PRACTICE QUIZ 2

① Verify the following identities.

(a) $\tan\theta\csc^2\theta - \tan\theta = \cot\theta$

(b) $1 - \tan^4\theta = 2\sec^2\theta - \sec^4\theta$

(c) $\dfrac{2 + \csc\theta\sec\theta}{\csc\theta\sec\theta} = (\sin\theta + \cos\theta)^2$

② Solve for <u>all</u> general solutions

(a) $\sec^2(2x) - 2\tan(2x) = 4$

(b) $4\sin^3 x + 2\sin^2 x - 2\sin x - 1 = 0$

(c) $-3\tan^2\left(\frac{x}{2}\right) + 3 = 0$

③ Graph the following (in radians)

(a) $y = -2\sin(4x) + 3$

(b) $y = 3\cos(\pi x) - 1$

ANSWERS

(1a) $\tan\theta\csc^2\theta - \tan\theta = \cot\theta$

(LHS) $\tan\theta(\csc^2\theta - 1) = \cot\theta$

$\tan\theta(\cot^2\theta) = \cot\theta$

$\tan\theta\left(\frac{1}{\tan^2\theta}\right) = \cot\theta$

$\frac{1}{\tan\theta} = \cot\theta$

$\cot\theta = \cot\theta$

$\frac{\sin^2}{\sin^2} + \frac{\cos^2}{\sin^2} = \frac{1}{\sin^2}$

$1 + \cot^2 = \csc^2$

$-1 \qquad -1$

$\cot^2 = \csc^2 - 1$

(1b) $1 - \tan^4\theta = 2\sec^2\theta - \sec^4\theta$

(LHS) $(1+\tan^2\theta)(1-\tan^2\theta) = 2\sec^2 - \sec^4\theta$

$(\sec^2)(1-\tan^2\theta) = 2\sec^2 - \sec^4\theta$

$(\sec^2)(1-(1+\sec^2)) = 2\sec^2 - \sec^4\theta$

$(\sec^2)(2-\sec^2) = 2\sec^2 - \sec^4\theta$

$2\sec^2\theta - \sec^4\theta = 2\sec^2\theta - \sec^4\theta$

$\frac{\sin^2}{\cos^2} + \frac{\cos^2}{\cos^2} = \frac{1}{\cos^2}$

$\tan^2 + 1 = \sec^2$

$\tan^2 = \sec^2 - 1$

(1c) $\frac{2 + \csc\theta\sec\theta}{\csc\theta\sec\theta} = (\sin\theta + \cos\theta)^2$

(LHS)

$\left(2 + \frac{1}{\sin\cdot\cos}\right)\sin\cos = (\sin+\cos)^2$

$2\sin\cos + 1 = (\sin+\cos)^2$

$2\sin\cos + \sin^2 + \cos^2 = (\sin+\cos)^2$

$(\sin+\cos)(\sin+\cos) = (\sin+\cos)^2$

$(\sin\theta+\cos\theta)^2 = (\sin\theta+\cos\theta)^2$

(2a) $\sec^2(2x) - 2\tan(2x) = 4$

$\tan^2(2x) + 1 - 2\tan(2x) = 4$

$\tan^2(2x) - 2\tan(2x) - 3 = 0$

$(\tan(2x) - 3)(\tan(2x) + 1) = 0$

$\tan(2x) = 3$ *use calculator

$\tan(2x) = -1$

$u = 2x$

$\tan u = -1$

$u = \frac{3\pi}{4} + \pi n \quad u = \frac{7\pi}{4} + \pi n$

$2x = \frac{3\pi}{4} + \pi n \quad 2x = \frac{7\pi}{4} + \pi n$

$\boxed{x = \frac{3\pi}{8} + \frac{\pi n}{2}} \quad \boxed{x = \frac{7\pi}{8} + \frac{\pi n}{2}}$

$\frac{\sin^2}{\cos^2} + \frac{\cos^2}{\cos^2} = \frac{1}{\cos^2}$

$\tan^2 + 1 = \sec^2$

(2b) $4\sin^3 x + 2\sin^2 x - 2\sin x - 1 = 0$

$2\sin^2 x(2\sin x + 1) - 1(2\sin x + 1) = 0$

$(2\sin^2 x - 1)(2\sin x + 1) = 0$

$\sin^2 x = \frac{1}{2} \quad \sin x = -\frac{1}{2}$

$\sin x = \pm\frac{\sqrt{2}}{2}$

$\boxed{x = \frac{7\pi}{6} + 2\pi n}$

$\boxed{x = \frac{11\pi}{6} + 2\pi n}$

$\boxed{x = \frac{\pi}{4} + 2\pi n}$

$\boxed{x = \frac{3\pi}{4} + 2\pi n}$

$\boxed{x = \frac{5\pi}{4} + 2\pi n}$

$\boxed{x = \frac{7\pi}{4} + 2\pi n}$

(2c) $-3\tan^2\left(\frac{x}{2}\right) + 3 = 0$

$\tan^2\left(\frac{x}{2}\right) = 1$

$\tan\left(\frac{x}{2}\right) = \pm 1$

$u = \frac{x}{2}$

$u = \frac{\pi}{4} + \pi n, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4} + \pi n$

