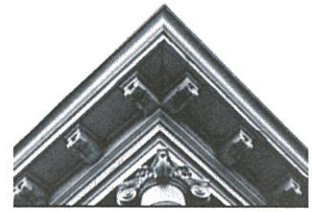
3.3H Warm Up

Special Rights – A Solidify Understanding Task



In previous courses you have studied the Pythagorean Theorem and right triangle trigonometry. Both of these mathematical tools are useful when trying to find missing sides of a right triangle.

1. What do you need to know about a right triangle in order to use the Pythagorean Theorem?

length of 2 sides

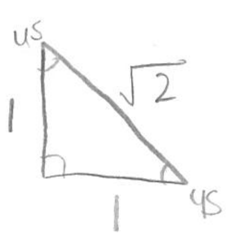
2. What do you need to know about a right triangle in order to use right triangle trigonometry?

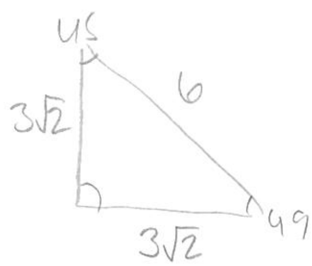
a side and an angle

While using the Pythagorean Theorem is fairly straight forward (you only have to keep track of the legs and hypotenuse of the triangle), right triangle trigonometry generally requires a calculator to look up values of different trigonometry ratios. There are some right triangles, however, for which knowing a side length and an angle measure is enough to calculate the value of the other sides without using trigonometry. These are known as ***special right triangles*** because their side lengths can be found by relating them to another geometric figure for which we know a great deal about its sides.

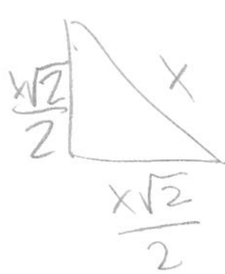
<u>One type of special right triangle is a 45° – 45° – 90° triangle</u>.

3. Draw a 45° – 45° – 90° triangle and assign a specific value to one of its sides. (For example, let one of the legs measure 5 cm, or choose to let the hypotenuse measure 8 inches. You will want to try both approaches to perfect your strategy.) Now that you have assigned a measurement to one of the sides of your triangle, find a way to calculate the exact measures of the other two sides. As part of your strategy, you may want to relate this triangle to another geometric figure that may be easier to think about.





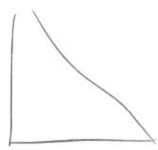
4. Generalize your strategy for a 45° – 45° – 90° triangle by letting one side of the triangle measure *x*. Show how the exact measures of the other two sides can be represented in terms of *x*. Make sure to consider cases where *x* is the length of a leg, as well as the case where *x* is the length of the hypotenuse.

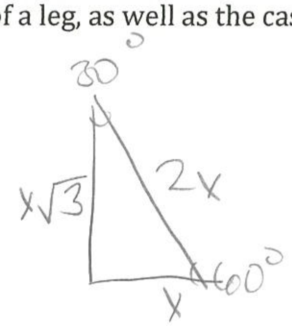


Another type of special right triangle is a 30° – 60° – 90° triangle.

5. Draw a 30° – 60° – 90° triangle and assign a specific value to one of its sides. Find a way to calculate the exact measures of the other two sides. As part of your strategy, you may want to relate this triangle to another geometric figure that may be easier to think about.

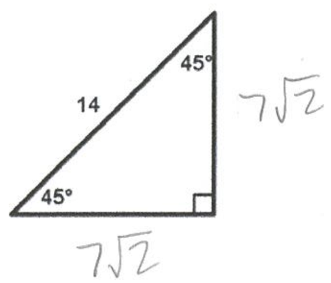
6. Generalize your strategy for 30° – 60° – 90° triangles by letting one side of the triangle measure *x*. Show how the exact measures of the other two sides can be represented in terms of *x*. Make sure to consider cases where *x* is the length of a leg, as well as the case where *x* is the length of the hypotenuse.





Find the missing sides of each special right triangle using the $45° - 45° - 90°$ or $30° - 60° - 90°$ triangle rules. Leave answers with simplified radicals, where necessary.

7.



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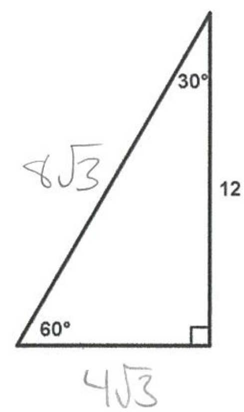

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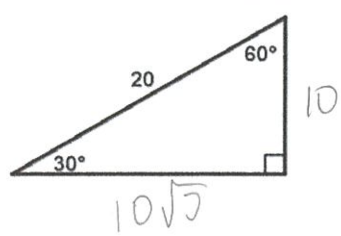

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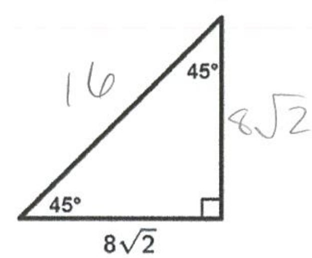

3.3H More Than Right
A Develop Understanding Task

We can use right triangle trigonometry and the Pythagorean Theorem to solve for missing sides and angles in a ***right triangle***. What about ***other*** triangles? How might we find unknown sides and angles in acute or obtuse triangles if we only know a few pieces of information about them?

In the previous homework and in today's warm up, we found it might be helpful to create right triangles by drawing an altitude in a non-right triangle. We can then apply trigonometry or the Pythagorean Theorem to the smaller right triangles, which may help us learn something about the sides and angles in the larger triangle.

See if you can devise a strategy for finding the missing sides and angles of each of these triangles.

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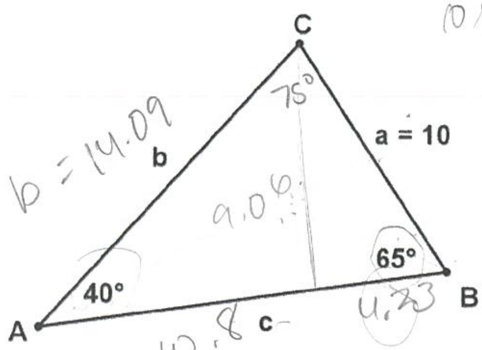


$c = 15.02$

$10 \times \sin 65 = \frac{x}{10} \times 10$

$x^2 + 9.06^2 = 100$

$x^2 = 17.86$

$x = 4.23$

$\sin 40 = \frac{9.06}{b}$

$\frac{9.06}{\sin 40} = b$

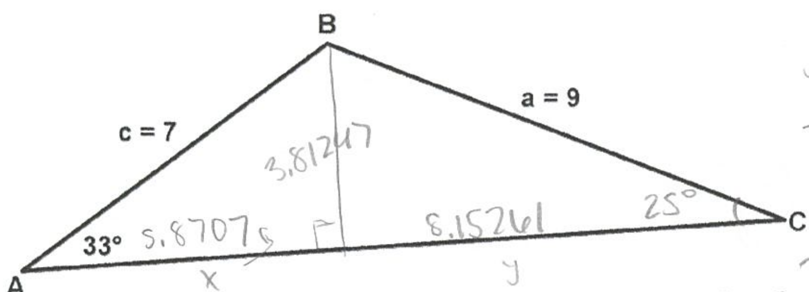
$b = 14.09$

$c^2 + 9.06^2 = 14.09^2$

$c^2 + 82.08 = 198.67$

$\boxed{\frac{\sin 40}{10} = \frac{\sin 65}{b}}$

2.



