
14. $y = 3\cot\theta$




15. $y = \sec\theta - 2$



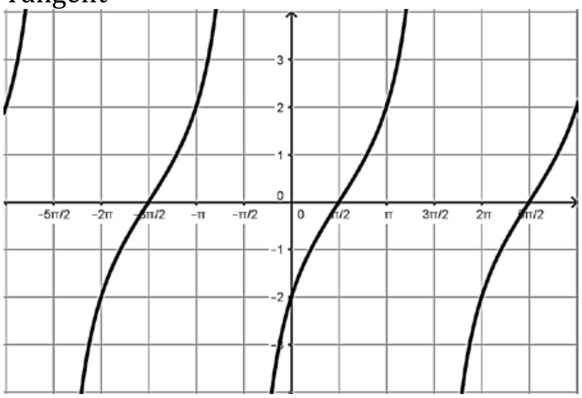

16. $y = \csc\theta + 1$



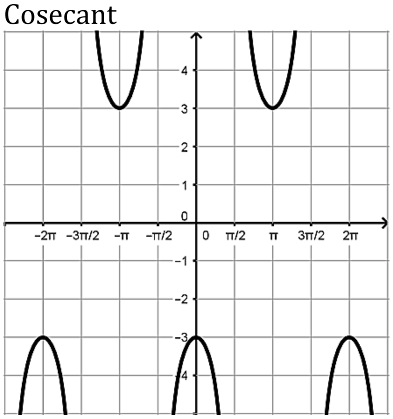
# 4.11H Warm Up
## *What's Your Equation?*

