IM3H Module 4 Review

Things you should know:

$$y = a\sin b(x-c)+d$$

$$y = a\cos b(x-c)+d$$

$a \rightarrow$ Scaling factor: Vertical stretch: $|a| > 1,$

Vertical shrink: $|a| < 1$

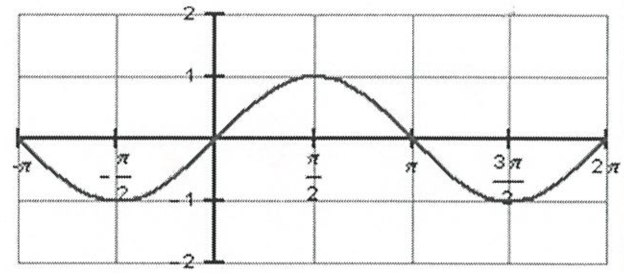
$-a \rightarrow$ Reflection over the x-axis

$b \rightarrow \dfrac{2\pi}{b} = period$

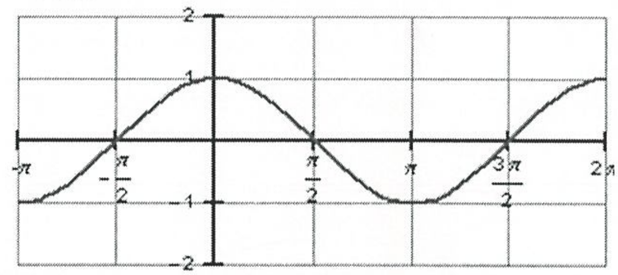
$c \rightarrow$ Horizontal translation (left or right

$d \rightarrow$ Vertical translation (up or down)

Graph of $y = sinx$



Graph of $y = cosx$



Half-Angle Formulas	$\sin\dfrac{u}{2} = \pm\sqrt{\dfrac{1-\cos u}{2}}$	$\cos\dfrac{u}{2} = \pm\sqrt{\dfrac{1+\cos u}{2}}$	$\tan\dfrac{u}{2} = \dfrac{1-\cos u}{\sin u} = \dfrac{\sin u}{1+\cos u}$
Double Angle Formulas	$\sin 2u = 2\sin u\cos u$ $\cos 2u = \cos^2 u - \sin^2 u = 2\cos^2 u - 1 = 1 - 2\sin^2 u$ $\tan 2u = \dfrac{2\tan u}{1-\tan^2 u}$		
Sum and Difference Formulas	$\sin(u+v) = \sin u\cos v + \cos u\sin v$ $\sin(u-v) = \sin u\cos v - \cos u\sin v$ $\cos(u+v) = \cos u\cos v - \sin u\sin v$ $\cos(u-v) = \cos u\cos v + \sin u\sin v$	$\tan(u+v) = \dfrac{\tan u + \tan v}{1-\tan u\tan v}$ $\tan(u-v) = \dfrac{\tan u - \tan v}{1+\tan u\tan v}$	

Given on the test

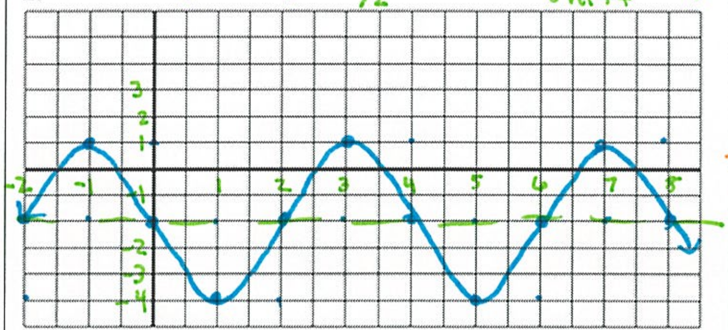
Even and Odd Trigonometric Functions	Definition: $Even \rightarrow f(-t) = f(t)$ $Odd \rightarrow f(-t) = -f(t)$	The cosine and secant functions are even. $\cos(-t) = \cos t \qquad \sec(-t) = \sec t$ The sine, cosecant, tangent, and cotangent functions are odd. $\sin(-t) = -\sin t \qquad \csc(-t) = -\csc t$ $\tan(-t) = -\tan t \qquad \cot(-t) = -\cot t$

