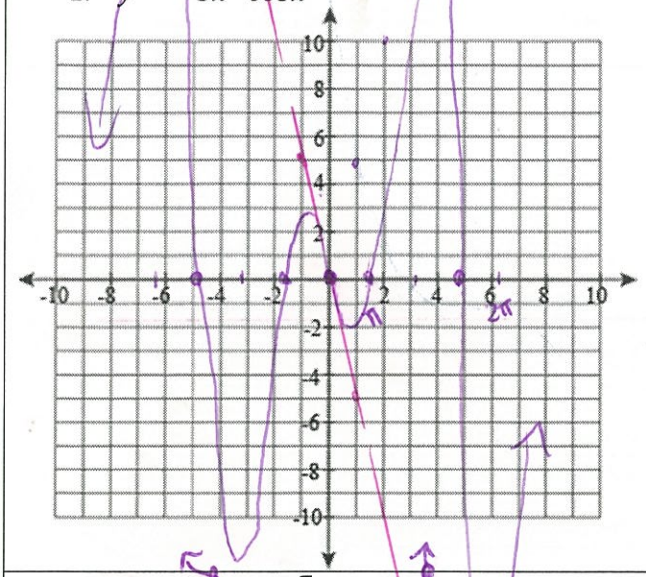
Module 5 Review

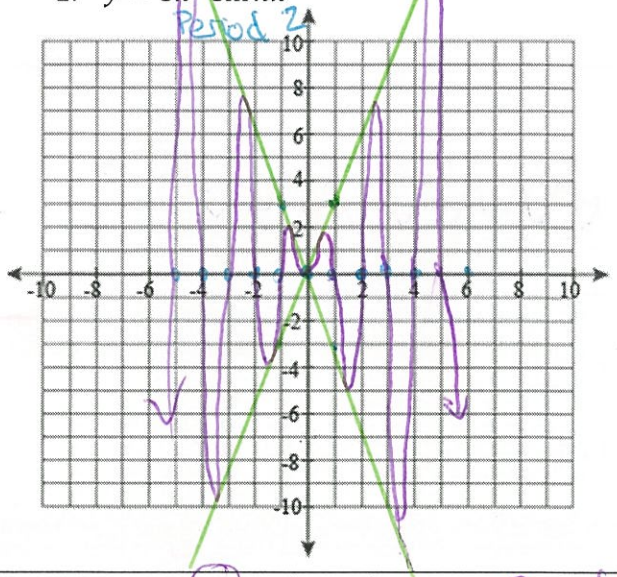
Graph the following functions:

1. $y = -5x \cdot cosx$



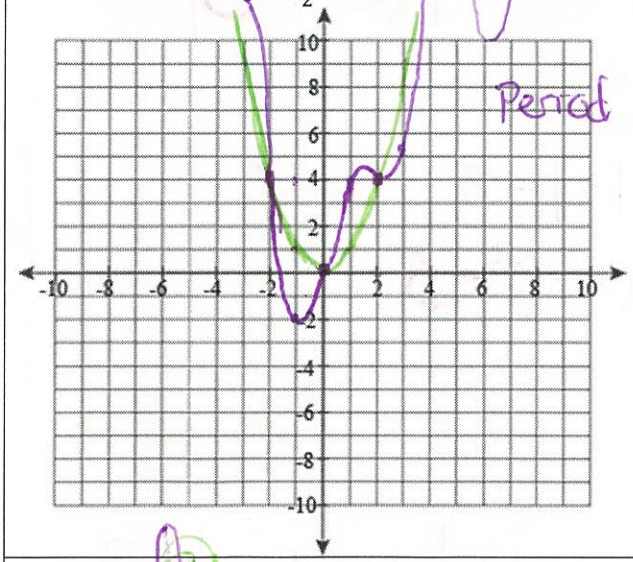
2. $y = 3x \cdot \sin \pi x$

Period 2



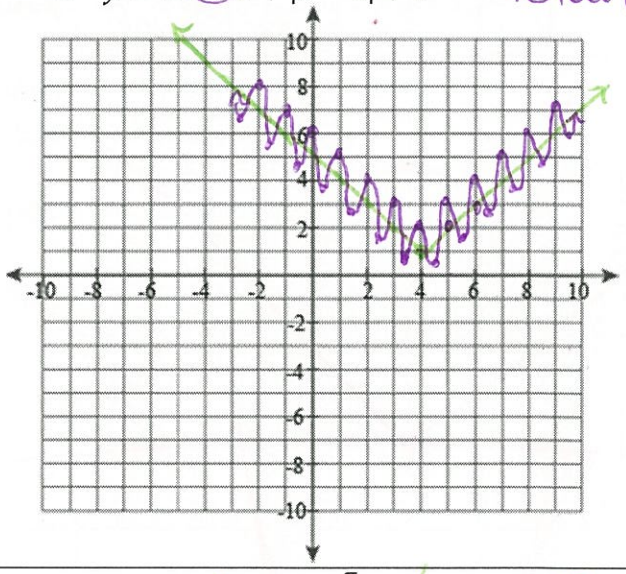
3. $y = x^2 + 3sin\frac{\pi}{2}x$

Period 4



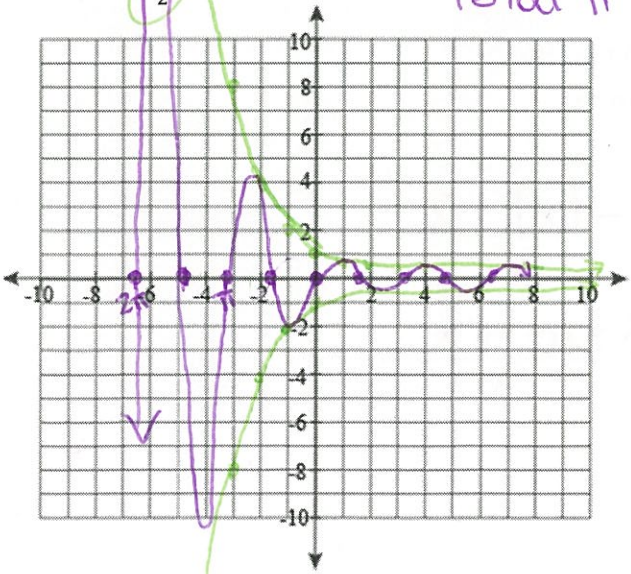
4. $y = \cos 2\pi x + |x - 4| + 1$

Period 1



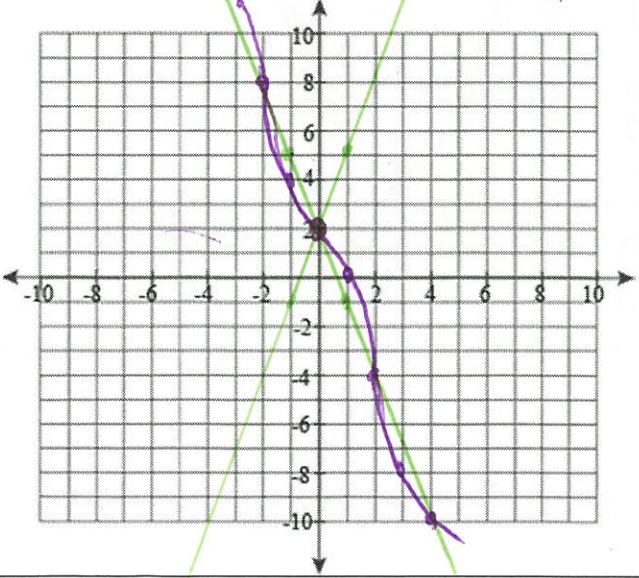
5. $y = \frac{1}{2}^x \cdot sin2x$

Period π

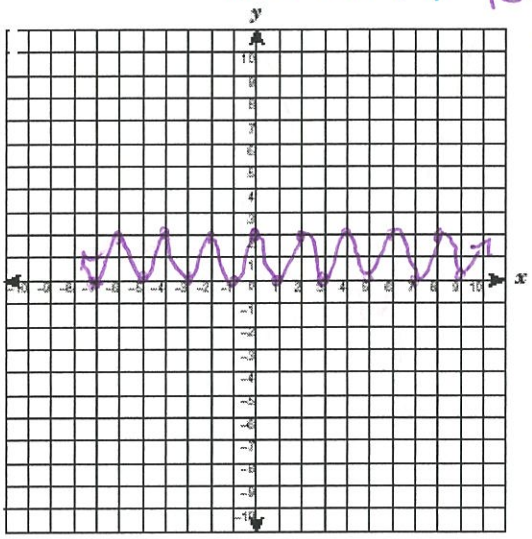


6. $y = -3x + 2 + sin\frac{\pi}{2}x$