**For each graph below, write the indicated function.**

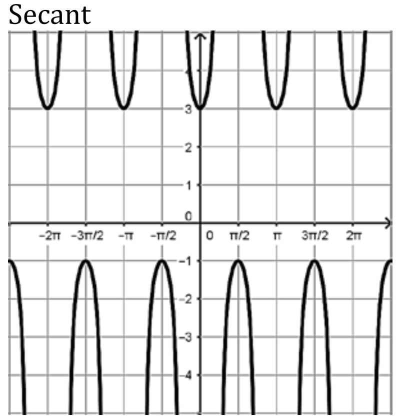
1. Tangent



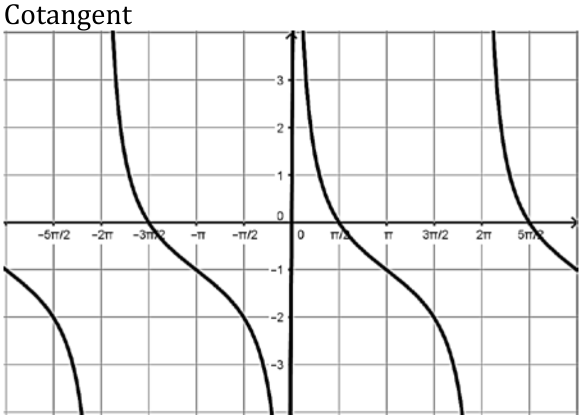
2. Cosecant




3. Secant




4. Cotangent

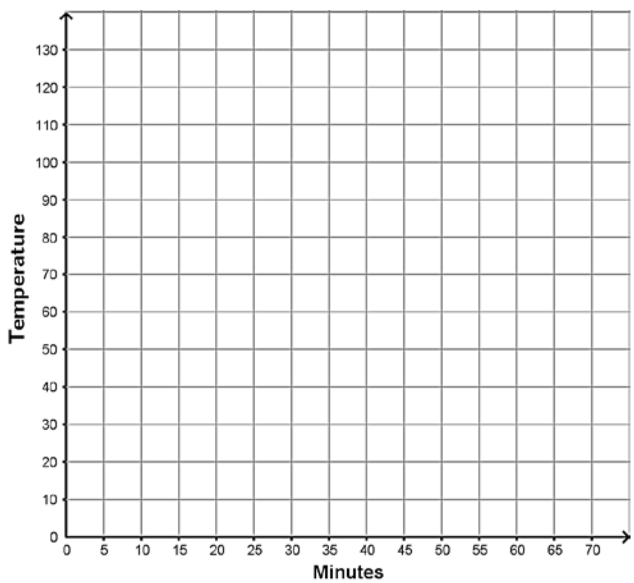


# 4.11H Modeling and Inverses
## *A Solidify and Practice Understanding Task*

**Part I:**

1. A team of biologists have discovered a new creature in the rain forest. They note the temperature of the animal appears to vary periodically over time. A maximum temperature of 125° occurs 15 minutes after they start their examination. A minimum temperature of 99° occurs 28 minutes later. The team would like to find a way to predict the animal's temperature over time in minutes.

   a. Create a graph of one full period.




   b. Write an equation of temperature as a function over time in minutes using sine.

   c. Write an equation of temperature as a function over time in minutes using cosine.

   d. Use your sine or cosine equations from parts a and b to write an equation to represent ***all*** the times when the temperature will be 108°.

   e. Give at least two times when the temperature will be 108°.

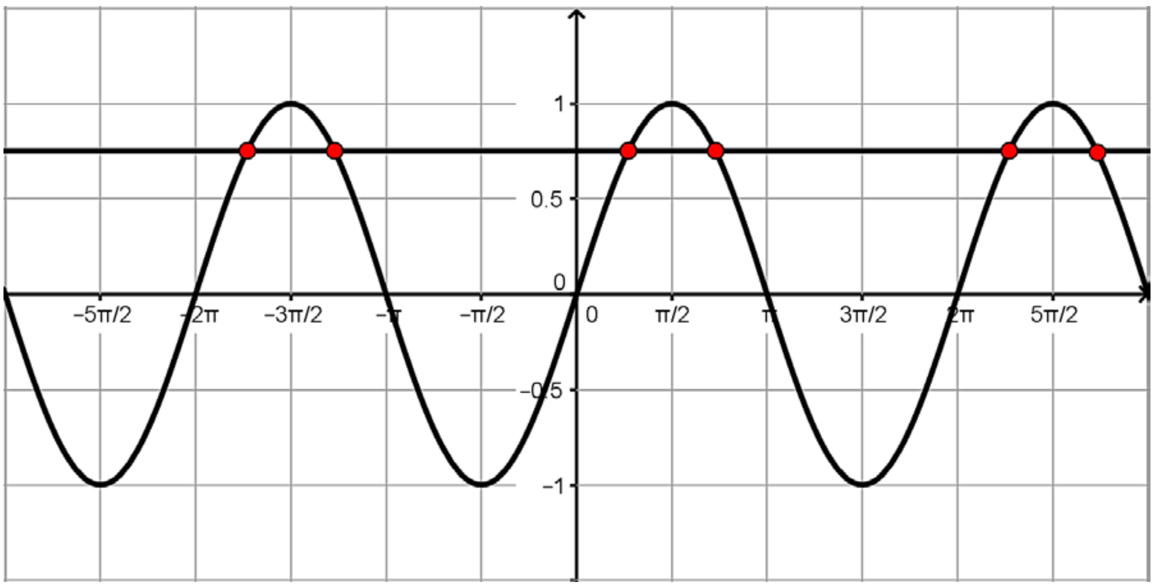
**Part II:**

Perhaps you thought about the unit circle or used a calculator to answer the previous question. For periodic functions, there are many answers to this type of question. Therefore, this question, by itself, does not define an inverse trigonometric function.

Suppose we have a simplified equation:

$$\sin\theta = 0.75$$

Using your calculator, $\sin^{-1} 0.75 = 0.848$ radians. However, the following graph indicates other values of $\theta$ for which $\sin\theta = 0.75$




1. Without tracing the graph or using any other calculator analysis tools, use the fact that $\sin^{-1} 0.75 = 0.848$ radians to find at least three other angles, $\theta$, where $\sin\theta = 0.75$. Each of these angles shows up as a point of intersection between the sine curve and the line $y = 0.75$ in the graph shown above.