Practice Problems

$y = 3\cos\left(\frac{\pi x}{2} + \frac{\pi}{2}\right) - 2$

Period $\frac{2\pi}{\pi/2} \cdot \frac{2}{\pi} = 4$ Phase shift: -1

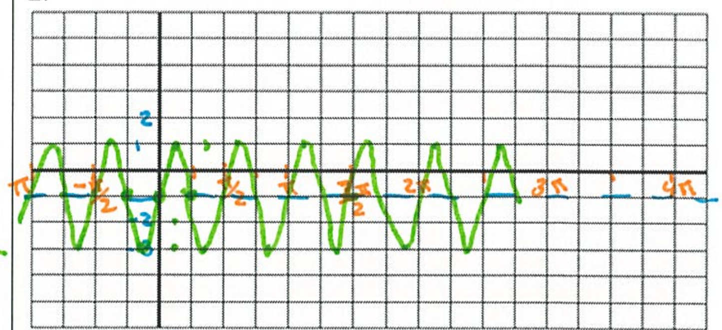
1.



$y = -2\sin(4x + \pi) - 1$

Period = $\pi/2$ Phase shift: $-\pi/4$

2.

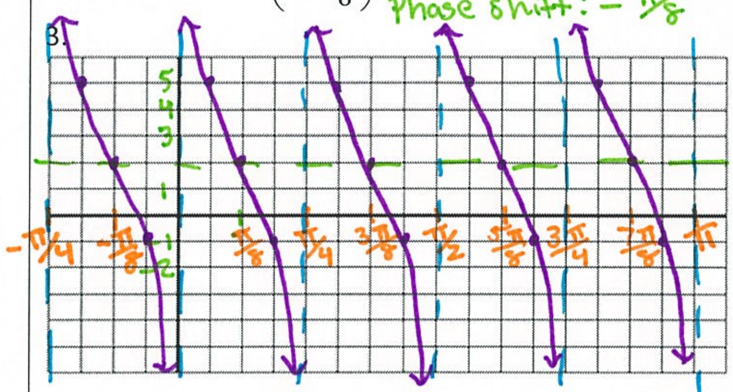


$f(x) = 2 - 3\tan 4\left(x + \frac{\pi}{8}\right)$

Period: $\pi/4$

Phase shift: $-\pi/8$

3.

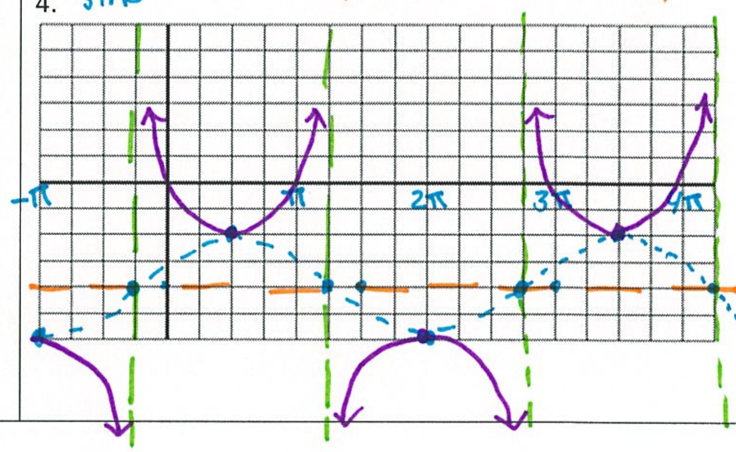


$y = 2\csc\frac{2}{3}\left(x + \frac{\pi}{4}\right) - 4$ $\frac{1}{\sin\theta}$

Period: $2\pi \cdot 3/2 = 3\pi$

phase shift: $-\pi/4$

4.



Find all solutions in the equation in the interval $[0, 2\pi)$.