Period 4

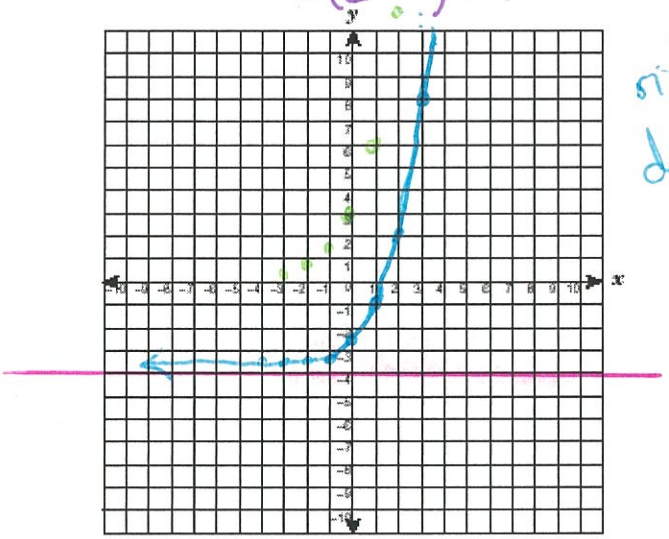


7. Given that $f(x) = x + 1$ *and* $g(x) = \cos \pi x$, graph $f(g(x))$ $= \cos \pi x + 1$ Period 2



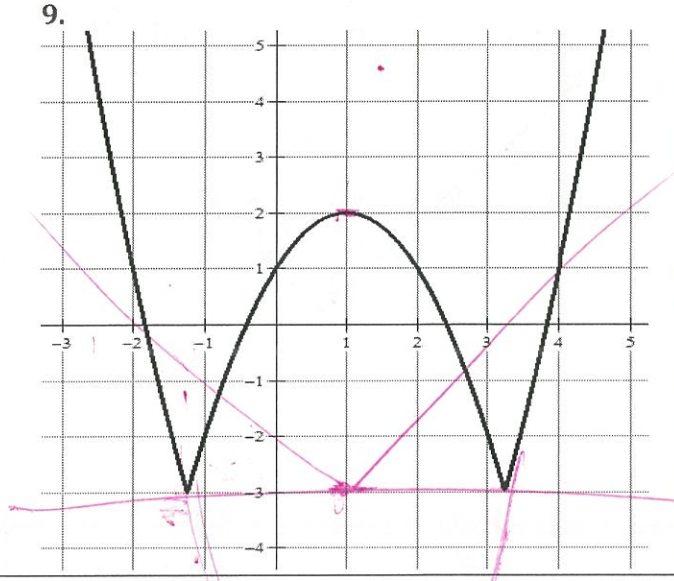
8. Given that $f(x) = 3x - 4$ *and* $g(x) = 2^{x-1}$ graph $f(g(x))$ $= 3(2^{x-1}) - 4$

right 1
down 4



For each graph below, write the function graphed and them write the function as a composition of two functions.

9.



