

5.3 Solving Trig Equations Practice Worksheet #1
Pre-calculus

Name: Key
Date: 0 Block:

Solve for the unknown variable on the interval $0 \leq x < 2\pi$.

1. $4 \cos^2 x - 3 = 0$

$\cos^2 x = \frac{3}{4}$

$\cos x = \pm \frac{\sqrt{3}}{2}$

$x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$

2. $\sqrt{2} \sin 2x = 1$

$\sin 2x = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

$2x = \frac{\pi}{4} + 2\pi n$

$2x = \frac{3\pi}{4} + 2\pi n$

$x = \frac{\pi}{8}, \frac{9\pi}{8}, \frac{3\pi}{8}, \frac{11\pi}{8}$ (with "add π " annotations)

5. $\sin x - 2 \sin x \cos x = 0$

$\sin x (1 - 2 \cos x) = 0$

$\sin x = 0$ $\cos x = \frac{1}{2}$

$x = 0, \pi, \frac{\pi}{3}, \frac{5\pi}{3}$

3. $3 \cot^2 x - 1 = 0$

$\cot^2 x = \frac{1}{3}$

$\cot x = \pm \frac{1}{\sqrt{3}}$

$\tan x = \pm \sqrt{3}$

$x = \frac{\pi}{3}, \frac{4\pi}{3}$

6. $2 \sin^2 x - \sin x - 3 = 0$

$(2 \sin x - 3)(\sin x + 1) = 0$

~~$\sin x = \frac{3}{2}$~~ $\sin x = -1$

$x = \frac{3\pi}{2}$

4. $\cos^3 x = \cos x$

$\cos^3 x - \cos x = 0$

$\cos x (\cos^2 x - 1) = 0$

$\cos x = 0$ $\cos x = \pm 1$

$x = \frac{\pi}{2}, \frac{3\pi}{2}, 0, \pi$

7. $\csc^2 x - \csc x - 2 = 0$

$(\csc x - 2)(\csc x + 1) = 0$

$\csc x = 2$ $\csc x = -1$
 $\sin x = \frac{1}{2}$ $\sin x = -1$

$x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{3\pi}{2}$

8. $\cos^2 x = 1 - \sin x$

$1 - \sin^2 x = 1 - \sin x$

$0 = \sin^2 x - \sin x$

$0 = \sin x (\sin x - 1)$

$\sin x = 0$ $\sin x = 1$

$x = 0, \pi, \frac{\pi}{2}$

Solve for the unknown variable on the given interval.

9. $\sqrt{3} + \tan(2x) = 0$ on $[0, 2\pi)$.

$\tan(2x) = -\sqrt{3}$

$2x = \frac{2\pi}{3} + 2\pi n$

$2x = \frac{5\pi}{3} + 2\pi n$

$x = \frac{\pi}{3} + \pi n$

$x = \frac{5\pi}{6} + \pi n$

$x = \frac{\pi}{3}$

$\frac{4\pi}{3}$

$\frac{5\pi}{6}$

$\frac{11\pi}{6}$

10. $\cos(\pi x) = 0.5$ on $[0, 2)$.

$\pi x = \frac{\pi}{3} + 2\pi n$

$\pi x = -\frac{\pi}{3} + 2\pi n$

$x = \frac{1}{3} + 2n$

$x = -\frac{1}{3} + 2n$

$x = \frac{1}{2}, \frac{5}{2}$

11. $\sin\left(\frac{x}{2}\right) - 1 = 0$ on $[0, 8\pi)$.

$\sin\left(\frac{x}{2}\right) = 1$

$\frac{x}{2} = \frac{\pi}{2} + 2\pi n$

$x = \pi + 4\pi n$

$x = \pi, 5\pi, \dots$

Part 1: Solve for the unknown variable. Give all of the exact general solutions.