5. $\tan^2 3x + \tan 3x = 0$

$\tan 3x\,(\tan 3x + 1) = 0$

$\tan 3x = 0$ $\quad$ $\tan 3x + 1 = 0$

$\tan 3x = -1$

$3x = 0$

$\boxed{x = 0}$

$3x = \frac{3\pi}{4}$

$\boxed{x = \pi/4}$

$3x = \pi$

$\boxed{x = \frac{\pi}{3}}$

$3x = \frac{7\pi}{4}$

$\boxed{x = \frac{7\pi}{12}}$

$\boxed{x = \frac{2\pi}{3}, \pi, \frac{4\pi}{3}, \frac{5\pi}{3}}$

6. $\sin 2x - \cos x = 0$

$2\sin x\cos x - \cos x = 0$

$\cos x\,(2\sin x - 1) = 0$

$\cos x = 0$ $\quad$ $\sin x = \frac{1}{2}$

$x = \pi/2$ $\quad$ $x = \pi/6$

$x = \frac{3\pi}{2}$ $\quad$ $x = 5\pi/6$

Find all general solutions in the equation

7. $4\cos^2 \frac{x}{2} - 3 = 0$

$4\cos^2\left(\frac{x}{2}\right) = 3$

$\cos^2\left(\frac{x}{2}\right) = \frac{3}{4}$

$\cos\left(\frac{x}{2}\right) = \pm\sqrt{3/4}$

$\cos\left(\frac{x}{2}\right) = \pm\frac{\sqrt{3}}{2}$

$\frac{x}{2} = \frac{\pi}{6}$ $\quad \frac{x}{2} = \frac{5\pi}{6}$ $\quad \frac{x}{2} = \frac{7\pi}{6}$ $\quad \frac{x}{2} = \frac{11\pi}{6}$

$\boxed{x = \pi/3,\ \frac{5\pi}{3}}$ $\quad x = \cancel{7\pi/3}$ $\quad x = \cancel{11\pi/3}$

8. $\csc^2 x - \csc x - 2 = 0$

$(\csc x - 2)(\csc x + 1) = 0$

$\csc x = 2 \qquad \csc x = -1$

$\sin x = \frac{1}{2} \qquad \sin x = -1$

$\boxed{x = \pi/6}$ $\quad \boxed{x = \frac{3\pi}{2}}$

$\boxed{x = 5\pi/6}$

9. Given $\sin\theta = \frac{\sqrt{5}}{5}$ and θ is in the inteval $[\frac{\pi}{2}, \pi]$. Find the exact values of cos2θ.

$\sqrt{5}^2 + b^2 = 25$

$b = \sqrt{20}$

$\cos 2\theta = \cos^2\theta - \sin^2\theta$

$= \left(\frac{\sqrt{20}}{5}\right)^2 - \left(\frac{\sqrt{5}}{5}\right)^2$

$= \frac{20}{25} - \frac{5}{25} = \frac{15}{25} = \boxed{3/5}$

10. Use the half-angle formulas to find the exact value of sin105°

$\sin\left(\frac{210}{2}\right)$

$= \pm\sqrt{\frac{1 - \cos 210}{2}}$

$= \pm\sqrt{\frac{1 - -\sqrt{3}/2}{2}}$

$= \pm\sqrt{\frac{\frac{2+\sqrt{3}}{2}}{2}} = \boxed{\pm\sqrt{2+\sqrt{3}}}$

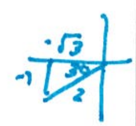
11. Use the sum formulas to find the exact value of tan255°.

$\tan(210 + 45)$

$= \frac{\tan 210 + \tan 45}{1 - \tan 210 \cdot \tan 45}$

$= \frac{1/\sqrt{3} + 1}{1 - 1/\sqrt{3} \cdot 1} = \frac{\sqrt{3}/3 + 3/3}{1 - \sqrt{3}/3}$

$= \frac{\sqrt{3}+3}{3} \cdot \frac{3}{3-\sqrt{3}} = \boxed{\frac{\sqrt{3}+3}{3-\sqrt{3}}}$



12 What is the period of the function $f(x) = 4 - 6\sin 2(x-1)$?

A. $\frac{\pi}{2}$ B. π C. $\frac{\pi}{3}$ D. 2π E. None of these

13. What is the amplitude of the function $f(x) = 4 - 6\sin 2(x-1)$?

A. 4 B. -6 C. 2 D. 6 E. None of these

Verify the identities:

14. $\sin^2\alpha - \sin^4\alpha = \cos^2\alpha - \cos^4\alpha$

$\sin^2\alpha(1 - \sin^2\alpha) =$

$\sin^2(\cos^2\alpha) =$

$(1 - \cos^2\alpha)(\cos^2\alpha) =$

$\cos^2\alpha - \cos^4\alpha =$ ✓

15. $\frac{1+\sin\theta}{\cos\theta} + \frac{\cos\theta}{1+\sin\theta} = 2\sec\theta$

$\frac{1 + 2\sin\theta + \sin^2\theta + \cos^2\theta}{\cos\theta(1+\sin\theta)} =$

$\frac{1 + 2\sin\theta + 1}{\cos\theta(1+\sin\theta)} =$

$\frac{2(1+\sin\theta)}{\ldots} =$

$\frac{2}{\cos\theta} =$

$2\sec\theta =$ ✓

16. $\tan\left(\frac{\pi}{4}-\theta\right)=\frac{1-\tan\theta}{1+\tan\theta}$

$$= \frac{\tan \pi/4 - \tan\theta}{1+\tan \pi/4 \cdot \tan\theta}$$

$$= \frac{1-\tan\theta}{1+\tan\theta} \quad ✓$$

17. $\sec^2\left(\frac{\pi}{2}-x\right)-1=\cot^2 x$

$$\tan^2\left(\frac{\pi}{2}-x\right) =$$

$$\left(\frac{\tan \pi/2 - \tan x}{1+\tan \pi/2 \cdot \tan x}\right)^2 =$$

$$\left(\frac{\frac{\sin \pi/2}{\cancel{\cos \pi/2}} - \tan x \cdot \frac{\cos \pi/2}{\cancel{\cos \pi/2}}}{\frac{\cos \pi/2}{\cancel{\cos \pi/2}} - \frac{\sin \pi/2}{\cancel{\cos \pi/2}} \cdot \tan x}\right)^2 =$$

$$\left(\frac{1-\cancel{\tan x}\cdot 0}{0-1\tan x}\right)^2 =$$

$$\left(\frac{1}{-\tan x}\right)^2 =$$

$$\cot^2 x = \quad ✓$$

18. Find the exact value using a sum or difference formula.

a. $\sin\frac{11\pi}{12} = \sin\left(\frac{8\pi}{12}+\frac{3\pi}{12}\right)$

$$\sin\left(\frac{2\pi}{3}+\frac{\pi}{4}\right)$$

$$= \sin\frac{2\pi}{3}\cdot\cos\frac{\pi}{4}+\cos\frac{2\pi}{3}\sin\frac{\pi}{4}$$

$$\left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right)+\left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$$

$$\boxed{=\frac{\sqrt{6}+\sqrt{2}}{4}}$$

b. $\cos\frac{11\pi}{12} = \left(\frac{2\pi}{3}+\frac{\pi}{4}\right)$

$$= \cos\frac{2\pi}{3}\cos\frac{\pi}{4} - \sin\frac{2\pi}{3}\sin\frac{\pi}{4}$$

$$\frac{1}{2}\left(\frac{\sqrt{2}}{2}\right)-\left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$$

$$\boxed{=\frac{\sqrt{2}-\sqrt{6}}{4}}$$

c. $\tan\frac{11\pi}{12} = \tan\left(\frac{2\pi}{3}+\frac{\pi}{4}\right)$

$$= \frac{\tan 2\pi/3 + \tan \pi/4}{1-\tan\frac{2\pi}{3}\cdot\tan \pi/4}$$

$$= \frac{-\sqrt{3}+1}{1- -\sqrt{3}\cdot 1}$$

$$\boxed{=\frac{1-\sqrt{3}}{1+\sqrt{3}}}$$

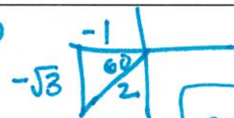
Evaluate:

19. $\cos^{-1}\left(\frac{1}{2}\right) =$ $\pi/3$	20. $\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right) =$ $-\pi/3$	21. $\csc^{-1}(-\sqrt{2}) =$ $-\pi/4$