$\sin 33 = \frac{x}{7}$

$7 \sin 33 = x$

$x = 3.81247$

$\sin C = \frac{3.81247}{9}$

$C = 25.06$

$x^2 + 3.81247^2 = 7^2$

$x^2 = 34.46507$

$x = 5.8707$

$(3.81247)^2 + y^2 = 81$

$y = 8.15261$

$\overline{AC} = 14.0233$

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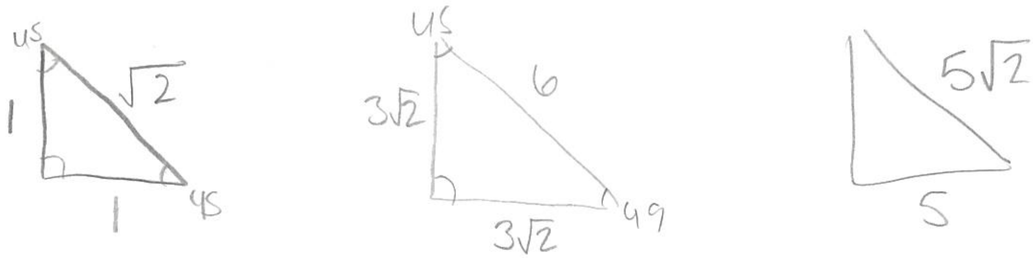

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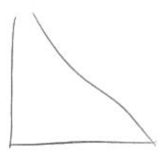

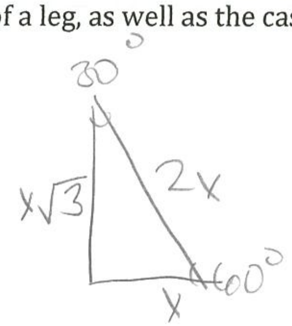

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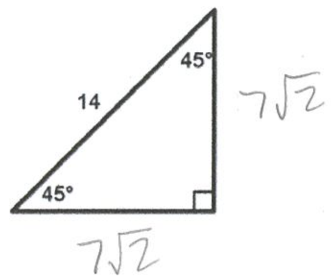

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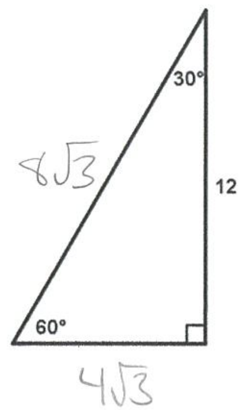

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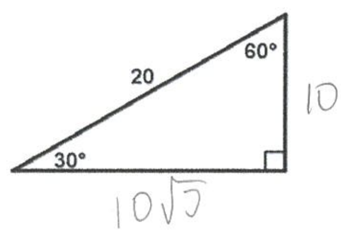

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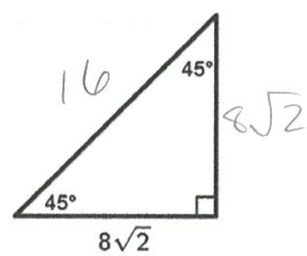

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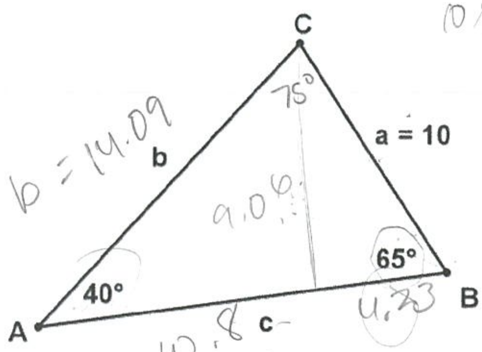

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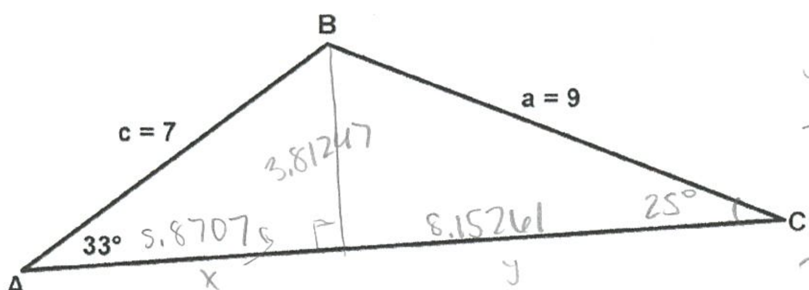

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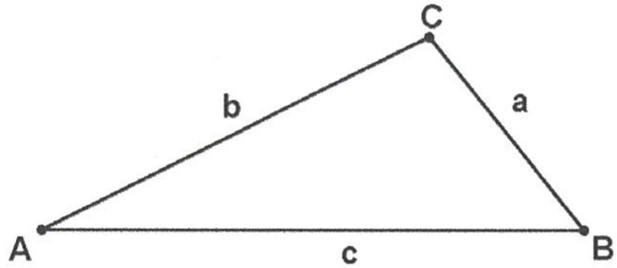
3. See if you can generalize the work you have done on questions 1 and 2 by finding relationships between sides and angles in the following diagram. Unlike the previous two questions, this triangle contains an ***obtuse angle*** at *C*. Find as many relationships as you can between sides *a, b,* and *c* and the related angles *A, B,* and *C.*



LAW OF SINES

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

LAW OF COSINES

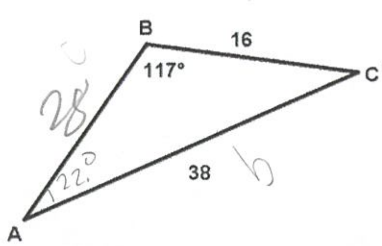
$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$b^2 = a^2 + c^2 - 2ac\cos B$$

$$c^2 = a^2 + b^2 - 2ab\cos C$$

Use the Law of Sines and/or the Law of Cosines to find the missing side lengths and angle measures in each triangle below. Round your answers to the nearest tenth.