Your calculator has been programmed to use the following definition for the inverse sine function, so that each time we find $\sin^{-1}$ of a number, we will get **exactly one** solution.

***Definition of the Inverse Sine Function*:**

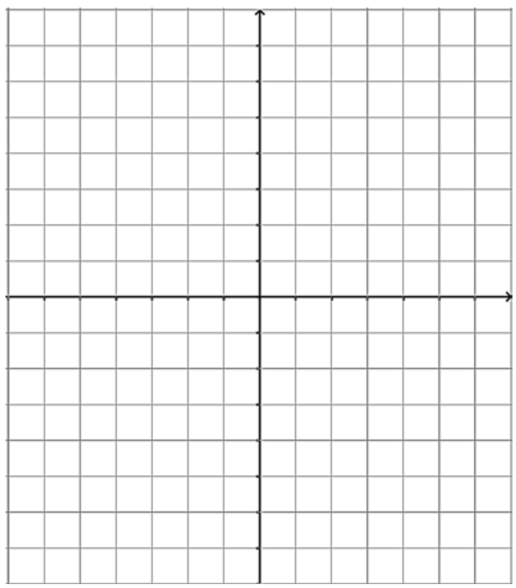
**$y = \sin^{-1} x$ means, "find the angle $y$, *on the interval* $-\frac{\pi}{2} \le y \le \frac{\pi}{2}$, such that $\sin y = x$."**

2. Based on the graph of the sine function, explain why defining the inverse trigonometric function in this way guarantees that it will have a single, unique output.

3. Based on this definition, what is the **domain** of this inverse trigonometric function?

4. Based on this definition, what is the **range** of this inverse trigonometric function?

5. Sketch a graph of the inverse sine function. Label your axes.



6a. Solve the equation.

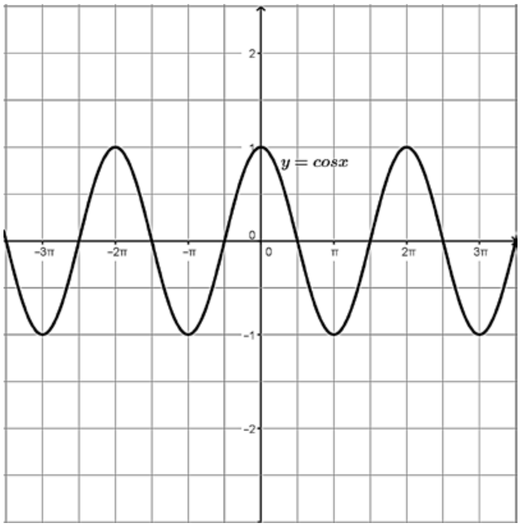
$\sin\theta = -\frac{1}{2}$.

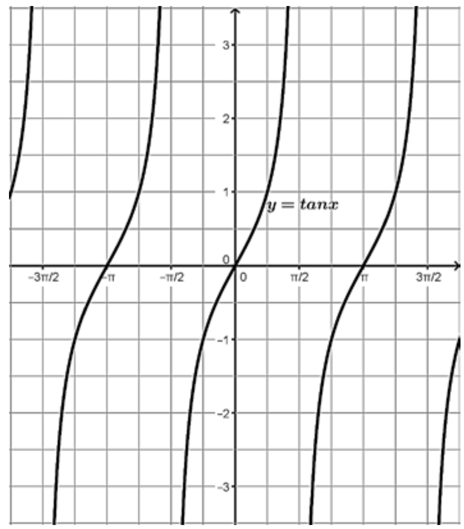
6b. Evaluate the expression, $\sin^{-1}\left(-\frac{1}{2}\right)$

6c. How are the answers to 6a and 6b different?

Examine the graphs of the cosine and tangent functions below. How would you restrict the domains of these trigonometric functions so that the inverse cosine function and the inverse tangent function can be constructed?