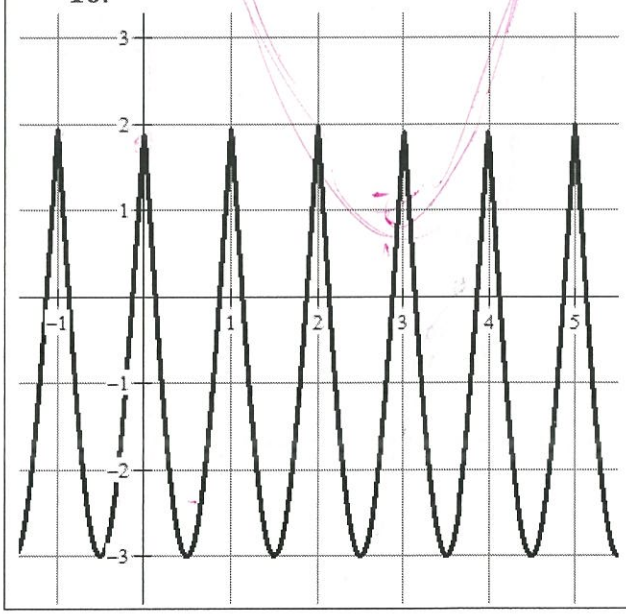
Function graphed: $|(x-1)^2 + 5| - 3$

Composed functions;
$k(x) = f(g(x))$

$f(x) =$ $|x| - 3$

$g(x) =$ $(x-1)^2 + 5$

10.



Function graphed: $-|5 \sin \pi x| + 2$

Composed functions;
$k(x) = f(g(x))$

$f(x) =$ $-|x| + 2$

$g(x) =$ $5 \sin \pi x$

Evaluate each composition using the following functions:

$f(x) = 5x - 4$ $g(x) = -x^2 + 3$ $h(x) = \sqrt{x-3}$

11. $f\left(g(h(12))\right)$

$f(g(3))$

$f(-6) = -34$

12. $(h \circ g \circ f)(-2)$

$h \circ g(-14)$

$h(-193) = 14i$

13. $g\left(h\left(f\left(\frac{11}{3}\right)\right)\right)$

$g\left(h\left(\frac{43}{3}\right)\right)$

$g\left(\sqrt{\frac{34}{3}}\right) = -\frac{25}{3}$

Given the following equations, answer the questions

$m(x) = \frac{x}{1000}$	$c(x) = \frac{150}{x}$	$w(x) = 60 \cdot \left(\frac{x-1500}{1500}\right)$	$a(x) = 4x + 100$	$p(x) = 30\cos(x+1)$

14. Find $m(a(x))$ when $x = 12$

$m(a(12)) = m(148) = \frac{148}{1000}$

15. Compose $c(p(x))$

$c(p(x)) = \frac{150}{30\cos(x+1)}$

16. Find $w(c(p(2))$

$w(c(30\cos 3)) = w\left(\frac{150}{30\cos 3}\right) = 60\left(\frac{\frac{150}{30\cos 3} - 1500}{1500}\right)$

17. Which composition creates the function, $t(x) = \frac{120\cos(x+1)+100}{1000}$

$t(x) = m(a(p(x)))$

18. Your computer's screen saver is an expanding circle. The circle starts as a dot in the middle of the screen and expands outward, changing colors as it grows. With a twenty-one inch screen, you have a viewing area with a 10-inch radius (measured from the center diagonally down to a corner). The circle reaches the corners in four seconds. Express the area of the circle (discounting the area cut off by the edges of the viewing area) as a function of time t in seconds.

$A(r) = \text{Area} = \pi r^2$ $r(t) = \frac{10}{4}t = \frac{5}{2}t$

$A(r(t)) = \pi\left(\frac{5}{2}t\right)^2$

19. In the mail, you receive a coupon for \$5 off of a pair of jeans. When you arrive at the store, you find that all jeans are 25% off. Let x represent the original cost of the jeans.

a. Write a function, f(x), that represents the effect of your original coupon. $f(x) = x - 5$

b. Write a function, g(x), that represents the effect of the 25% discount at the store. $g(x) = .75x$

c. Write a function, h(x), that represents how much you would pay if you use the mail coupon first followed by applying the discount from the store. $h(x) = g(f(x)) = .75(x-5)$

d. Write a function, j(x), that represents how much you would pay if you use the store discount first, followed by the mail coupon. $j(x) = f(g(x)) = .75x - 5$

e. You find a pair of jeans for \$36. How much would you pay for it using both functions h(x) and J(x). If you only have \$40 with you, what's the most expensive pair of jeans you can purchase? (do not consider tax).

