Practice Test

① Use 5 right hand rectangles to approximate $\int_{1}^{4} \sqrt[3]{x} + 1 \, dx$
Round your final answer to 2 decimal places.

a) Find Area.

~~Left Hand~~

$f(1) = 2$

$f(1.6) = 2.169607$
$f(2.2) = 2.300591$
$f(2.8) = 2.4094597$
$f(3.4) = 2.5036946$ $f(4) = 2.587401$

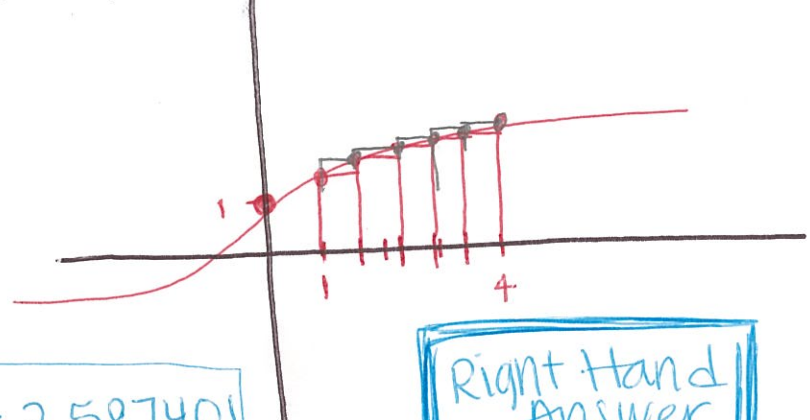
R. H.

~~$(11.9907533) \cdot .6$~~

$(11.3833523) \cdot .6 = \boxed{6.83}$ Left Hand ← Answer

b) Graph.

$W = \frac{4-1}{5} = \frac{3}{5} = 0.6$



Right Hand Answer 7.18

c) Use Sigma Notation to represent the area.

$$\sum_{i=1}^{5} 0.6\left(\sqrt[3]{.6i+.4} + 1\right)$$

i	x_i	
1	1	
2	1.6	2+.6
3	2.2	2+.6
4	2.8	2+.6
5	3.4	2+.6

$x_i = .6i + .4$

② Barb pulled the plug on her bubble bath and the water started to drain. The amount of water in the bathtub as it drains is represented by the equation $f(x) = -5x^2 + 33x + 14$.

Determine the speed at the exact moment the tub was completely drained.

($f(x)$ represents gallons of water remaining, x represents time in minutes.)

$0 = -5x^2 + 33x + 14$
$0 = -(5x^2 - 33x - 14)$
$0 = -(5x + 2)(x - 7)$

$\cancel{x = -\frac{2}{5}}$ $\boxed{x = 7}$

$f(7) = 0$
$f(6.999) = -244.93005 + 230.967 + 14$
$= -0.03695$

$$f'(c) = \frac{f(7) - f(6.999)}{7 - 6.999} = \frac{0.03695}{.001} = 36.95 \ \frac{\text{gallons}}{\text{min}}$$

③ Given $f(x) = 2x^2 + 8x - 3$, find the equation of the tangent line at $x = 3$.

point $(3, 39)$

$f'(3) = \dfrac{f(2.999) - f(3)}{2.999 - 3} = \dfrac{-0.019998}{-.001}$

$= 19.998$

≈ 20

$f(2.999) = 17.988002 + 23.992 - 3 = 38.980002$

$y = m(x - x_1) + y_1$

$\boxed{y = 20(x - 3) + 39}$

④ Given $h(3) = 6$, $h'(3) = -2$, find the equation of the tangent line at $x = 3$.

point $(3, 6)$ $\quad m = -2 \Rightarrow y - 6 = -2(x - 3)$

$\boxed{y = -2(x - 3) + 6}$

⑤ At what values, does the function $g(x) = \dfrac{3}{x+4}$ have a slope of $-\frac{1}{3}$

$g'(x) = \lim_{h \to 0} \dfrac{\frac{3}{x+h+4} - \frac{3}{x+4}}{h} \Rightarrow \dfrac{\cancel{3x+12} - \cancel{3x} - 3h - \cancel{12}}{(x+h+4)(x+4)\cancel{h}}$