4.



$\frac{c}{\sin 41} = \frac{38}{\sin 117}$

$\frac{c}{\sin 41} = 42.6484$

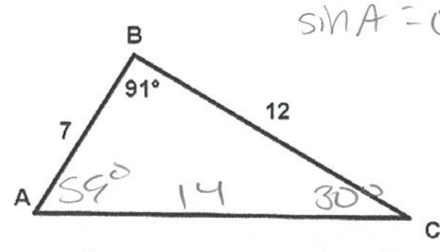
$c = 28$

$\frac{\sin A}{16} = \frac{\sin 117}{38}$

$\sin A = .37516$

$\boxed{A = 22.03}$

5.



$\frac{\sin A}{12} = \frac{\sin 91}{14}$

$\sin A = 0.85701$

$b^2 = 144 + 49 - (168)(-.01745)$

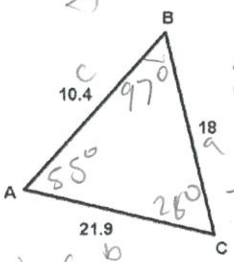
$b^2 = 144 + 49 + 2.932$

$\sqrt{b^2} = 195.932$

$b = 14$

use law of cosines for each

6.



$324 = 479.61 + 108.16 - 455.52(\cos A)$

$324 = 587.77 - 455.52(\cos A)$

$\dfrac{-263.77 = -455.52(\cos A)}{-455.52}$

$.57905 = \cos A$

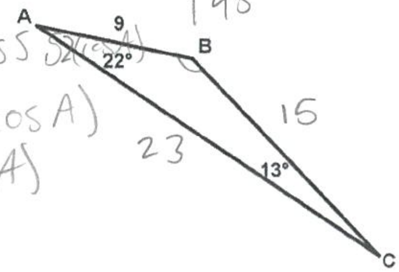
$A = 54.6$

$\dfrac{\sin 55}{18} = \dfrac{\sin C}{10.4}$

$.4732 = \sin C$

$C = 28.2$

7.



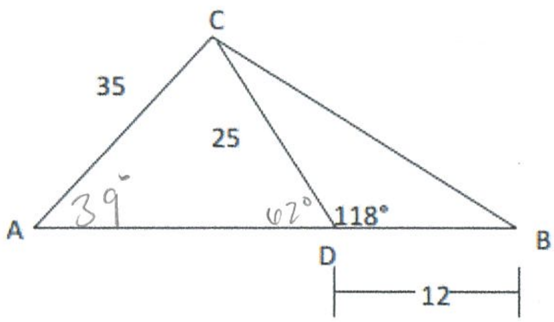
$\dfrac{9}{\sin 13} = \dfrac{a}{\sin 22}$

$a = 15$

$\dfrac{b}{\sin 145} = \dfrac{9}{\sin 13}$

$b = 23$

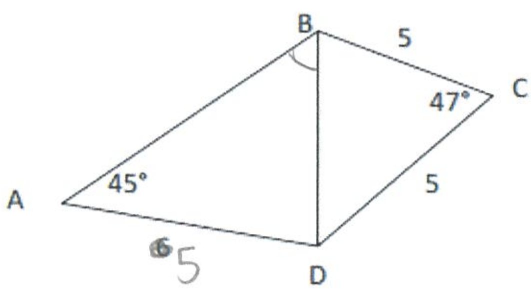
8. Find angle A.



$\dfrac{\sin 62}{35} = \dfrac{\sin A}{25}$

$\sin A = .63068$

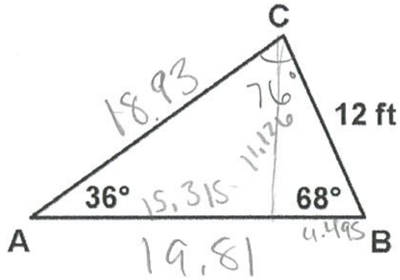
9. Find angle ABD.



3.5H Triangle Areas by Trig
A Practice Understanding Task

Find the area of the following two triangles using the strategies and procedures you have developed in the past few tasks. For example, draw an altitude as an auxiliary line, use right triangle trigonometry, use the Pythagorean Theorem, or use the Law of Sines or the Law of Cosines to find needed information.

1. Find the area of this triangle.



$\frac{12}{\sin 36} = \frac{b}{\sin 68}$

$b = 18.93$

$\frac{12}{\sin 36} = \frac{c}{\sin 76}$

$c = 19.81$

$12(\sin 68) = 11.126$

$12(\cos 68) = 4.495$

$\frac{1}{2}(15.315)(11.126)$

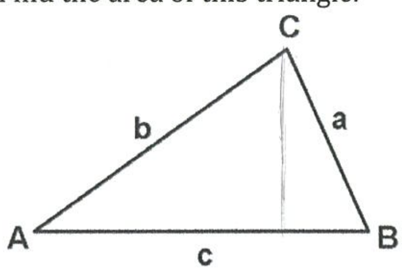
85.197

$\frac{1}{2}(4.495)(11.126)$

25.006

110.203

2. Find the area of this triangle.



$\frac{c \cdot (\sin A)(B)}{2}$

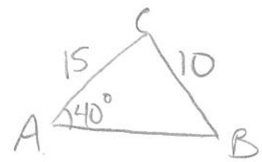
$\frac{1}{2} ab \sin C$

side side angle between

Jumal and Jabari are helping Jumal's father with a construction project. He needs to build a triangular frame as a component of the project, but he has not been given all the information he needs to cut and assemble the pieces of the frame. He is even having a hard time envisioning the shape of the triangle from the information he has been given.

Here is the information about the triangle that Jumal's father has been given:

- Side $a = 10.00$ meters
- Side $b = 15.00$ meters
- $\angle A = 40.0°$



Jumal's father has asked Jumal and Jabari to help him find the measure of the other two angles and the missing side of this triangle.

3. Carry out each student's described strategy. Then draw a diagram showing the shape and dimensions of the triangle that Jumal's father should construct. (Note: To provide as accurate information as possible, Jumal and Jarbari decide to round all calculated sides to the nearest centimeter, that is, to the nearest hundredth of a meter, and all angle measures to the nearest tenth of a degree.