$x = 36$

$h(36) = .75(31) = \$23.25$

$40 = h(x) \rightarrow 40 = .75(x-5)$

$x = \$58.33$

$40 = j(x) = \rightarrow 40 = .75x - 5$

20. Given the following functions, find a composition of functions with each feature listed below.

$a(x) = \frac{x-1}{x-2},\ b(x) = \log_2 x,\ c(x) = |3x|,\ d(x) = 2x^2 + 4,\ e(x) = x - 4,\ f(x) = x^2 + 5x - 4$

a. A composition of functions with a range of $[32, \infty)$ $f(d(x)) = (2x^2+4)^2 + 5(2x^2+4) - 4$ $16 + 20 - 4 = 32$ ✓

b. A composition of functions with no roots $c(d(x)),\ f(d(x))$

c. A composition of functions with an asymptote at $x = 4$ $b(e(x))$

d. A composition of functions with end behavior: $As\ x \to \infty, y \to 1$ $a(e(x))$

21. Given $f(x) = 2x - 3,\quad g(x) = x^2 - 2x,\quad and\quad h(x) = -5x$. Find $\boldsymbol{g\left(f(h(x))\right)}$.

$f(h(x)) = 2(-5x) - 3 = -10x - 3$ $g(f(h(x))) = (-10x-3)^2 - 2(-10x-3)$

$100x^2 + 60x + 9 + 20x + 6$

$= 100x^2 + 80x + 15$

22. Given $f(x) = 2x^3 - 9x^2 + x + 12,\quad g(x) = 2x - 3,\quad and\quad h(x) = x + 1.$

a. Find $g(x) - f(x)$ $= 2x - 3 - (2x^3 - 9x^2 + x + 12)$

$= -2x^3 - 9x^2 + x - 15$

b. Find $g(x) \cdot h(x)$ $(2x-3)(x+1) = 2x^2 - x - 3$

c. Find $\frac{f(x)}{g(x)\cdot h(x)}$ $\dfrac{x^2+5x-4}{2x^2-x-3} =$

23. Given:

$f(x) = 2x - 4,$

$g(x) = \cos\frac{\pi}{4}x,\quad and$

~~$h(x) = -5x$~~

Graph the following:

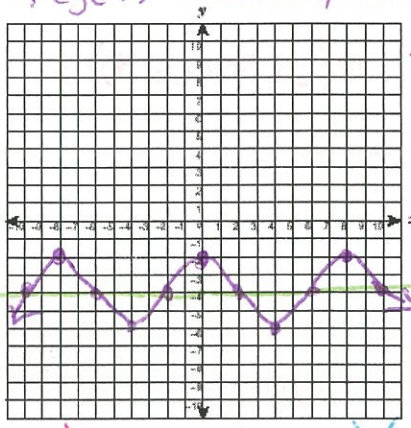
a. $f(g(x))$

b. $(f + g)(x)$

c. $f \cdot g(x)$

$f(g(x)) = 2\cos\frac{\pi}{4}x - 4$

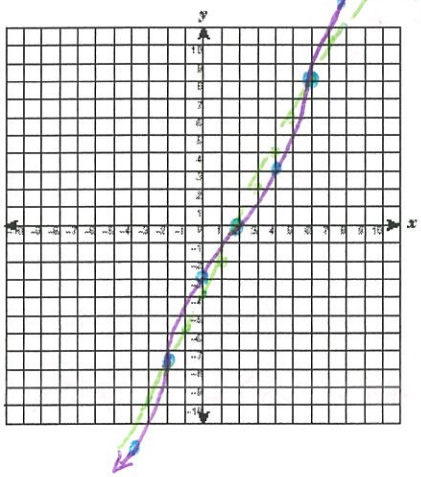
a.



Period $2\pi \cdot \frac{4}{\pi} =$

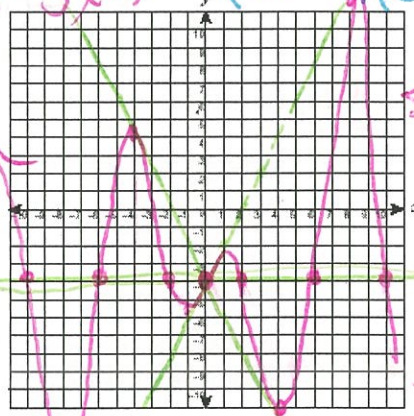
$(f+g)(x) = 2x - 4 + \cos\frac{\pi}{4}x$ Period 8

b.



$(f \cdot g)(x) = (2x-4)(\cos\frac{\pi}{4}x)$

c.



Period 8

at $x=2$

$x=4$

$\cos\theta = 0$