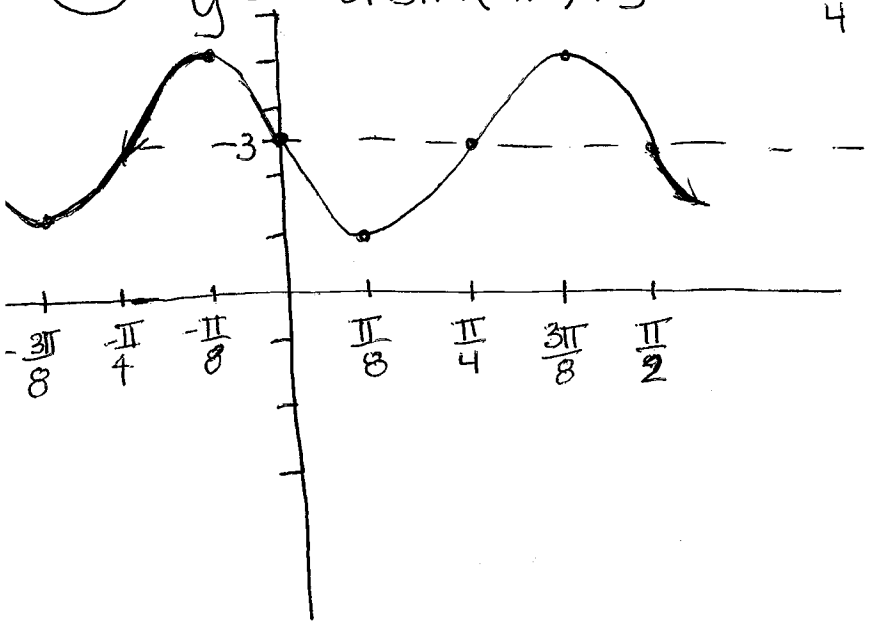
$\frac{x}{2} = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4} + \pi n$

$\boxed{x = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2} + 2\pi n}$

(3a) $y = -2\sin(4x) + 3$

period: $\frac{2\pi}{4} = \frac{\pi}{2}$

scale: $\frac{\pi}{2} \cdot \frac{1}{4} = \frac{\pi}{8}$



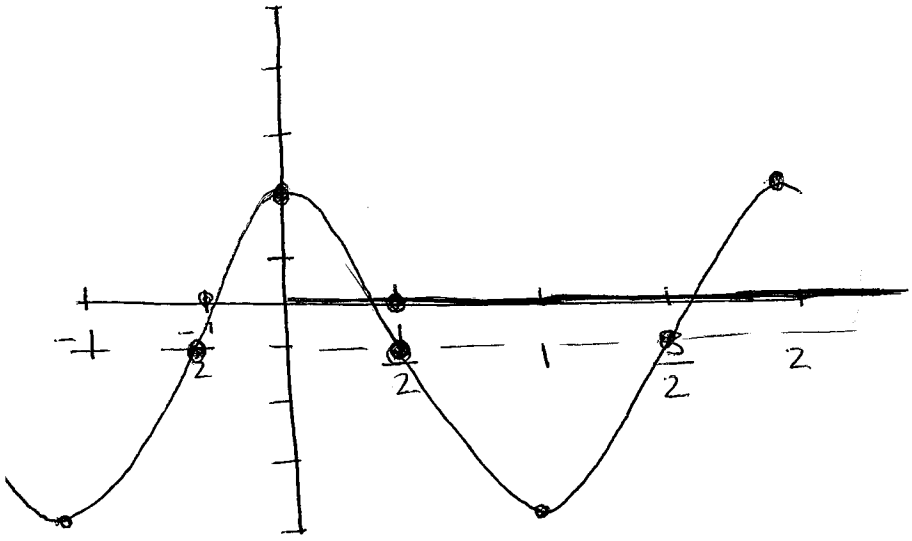
amplitude = 2

midline = 3 (vertical shift up 1)

(3b) $y = 3\cos(\pi x) - 1$

period = $\frac{2\pi}{\pi} = 2$

scale: $\frac{2}{4} = \frac{1}{2}$



amplitude = 3

midline = -1 (vertical shift down 1)