<u>Jumal's Approach</u>

- Find the measure of $\angle B$ using the Law of Sines

$$\frac{\sin 40}{10} = \frac{\sin B}{15}$$

$\sin B$

$B = 74.6$

- Find the measure of the third $\angle C$

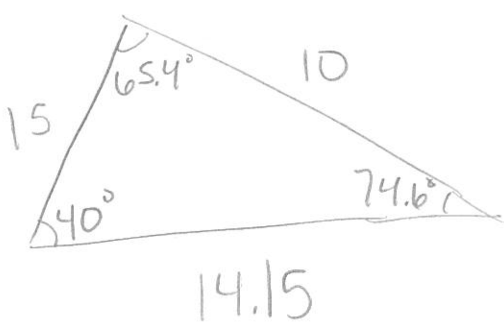
$65.4°$

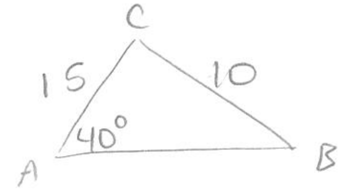
- Find the measure of side c using the Law of Sines

$$\frac{10}{\sin 40} = \frac{c}{\sin 65.4}$$

$c = 14.15$

- Draw the triangle





Jabari's Approach

- Solve for c using the Law of Cosines: $a^2 = b^2 + c^2 - 2bc\cos A$

$100 = 225 + c^2 - 30c(\cos 40)$

$100 = 225 + c^2 - 29.92692c$

$-125 = c^2 - 30(\cos 40)c + 125$

$0 = c^2 - 22.98133c + 125$

$c = \dfrac{22.98 \pm \sqrt{528.14168 - 500}}{2}$

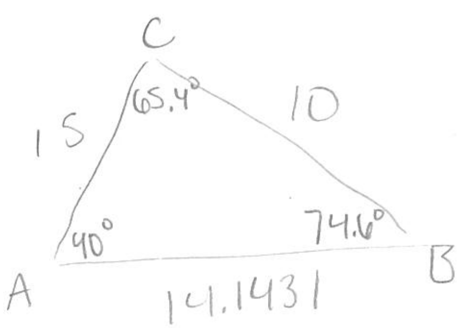
$c = \dfrac{22.98133 \pm 5.30487}{2}$

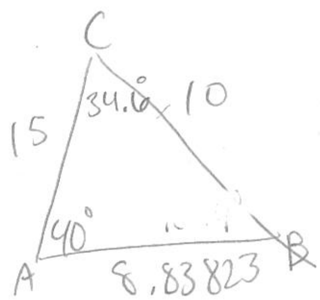
$c = 8.83823$

$\boxed{c \approx 14.1431}$

- Jabari is surprised that this approach leads to a quadratic equation, which he solves with the quadratic formula. He is even more surprised when he finds **two reasonable solutions** for the length of side *c*.

Draw both possible triangles and find the two missing angles of each using the Law of Sines.





$\dfrac{\sin 40}{10} = \dfrac{\sin C}{14.1431}$

$\dfrac{\sin 40}{10} = \dfrac{\sin C}{8.83823}$

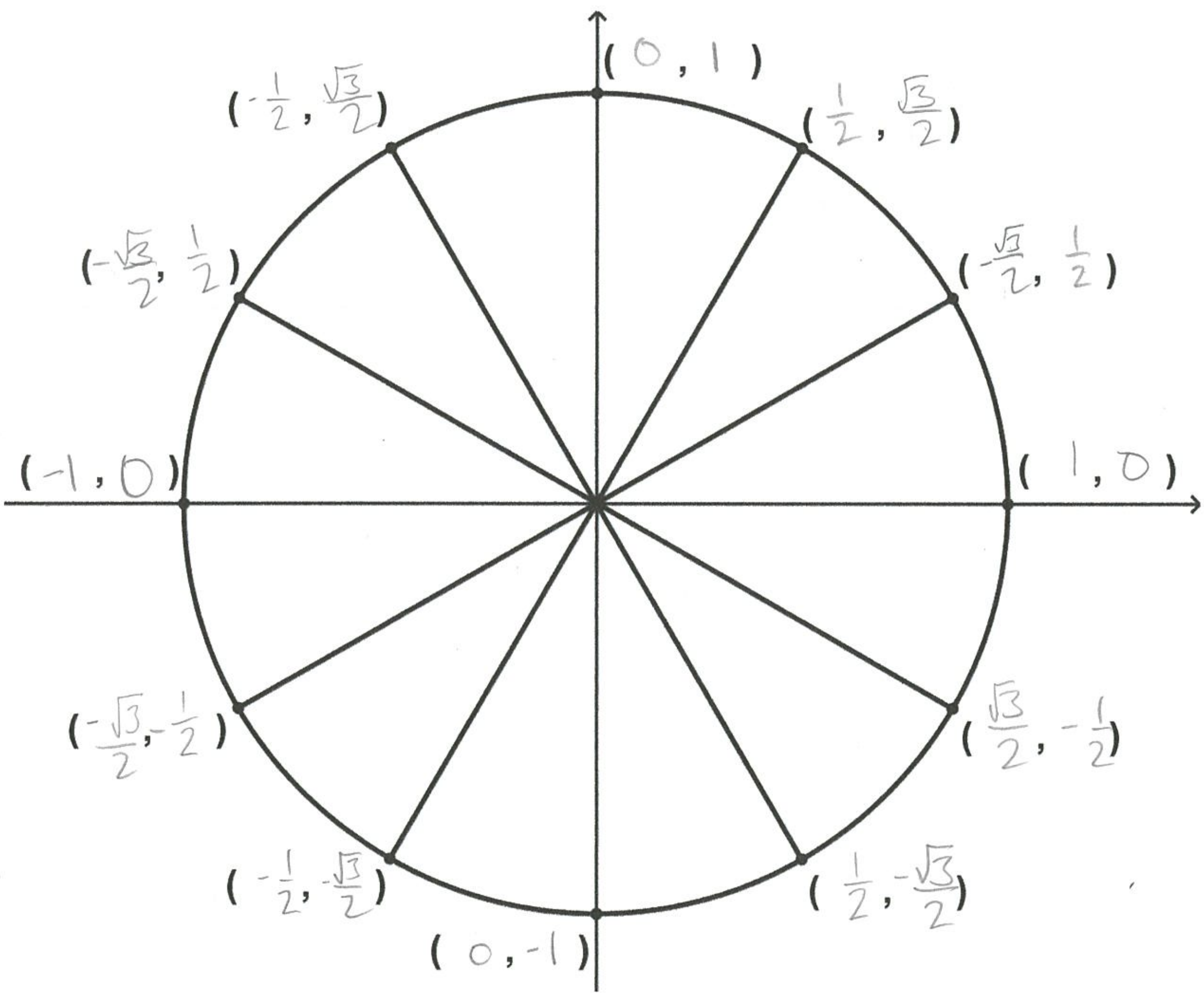
$\dfrac{\sin 40}{10} = \dfrac{\sin B}{15}$?

$B = 74.6$?

4.1H Warm Up
Trigonometric Functions – finding exact values

Explain the relationship between degrees and radians

Complete the unit circle below using your understanding of degrees, radians, and special right triangles



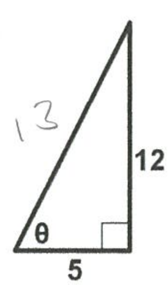
Review Trigonometric Ratios

Find the trigonometric ratios for the given triangles. Simplify all answers completely.