$g'(x) = \lim_{h \to 0} \dfrac{-3}{(x+h+4)(x+4)}$

$g'(x) = \dfrac{-3}{(x+4)^2}$

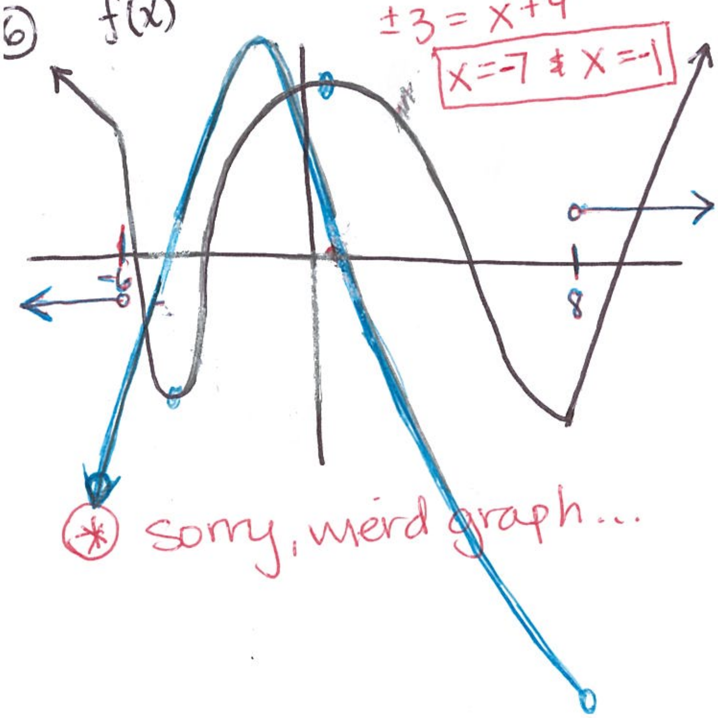
$\dfrac{-3}{(x+4)^2} = -\dfrac{1}{3}$

$\sqrt{9} = \sqrt{(x+4)^2}$

$\pm 3 = x + 4$

$\boxed{x = -7 \ \& \ x = -1}$

⑥ $f(x)$



⊛ sorry, weird graph...

a. Sketch the derivative

Make a scale.

b. Find when $f'(x) > 0$ (use interval notation)

$(-5, 1) \cup (8, \infty)$

c. Find when $f'(x) < 0$

$(-\infty, -6) \cup (-6, -5) \cup (1, 8)$

d. when is $f'(x)$ the greatest value?

about $x = -5.5$

(when it's the steepest)

⑦ Find the interval when the following graph is continuous.

hole at $x=-1$
asymptote at $x=1$
can't be negative so $x \geq -\frac{1}{2}$

$$g(x) = \frac{(x+1)(\sqrt{2x+1})}{x^2-1} = \frac{\cancel{(x+1)}(\sqrt{2x+1})}{\cancel{(x+1)}(x-1)}$$

$\boxed{[-\frac{1}{2}, 1) \cup (1, \infty)}$

⑧ Find variables a and b to make the function continuous.

$$g(x) = \begin{cases} ax+b & x>3 \\ x^2+b-a-1 & 3 \leq x < 7 \\ x+43 & x \geq 7 \end{cases}$$

$3a+b = 9+b-a-1$
$3a+\cancel{b} = 8+\cancel{b}-a$
$4a = 8$
$\boxed{a=2}$

$49+b-2-1 = 50$
$46+b = 50$
$\boxed{b=4}$

If continuous there will be no holes

⑨ Find all limits (solve algebraically when possible)

ⓐ $\lim_{x \to -5} \frac{5x+1}{x+5}$

$\lim_{x \to -5^-} = \infty$

$\lim_{x \to -5^+} = -\infty$

$\boxed{DNE}$

ⓑ $\lim_{x \to -\infty} \frac{5x^3+2x^2+1}{x^2-x^3}$

$\frac{5x^3}{-x^3}$ $\quad \frac{-5}{1}$

$\boxed{-5}$

ⓒ $\lim_{x \to 16^-} \frac{(4-\sqrt{x})(16-x)}{x^2-32x+256}$

$-\cancel{(x-16)}$

$= \frac{-(4-\sqrt{x})}{x-16} \cdot \frac{-4-\sqrt{x}}{-4-\sqrt{x}}$

$-4+\sqrt{x}$

$(x-16)\cancel{(x-16)}$

$= \frac{16-x}{(x-16)(-4-\sqrt{x})} = \frac{-\cancel{(x-16)}}{\cancel{(x-16)}(-4-\sqrt{x})}$

$\lim_{x \to 16^-} \frac{-1}{-4-\sqrt{x}} = \frac{-1}{-4-4} = \boxed{\frac{1}{8}}$