$\sin^{-1}\left(-\frac{\sqrt{2}}{2}\right)$

22. Find $\cos\theta$ if $\cot\theta=\frac{\sqrt{3}}{3}$ and $\csc\theta<0$.

$\sin\theta < 0$



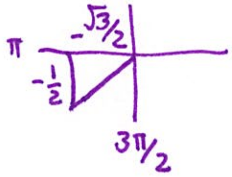
$$\tan\theta = \frac{3}{\sqrt{3}} = \sqrt{3}$$

$$\boxed{\cos\theta = -\frac{1}{2}}$$

23. Find $\sin 2x$, if $\csc x=-2$, and $\pi\le x\le\frac{3\pi}{2}$.

$\sin 2x = 2\sin x\cdot\cos x$

$\sin x = -\frac{1}{2}$

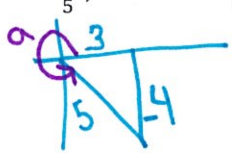


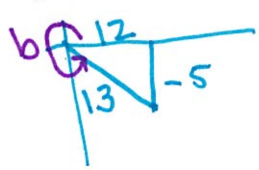
$$2\left(-\frac{1}{2}\right)\left(-\frac{\sqrt{3}}{2}\right) = \boxed{\frac{\sqrt{3}}{2}}$$

$$= \frac{36}{65} - \frac{20}{65}$$

$$= \frac{16}{65}$$

24. Find $\cos(a-b)$, if $\cos a=\frac{3}{5}$, $\sin b=-\frac{5}{13}$, and *angle a and b* are in the same quadrant.





$\cos(a-b)$

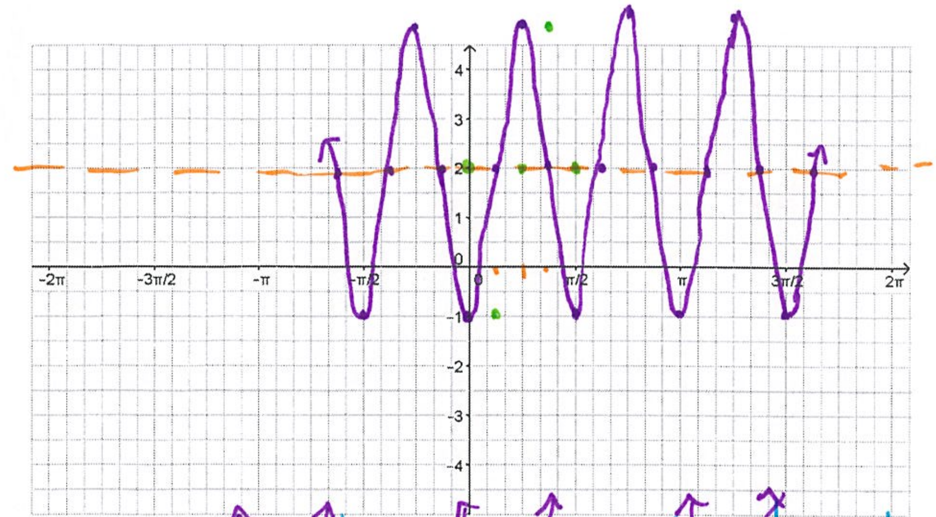
$= \cos a\cdot\cos b - \sin a\cdot\sin b$

$\left(\frac{3}{5}\right)\left(\frac{12}{13}\right) - \left(-\frac{4}{5}\right)\left(-\frac{5}{13}\right)$