$\sin\theta = \frac{12}{13}$

$\cos\theta = \frac{5}{12}$

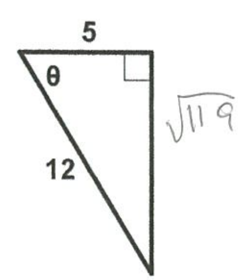
$\tan\theta = \frac{12}{5}$



$\csc\theta = \frac{12\sqrt{119}}{12}$

$\sec\theta = \frac{12}{5}$

$\cot\theta = \frac{5\sqrt{119}}{119}$



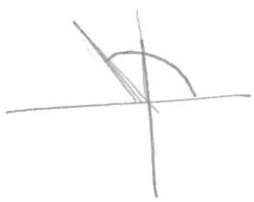
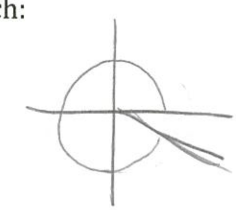
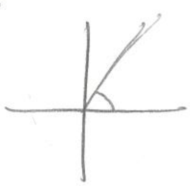
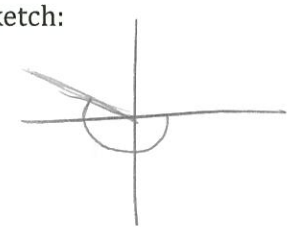
An angle is in **standard position** if its vertex is located at the origin and one ray is on the positive x-axis. The ray on the x-axis is called the **initial side** and the other ray is called the **terminal side.**

An angle, in standard position, that is rotated counter clockwise has a **positive measurement**. An angle, in standard position, that is rotated clockwise has a **negative measurement**.

If θ is an angle in standard position, its **reference angle** is the acute angle formed by the terminal side of θ and the x-axis.

Two angles, in standard position, are **coterminal** if they share a terminal side.

For each angle measurement below, (A) sketch the angle in standard position, (B) identify the measure of the reference angle, and (C) identify the measure of an angle that is coterminal with the given angle.

5. 110° (A) Sketch:  (B) Reference Angle: 70° (C) Coterminal Angle: -250°	6. 345° (A) Sketch:  (B) Reference Angle: 15° (C) Coterminal Angle: -15°
7. 76° (A) Sketch:  (B) Reference Angle: 76° (C) Coterminal Angle: -284°	8. −192° (A) Sketch:  (B) Reference Angle: 12° (C) Coterminal Angle: 168°

α = reference angle
- always positive
- always acute
- always touching x-axis

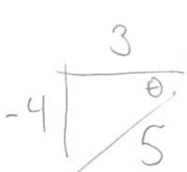
Find the exact values of the following.

1. cos(210) $-\frac{\sqrt{3}}{2}$	2. sin (330) $-\frac{1}{2}$	3. tan (−750) $\sqrt{3}$
4. $\sin\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}$	5. $\cos\left(\frac{3\pi}{2}\right) = 0$	6. $\tan(\pi) = 0$
7. $\tan\left(\frac{\pi}{4}\right) = 1$	8. $\sin\left(\frac{5\pi}{4}\right) = -\frac{\sqrt{2}}{2}$	9. $\cos\left(\frac{3\pi}{4}\right) = -\frac{\sqrt{2}}{2}$
10. $\sin\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$	11. $\cos\left(-\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$	12. $\tan\left(\frac{\pi}{2}\right) =$ undefined
13. $\cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$	14. $\tan -\frac{11\pi}{6} = \frac{\sqrt{3}}{3}$	15. $\sin\frac{7\pi}{3} = \frac{\sqrt{3}}{2}$
16. $\sin\left(\frac{\pi}{6}\right) = \frac{1}{2}$	17.$\tan\left(\frac{5\pi}{6}\right) = -\frac{\sqrt{3}}{3}$	18. $\cos\left(\frac{13\pi}{6}\right) = \frac{\sqrt{3}}{2}$

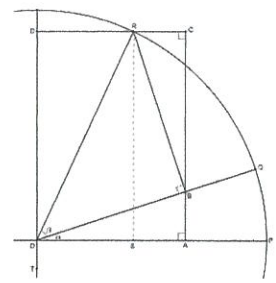
$\frac{1}{2} \cdot \frac{2}{\sqrt{3}} =$

Plugin 180 for π to convert radians to degrees (or multiply by $\frac{180}{\pi}$)

Find the exact values of the following.

19. Given $\sin\theta = -\frac{4}{5}$ and $sec\theta > 0$; find	20. Given $\tan\theta = \frac{3}{5}$ and $csc\theta < 0$; find
 $\sin\theta = -\frac{4}{5}$ $\csc\theta = -\frac{5}{4}$ $\cos\theta = \frac{3}{5}$ $\sec\theta = \frac{5}{3}$ $\tan\theta = -\frac{4}{3}$ $\cot\theta = -\frac{3}{4}$	 $\sin\theta = \frac{3\sqrt{34}}{34}$ $\csc\theta = \frac{\sqrt{34}}{3}$ $\cos\theta = \frac{5\sqrt{34}}{34}$ $\sec\theta = \frac{\sqrt{34}}{5}$ $\tan\theta = \frac{3}{5}$ $\cot\theta = \frac{5}{3}$

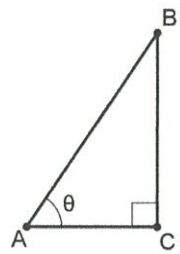
4.1H Understanding Identities
A Practice Understanding Task



Part I: Fundamental Trigonometric Identities
Right triangles and the unit circle provide images that can be used to derive, explain, and justify a variety of trigonometric identities.

For example, how might the right triangle diagram below help you justify why the following identity is true for all angles θ between 0° and 90°?

$$\sin\theta = \cos(90° - \theta)$$

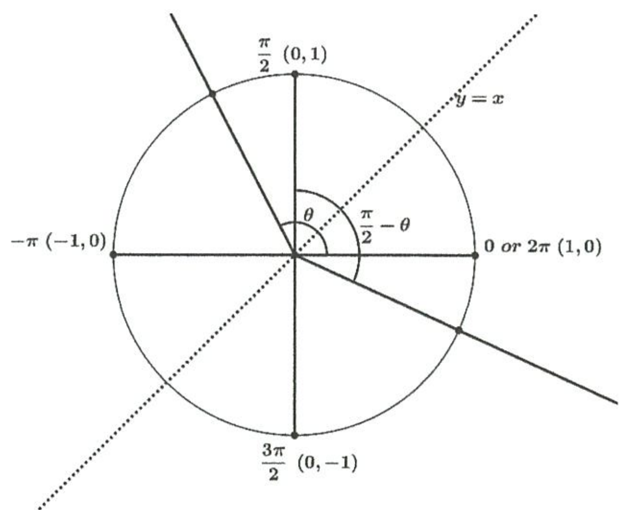


Since we have extended our definition of the sine to include angles of rotation, rather than just the acute angles in a right triangle, we might wonder if this identity is true for all angles θ, not just those that measure between 0° and 90°?