ⓓ $\lim_{x \to -3} \frac{5x^3+15x^2-3x-9}{x^2-9}$

$5x^2(x+3)-3(x+3)$

$(x+3)(x-3)$

$\frac{(5x^2-3)\cancel{(x+3)}}{\cancel{(x+3)}(x-3)}$

$\lim_{x \to -3} \frac{5x^2-3}{x-3} = \frac{5(-3)^2-3}{-3-3}$

$\frac{42}{-6} = \boxed{-7}$

⑩ Using the definition of derivative, find the derivative of

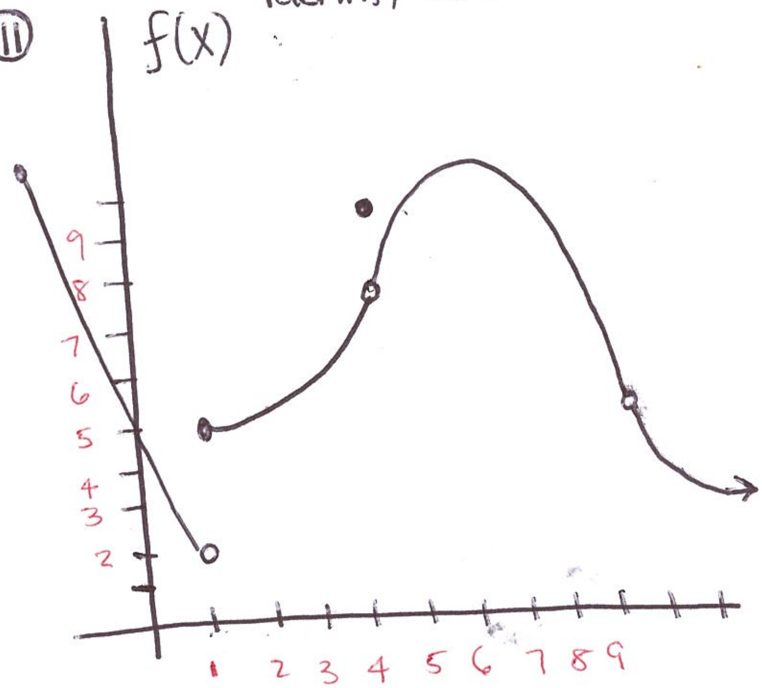
$\frac{1}{5}\sqrt{2x-3}$

$$f'(x)=\lim_{h\to 0}\frac{\frac{1}{5}(\sqrt{2(x+h)-3})-\frac{1}{5}(\sqrt{2x-3})}{h} \Rightarrow \frac{\cancel{\frac{1}{5}}(\sqrt{2x+2h-3}-\sqrt{2x-3})}{5h}\cdot\frac{\sqrt{2x+2h-3}+\sqrt{2x-3}}{\sqrt{2x+2h-3}+\sqrt{2x}}$$

$$\frac{\cancel{2x}+2\cancel{h}\cancel{-3}\cancel{-2x}\cancel{+3}}{5\cancel{h}(\sqrt{2x+2h-3}+\sqrt{2x-3})} \Rightarrow f'(x)=\lim_{h\to 0}\frac{2}{5(\sqrt{2x+2h-3}+\sqrt{2x-3})} \Rightarrow f'(x)=\frac{2}{10\sqrt{2x-3}}=\boxed{\frac{1}{5\sqrt{2x}}}$$

Identify continuous or discontinuous. If discontinuous, state type and reason it's <u>not</u> continuous.

⑪



$\lim_{x\to 1}$ = DNE — Non Removable Discontinuous

$\lim_{x\to 1^-}$ = 2 — $\lim_{x\to 1} f(x)$ DNE

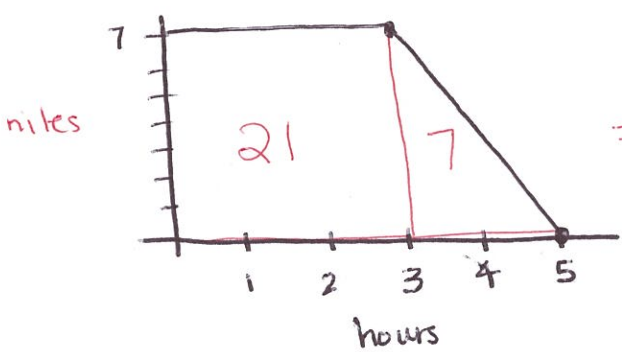
$\lim_{x\to 1^+}$ = 5

$\lim_{x\to 4}$ = 8 — Removable Discontinuous $\lim_{x\to 4} f(x) \neq f(4)$

$\lim_{x\to 9}$ = 5 — f(9) doesn't exist

$\lim_{x\to 2}$ = 5.5 (about) continuous :)

⑫ The speed Ms. Shultis runs a race is demonstrated below. Find the total distance she traveled.



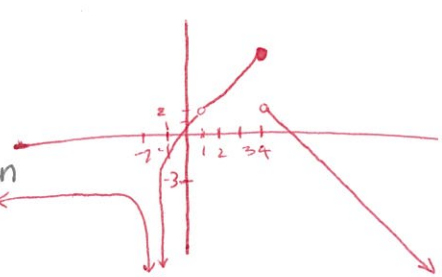
= 28 miles

⑬ Sketch a graph with all the following:

• $\lim_{x\to -\infty} f(x) = -3$ • $\lim_{x\to 1} f(x) = 2$ • f(1) is undefined

• f'(x) > 0 <u>only</u> on the interval (-2, 4) • Non removable discontinuity at x = 4

• f'(x) < 0 & constant only when x > 4



• f'(x) < 0 & constant only when x > 4)