25. Graph the function: $f(x) = 2 - 3\sin 4\left(x + \frac{\pi}{8}\right)$

Period: $\frac{2\pi}{4} = \pi/2$

Phase shift: $-\pi/8$



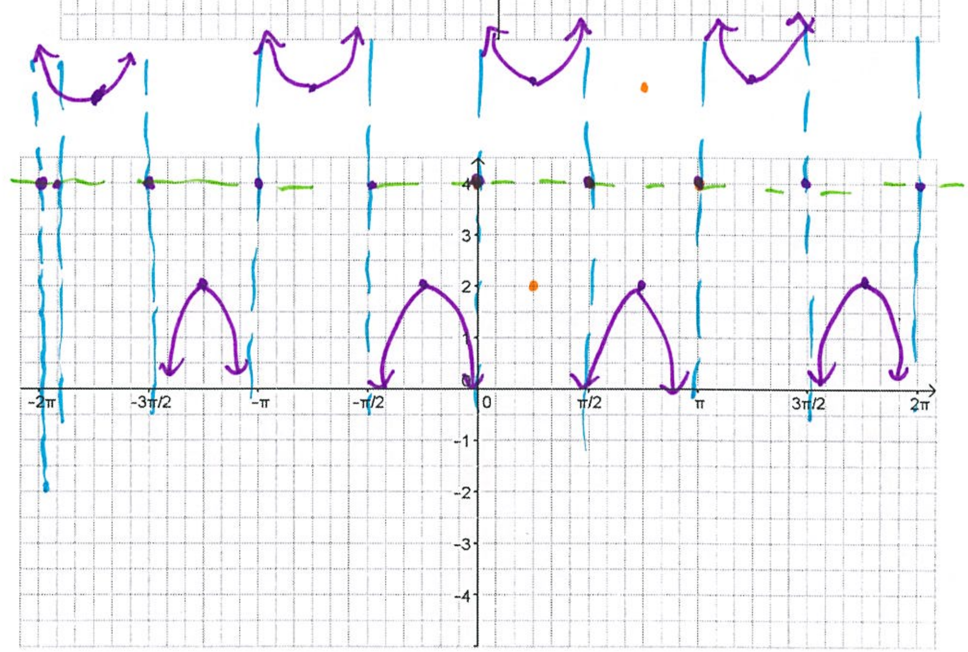
26. Graph the function:

$f(x) = 4 + 2\csc(2x - \pi)$

$2(x - \pi/2)$

Period: $\frac{2\pi}{2} = \pi$

Phase shift $= +\ \pi/2$



27. Write a sine and a cosine equation for the graph below.

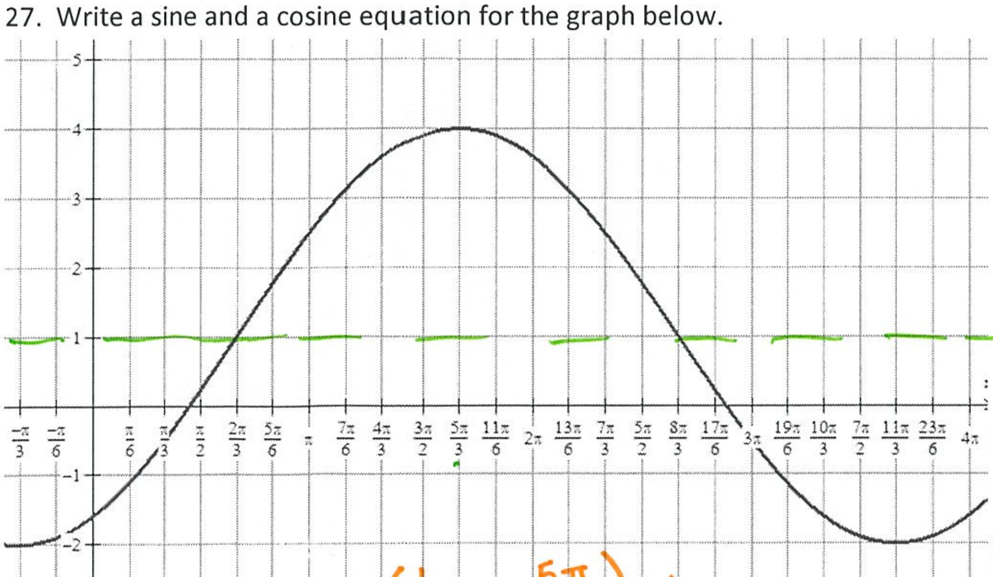
Min - Max

$\frac{11\pi}{3} - \frac{5\pi}{3} = \frac{6\pi}{3} = 2\pi \cdot 2$

Period: 4π

$B = \frac{2\pi}{4\pi} = \frac{1}{2}$

Phase shift: $+\ \frac{2\pi}{3}$



$y = 3\sin\left(\frac{1}{2}\left(x - \frac{2\pi}{3}\right)\right) + 1$

$y = 3\sin\left(\frac{1}{2}x - \pi/3\right) + 1$

$y = 3\cos\left(\frac{1}{2}x - \frac{5\pi}{6}\right) + 1$

28. REVIEW FERRIS WHEEL PROBLEMS!!!! $y = 3\cos\left(\frac{1}{2}x - \pi/3 - \pi/2\right) + 1$