A version of this identity that uses radian rather than degree measure would look like this:

$$\sin\theta = \cos\left(\frac{\pi}{2} - \theta\right)$$

How might you use this unit circle diagram to justify why this identity is true for all angles θ?



Here are some additional trigonometric identities. Use either a right triangle diagram or a unit circle diagram to justify why each is true.

1. $\sin(-\theta) = -\sin\theta$

2. $\cos(-\theta) = \cos\theta$

3. $\sin^2\theta + \cos^2\theta = 1$ [Note: This is the preferred notation for $(\sin\theta)^2 + (\cos\theta)^2 = 1$]

4. $\frac{\sin\theta}{\cos\theta} = \tan\theta$

5. $\tan(-\theta) = -\tan\theta$

Part II: More Trigonometric Identities

Luna and Happ are in math class trying to solve sin(105°), they decide that there is no way they can solve this problem without using a calculator. Shultis joins their group and realizes 105° could be broken up into two angles that they know how to find the exact value of using a sum.

Which two angles add up to 105° so sin(105°) can be found without using a calculator?

Shultis gives Luna and Happ the formula sheet they forgot to pick up on the way into class.

Half-Angle Formulas	$\sin\frac{u}{2} = \pm\sqrt{\frac{1-\cos u}{2}}$	$\cos\frac{u}{2} = \pm\sqrt{\frac{1+\cos u}{2}}$	$\tan\frac{u}{2} = \frac{1-\cos u}{\sin u} = \frac{\sin u}{1+\cos u}$
Double Angle Formulas	$\sin 2u = 2\sin u\cos u$ $\cos 2u = \cos^2 u - \sin^2 u = 2\cos^2 u - 1 = 1 - 2\sin^2 u$ $\tan 2u = \frac{2\tan u}{1-\tan^2 u}$		
Sum and Difference Formulas	$\sin(u+v) = \sin u\cos v + \cos u\sin v$ $\sin(u-v) = \sin u\cos v - \cos u\sin v$ $\cos(u+v) = \cos u\cos v - \sin u\sin v$ $\cos(u-v) = \cos u\cos v + \sin u\sin v$	$\tan(u+v) = \frac{\tan u+\tan v}{1-\tan u\tan v}$ $\tan(u-v) = \frac{\tan u-\tan v}{1+\tan u\tan v}$	

Using the formula $\sin(u+v) = \sin u\cos v + \cos u\sin v$, Shultis solves the problem:

$$\sin(60+45) = \sin 60\cos 45 + \cos 60\sin 45$$

$$\sin(60+45) = \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) + \left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$$

$$\sin(60+45) = \left(\frac{\sqrt{6}}{4}\right) + \left(\frac{\sqrt{2}}{4}\right) = \left(\frac{\sqrt{6}+\sqrt{2}}{2}\right)$$

However, using the calculator Happ and Luna got 0.9659258...

Did they get the same answer?

Use the formulas above to solve the following problems.

1. Use the angle sum or difference of angles formulas to answer the following questions.

   a. Find $\sin(75°)$

   $\sin(30+45) = \sin 30 \cos 45 + \cos 30 \sin 45$

   $= (\frac{1}{2})(\frac{\sqrt{2}}{2}) + (\frac{3}{2})(\frac{\sqrt{2}}{2}) = \frac{\sqrt{2}+\sqrt{6}}{4}$

   b. Find $\cos(195°)$

   $= \cos(150+45)°$

   $= \cos 150 \cos 45 - \sin 150 \sin 45$

   $= (-\frac{\sqrt{3}}{2})(\frac{\sqrt{2}}{2}) - (\frac{1}{2})(\frac{\sqrt{2}}{2})$

   $= \frac{-\sqrt{6}-\sqrt{2}}{4}$

The group continues with the classwork and comes across the questions $\sin(165)$. Luna uses the same strategy as before

Luna's work:

$$\sin(120 + 45) = \sin 120 \cos 45 + \cos 120 \sin 45$$

Happ says, I agree but we could also use the half angle formula since $\frac{330}{2}$=165

Happ's work:

$$sin\frac{330}{2} = \pm\sqrt{\frac{1-\cos(330)}{2}}$$

$$sin\frac{330}{2} = \pm\sqrt{\frac{1-\frac{\sqrt{3}}{2}}{2}} =$$

2. If $\sin A = -\frac{3}{5}$ with $\pi < A < \frac{3\pi}{2}$ and $\cos B = \frac{12}{13}$ with $0 < B < \frac{\pi}{2}$, find...

   a. $\cos(A + B)$

   $= \cos A \cos B - \sin A \sin B$

   $= (-\frac{4}{5})(\frac{12}{13}) + (-\frac{3}{5})(\frac{5}{13})$

   $= -\frac{33}{65}$

   b. $\cos(2A)$

   $= 1 - 2\sin^2 A$

   $= 1 - 2(\frac{9}{25})$

   $= 1 - \frac{18}{25}$

   $= \frac{7}{25}$

   c. $\sin(A - B)$

   $= \sin A \cos B - \cos A \sin B$

   $= (-\frac{3}{5})(\frac{12}{13}) - (-\frac{4}{5})(\frac{5}{13})$

   $= -\frac{16}{65}$

   d. $\sin(2A)$

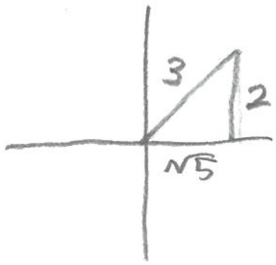
   $= 2\sin A \cos A$

   $= 2(-\frac{3}{5})(-\frac{4}{5})$

   $= \frac{24}{25}$

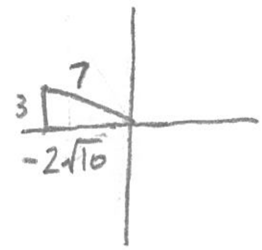
4.2H Warm-Up
Finding trig expressions

1. Find $\cos\theta$ if $\sin\theta = \frac{2}{3}$ and $0 \le \theta \le \frac{\pi}{2}$



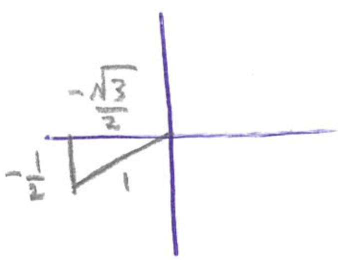
$\cos\theta = \boxed{\frac{\sqrt{5}}{3}}$