1. $\sin \theta = \frac{\sqrt{2}}{2}$

$$\theta = \frac{\pi}{4} + 2\pi n$$

$$= \frac{3\pi}{4} + 2\pi n$$

4. $1 + \sin \theta = 2 \cos^2 \theta$

$$1 + \sin \theta = 2 - 2 \sin^2 \theta$$

$$2 \sin^2 \theta + \sin \theta - 1 = 0$$

$$(2 \sin \theta - 1)(\sin \theta + 1) = 0$$

$$\sin \theta = \frac{1}{2} \quad \sin \theta = -1$$

$$\theta = \frac{\pi}{6} + 2\pi n \quad \theta = \frac{3\pi}{2} + 2\pi n$$

$$\theta = \frac{5\pi}{6} + 2\pi n$$

7. $\sin^2 \theta - 1 = 0$

$$\sin^2 \theta = 1$$

$$\sin \theta = \pm 1$$

$$\theta = \frac{\pi}{2} + 2\pi n$$

$$\theta = \frac{3\pi}{2} + 2\pi n$$

10. $\tan 4\theta = -1$

$$4\theta = \frac{3\pi}{4} + 2\pi n$$

$$4\theta = \frac{7\pi}{4} + 2\pi n$$

$$\theta = \frac{3\pi}{16} + \frac{\pi}{2} n$$

$$\theta = \frac{7\pi}{16} + \frac{\pi}{2} n$$

2. $\cos \theta = \sin \theta$

$$\theta = \frac{\pi}{4} + 2\pi n$$

5. $2 \cos^2 \theta + \cos \theta = 0$

$$\cos \theta (2 \cos \theta + 1) = 0$$

$$\cos \theta = 0 \quad \cos \theta = -\frac{1}{2}$$

$$\theta = \frac{\pi}{2} + 2\pi n, \quad \theta = \frac{2\pi}{3} + 2\pi n$$

$$= \frac{3\pi}{2} + 2\pi n, \quad \theta = \frac{4\pi}{3} + 2\pi n$$

8. $\cos 2\theta = \frac{1}{2}$

$$2\theta = \frac{\pi}{3} + 2\pi n$$

$$2\theta = -\frac{\pi}{3} + 2\pi n$$

$$\theta = \frac{\pi}{6} + \pi n$$

$$\theta = -\frac{\pi}{6} + \pi n$$

11. $\tan^2 3x = 3$

$$\tan 3x = \pm \sqrt{3}$$

$$3x = \frac{\pi}{3} + 2\pi n = \frac{\pi}{9} + \frac{2\pi}{3} n$$

$$3x = \frac{2\pi}{3} + 2\pi n = \frac{2\pi}{9} + \frac{2\pi}{3} n$$

$$3x = \frac{4\pi}{3} + 2\pi n = \frac{4\pi}{9} + \frac{2\pi}{3} n$$

$$3x = \frac{5\pi}{3} + 2\pi n = \frac{5\pi}{9} + \frac{2\pi}{3} n$$

3. $\tan \theta = 1$

$$\theta = \frac{\pi}{4} + 2\pi n$$

$$= \frac{5\pi}{4} + 2\pi n$$

6. $\sin 3\theta = -1$

$$3\theta = \frac{3\pi}{2} + 2\pi n$$

$$\theta = \frac{3\pi}{6} + \frac{2\pi}{3} n$$

$$\theta = \frac{\pi}{2} + \frac{2\pi}{3} n$$

9. $2 \sin^2 \theta - \sin \theta - 1 = 0$

$$(2 \sin \theta + 1)(\sin \theta - 1) = 0$$

$$\sin \theta = -\frac{1}{2} \quad \sin \theta = 1$$

$$\theta = \frac{7\pi}{6} + 2\pi n \quad \theta = \frac{\pi}{2} + 2\pi n$$

$$\theta = -\frac{\pi}{6} + 2\pi n$$

12. $\cos \frac{x}{2} = \frac{\sqrt{2}}{2}$

$$\frac{x}{2} = \frac{\pi}{4} + 2\pi n$$

$$\frac{x}{2} = -\frac{\pi}{4} + 2\pi n$$

$$x = \frac{\pi}{2} + 4\pi n$$

$$x = -\frac{\pi}{2} + 4\pi n$$