Complete the definitions of the inverse cosine function and the inverse tangent function below. State the domain and range of each function, and sketch its graph.

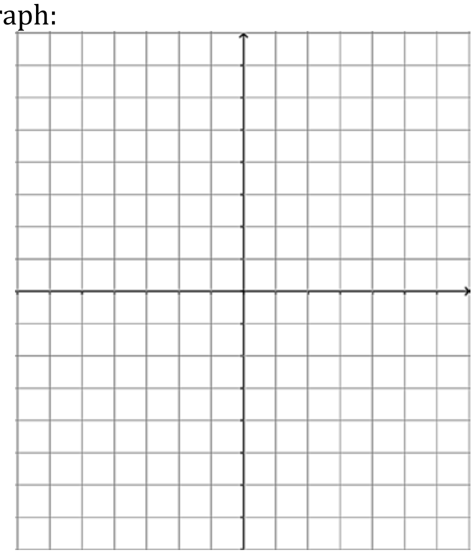



***<u>Definition of the Inverse Cosine Function</u>***:

Domain: Range:

Graph:

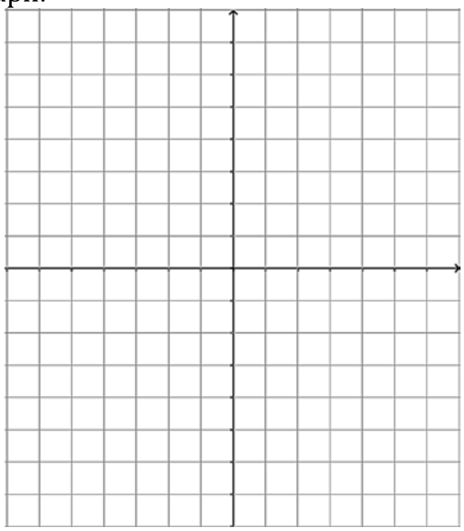


7. Find: $\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)$ $\cos^{-1}(-1)$ $\cos^{-1}\left(\frac{1}{2}\right)$

***Definition of the Inverse Tangent Function*:**

Domain: Range:

Graph:



8. Find: $\tan^{-1}(-\sqrt{3})$ $\tan^{-1}(-1)$ $\tan^{-1}(1)$

Another way to write the inverse trigonometric functions is by using the following notation:

$$\sin^{-1}(x) = \arcsin(x)$$

$$\cos^{-1}(x) = \arccos(x)$$

$$\tan^{-1}(x) = \arctan(x)$$

**Find the exact values of the following:**

9. $\arccos\left(-\frac{\sqrt{2}}{2}\right)$ 10. $\arctan\left(\frac{\sqrt{3}}{3}\right)$ 11. $\arcsin\left(\frac{1}{2}\right)$

**Find the values of the following correct to four decimal places:**

12. $\arctan(-\sqrt{473})$ 13. $\arcsin(-0.625)$ 14. $\arccos\left(\frac{\sqrt{7}}{10}\right)$

**Practice:**

**Find the exact value without a calculator.**

1. $cos\left(sin^{-1}\left(\frac{1}{2}\right)\right)$

2. $sin\left(cos^{-1}\left(\frac{\sqrt{2}}{2}\right)\right)$

3. $cos^{-1}\left(sin\left(\frac{\pi}{3}\right)\right)$

4. $sin^{-1}\left(cos\left(\frac{\pi}{6}\right)\right)$

5. $sin^{-1}\left(sin\left(\frac{7\pi}{3}\right)\right)$

6. $cos^{-1}\left(sin\left(\frac{5\pi}{3}\right)\right)$

7. $tan^{-1}(sin(\pi))$

8. $sin\left(arctan(\sqrt{3})\right)$

9. $cos\left(arctan(-1)\right)$

10. $arccos\left(sin\left(-\frac{7\pi}{4}\right)\right)$

11. $sin\left(arccos\left(\frac{5}{12}\right)\right)$

12. $sin\left(arctan\left(-\frac{7}{11}\right)\right)$