2. Find $\tan\theta$ if $\sin\theta = \frac{3}{7}$ and $\frac{\pi}{2} \le \theta \le \pi$



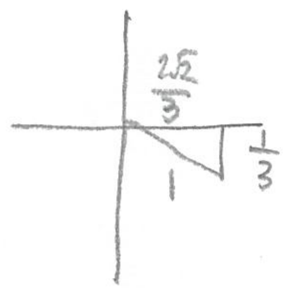
$\tan\theta = \frac{-3}{2\sqrt{10}} = \boxed{-\frac{3\sqrt{10}}{20}}$

3. Find $\csc\theta$ if $\cos\theta = \frac{-\sqrt{3}}{2}$ and $\pi \le \theta \le \frac{3\pi}{2}$



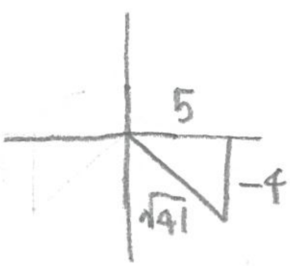
$\csc\theta = \frac{1}{\sin\theta} = \frac{1}{-\frac{1}{2}} = \boxed{-2}$

4. Find $\sec\theta$ if $\sin\theta = \frac{1}{3}$ and $\frac{3\pi}{2} \le \theta \le 2\pi$



$\sec\theta = \frac{1}{\cos\theta} = \frac{1}{\frac{2\sqrt{2}}{3}} = \boxed{\frac{3\sqrt{2}}{4}}$

5. If $\tan\theta = -\frac{4}{5}$, $270° < \theta < 360°$, find all the remaining functions of θ.

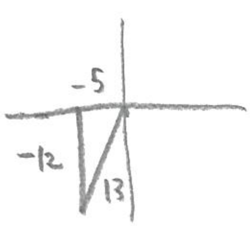


$\sin\theta = -\frac{4\sqrt{41}}{41}$ $\quad \sec\theta = \frac{\sqrt{41}}{5}$

$\cos\theta = \frac{5\sqrt{41}}{41}$ $\quad \cot\theta = -\frac{5}{4}$

$\csc\theta = -\frac{\sqrt{41}}{4}$

6. Find the values of the six trig. functions of θ, if θ is an angle in standard position with the point $(-5, -12)$ on its terminal ray.



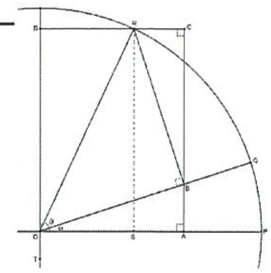
$\sin\theta = -\frac{12}{13}$ $\quad \csc\theta = -\frac{13}{12}$

$\cos\theta = -\frac{5}{13}$ $\quad \sec\theta = -\frac{13}{5}$

$\tan\theta = \frac{12}{5}$ $\quad \cot\theta = \frac{5}{12}$

4.2H More Identities

A Practice Understanding Task



Simplify the following:

1. $cos\,(270° - x) =$ $\overset{(0)}{\cos 270}\cos x + \sin 270 \sin x$

$= -\sin x$

2. $sin\,(x + \frac{\pi}{2}) =$ $\sin x \underset{(0)}{\cos\frac{\pi}{2}} + \cos x \sin\frac{\pi}{2}$

$= \cos x$

Verify the following trigonometric identities.

3. $\cot x + \tan x = \frac{1}{\sin x \cos x}$

$\frac{\cos}{\cos}\Big)\frac{\cos}{\sin} + \frac{\sin}{\cos}\Big(\frac{\sin}{\sin} = \frac{1}{\sin x \cos x}$

$\frac{\cos^2 + \sin^2}{\sin\cos} = \frac{1}{\sin x \cos x}$

$\frac{1}{\sin x \cos x} = \frac{1}{\sin x \cos x}$

4. $\frac{\sin^4 x - \cos^4 x}{\sin^2 x - \cos^2 x} = 1$

$\frac{(\sin^2 + \cos^2)\cancel{(\sin^2 - \cos^2)}}{\cancel{\sin^2 - \cos^2}} = 1$

$\sin^2 + \cos^2 = 1$

$1 = 1$

$\sin^2 + \cos^2 = 1$

5. $\frac{\tan^2 x}{\tan^2 x + 1} = \sin^2 x$

$\frac{\cos^2}{\cos^2} + \frac{\sin^2}{\cos^2} = \frac{1}{\cos^2}$

$1 + \tan^2 = \sec^2$

$\frac{\frac{\sin^2}{\cos^2}}{\sec^2} = \sin^2$

$\frac{\sin^2}{\cancel{\cos^2}} \cdot \cancel{\cos^2} = \sin^2$

$\sin^2 = \sin^2$

6. $\frac{\sin^2 x + 4\sin x + 3}{\cos^2 x} = \frac{3 + \sin x}{1 - \sin x}$

$\frac{(\sin + 3)(\sin + 1)}{1 - \sin^2} = \frac{3 + \sin}{1 - \sin}$

$\frac{(\sin + 3)\cancel{(\sin + 1)}}{\cancel{(1 + \sin)}(1 - \sin)} = \frac{3 + \sin}{1 - \sin}$

$\frac{\sin + 3}{1 - \sin} = \frac{3 + \sin}{1 - \sin}$

$2\sin\cos$

7. $\sin 2x = \frac{2\tan x}{1+\tan^2 x}$

$$\sin 2x = \frac{2\frac{\sin x}{\cos x}}{\sec^2}$$

$$\sin 2x = \frac{2\sin x}{\cancel{\cos x}} \cdot \cos^{\cancel{2}}$$

$$\sin 2x = 2\sin x\cos x$$

$\cancel{2\sin x}$ $\sin 2x = \sin 2x$

8. $1 + \sin 2x = (\sin x + \cos x)^2$

$$\frac{1+\sin 2x}{\cancel{1+2\sin\cos}} = \sin x^2 + 2\sin x\cos x + \cos$$

$$1+\sin 2x = \cos^2 + \sin^2 + 2\sin\cos$$

$$1+\sin 2x = 1+\sin 2x$$

9. $\sin 2x = (\tan x)(1 + \cos 2x)$

$$\left(\frac{\sin}{\cos}\right)\left(1+(1-2\sin^2)\right)$$

$$\frac{\sin}{\cos}(2-2\sin^2)$$

$$\frac{2\sin}{\cos}(1-\sin^2)$$

$$\frac{2\sin}{\cos}(\cos^2)$$

$$\cancel{2}\sin 2x = 2\sin\cos$$

$$\cancel{2}\sin 2x = \sin 2x$$

10. $\cos 2x = \frac{1-\tan^2 x}{1+\tan^2 x}$

$$\cos 2x = \frac{1-\tan^2}{\sec^2}$$

$$\cos 2x = \left(1 - \frac{\sin^2}{\cos^2}\right) \cdot \cos^2$$

$$\cos 2x = \cos^2 - \sin^2$$

$$\cos 2x = \cos 2x$$

$\tan A = \frac{12}{13} \cdot \frac{13}{5} = \frac{12}{5}$

4.3H Warm-Up

$\tan B = -\frac{15}{17} \cdot -\frac{17}{8} = \frac{15}{8}$

Using sum, difference, double and half angle Identities

Find each of the following numbers:

$\cos A = \frac{5}{13}$ $\qquad$ $\sin B = -\frac{15}{17}$

If $\sin A = \frac{12}{13}$, $0 < A < \frac{\pi}{2}$ and $\cos B = -\frac{8}{17}$, $\pi < B < \frac{3}{2}\pi$

1. $\sin(A+B) = \sin A \cos B + \cos A \sin B$

$$\left(\frac{12}{13}\right)\left(-\frac{8}{17}\right) + \left(\frac{5}{13}\right)\left(-\frac{15}{17}\right) = \left(\frac{-96}{221}\right) + \left(\frac{-75}{221}\right) = \left(-\frac{171}{221}\right)$$

2. $\cos(A-B)$ $\cos A \cos B + \sin A \sin B$

$$\left(\frac{5}{13}\right)\left(-\frac{8}{17}\right) + \left(\frac{12}{13}\right)\left(-\frac{15}{17}\right) = \frac{-40}{221} + \frac{-180}{221} = \frac{-220}{221}$$

3. $\tan(A+B) = \dfrac{\tan A + \tan B}{1 - \tan A \tan B} = \dfrac{\frac{12}{5} + \frac{15}{8}}{1 - \left(\frac{12}{5}\right)\left(\frac{15}{8}\right)} = \dfrac{\frac{171}{40}}{1 - \frac{180}{40}} = -\dfrac{171}{140}$

4. Simplify:

67.5° in the first quadrant

(a) $\sin(67\tfrac{1}{2}°) = \sin\frac{135}{2} = \pm\sqrt{\frac{1-\cos 135}{2}} = \pm\sqrt{\frac{1+\frac{\sqrt{2}}{2}}{2}} = +\sqrt{\frac{1}{2}+\frac{\sqrt{2}}{4}}$

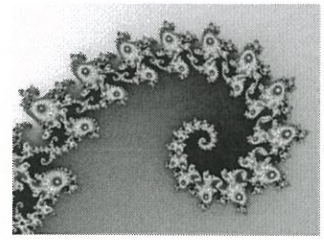
(b) $\cos\left(\frac{\pi}{8}\right) = \cos\left(\frac{45}{2}\right) = \pm\sqrt{\frac{1+\cos 45}{2}} = \pm\sqrt{\frac{1+\frac{\sqrt{2}}{2}}{2}} = \pm\sqrt{\frac{2+\sqrt{2}}{4}} = +\sqrt{\frac{2+\sqrt{2}}{4}}$

(c) $\sin\left(\frac{5\pi}{8}\right) = \sin\left(\frac{225}{2}\right) = \pm\sqrt{\frac{1-\cos 225}{2}} = \pm\sqrt{\frac{1+\frac{\sqrt{2}}{2}}{2}} = +\sqrt{\frac{2+\sqrt{2}}{4}}$

(d) $\cos(202\tfrac{1}{2}°) = \cos\left(\frac{405}{2}\right) = \pm\sqrt{\frac{1+\cos 405}{2}} = \pm\sqrt{\frac{1+\frac{\sqrt{2}}{2}}{2}} = -\sqrt{\frac{2+\sqrt{2}}{4}}$

4.3H How Many Solutions Can There Be?

A Practice Understanding Task



Throughout this module, you will investigate various periodic phenomena: finding the height of a rider on a Ferris Wheel, describing the high and low tide, and examining the body temperature of a newly discovered animal, just to name a few.

All of these situations involve the use of trigonometric equations to find particular solutions. Many times, these contexts offer an infinite amount of solutions. These solutions were typically evenly spaced. Many trigonometric equations have an infinite amount of solutions and some have no solutions. Furthermore, you have explored trigonometric identities, which have solutions for *any* given angle value.

Below are some basic trigonometric equations for you to solve on the interval $[0, 2\pi)$.

1. $\sin x = \frac{\sqrt{3}}{2}$

   $\frac{\pi}{3}, \frac{2\pi}{3}$ $(60°, 120°)$

2. $\cos x = \frac{\sqrt{2}}{2}$

   $\frac{\pi}{4}, \frac{7\pi}{4}$ $(45°, 315°)$

3. $\csc x = -2$ $\rightarrow \frac{1}{\sin x} = -\frac{1}{2}$

   $\frac{7\pi}{6}, \frac{11\pi}{6}$ $(210°, 330°)$

4. $\tan x = -1$

   $\frac{7\pi}{4}, \frac{3\pi}{4}$ $(135°, 315°)$

5. $\cos x = 1$

   0 $(0,1)$ $(-1,0)$ $(1,0)$ $(0,-1)$

6. $\sin x\,(1 + \cos x) = 0$

   $\sin x = 0$ $\quad \cos x = -1$

   $0, \pi$ $\quad \pi$

   $(0°, 180°)$

Because of the restricted domain given for the equations above, there were only a **finite number** of solutions for each problem. If there was no restriction on the domain, there would likely be an **infinite amount** of solutions.

7. Consider the equation: $\sin x = \frac{\sqrt{2}}{2}$

   a. What are the solutions in the domain $[0, 2\pi)$?

   $\frac{\pi}{4}, \frac{3\pi}{4}$ $(45°, 135°)$

   b. One of the solutions from part a is in the first quadrant on the unit circle. Find four more solutions that are also in the first quadrant if there is no domain restriction. (Hint: think of **coterminal** angles)

   $\frac{\pi}{4}, \frac{9\pi}{4}, \frac{17\pi}{4}, \frac{25\pi}{4}, \frac{33\pi}{4}$ $(+2\pi n)$

   c. In what quadrant are the other solutions located? List four of these solutions.

   Q II $\quad \frac{3\pi}{4} + 2\pi n$

   d. Generalize all solutions to the equation $\sin x = \frac{\sqrt{2}}{2}$.

   $\frac{\pi}{4} + 2\pi n, \frac{3\pi}{4} + 2\pi n$

Find all solutions to the following trigonometric equations. Be resourceful in your methods – you may want to consider factoring and using a trigonometric identity along the way.

8. $7\cos x + 9 = -2\cos x$

$7\cos x + 2\cos x + 9 = 0$

$9\cos x + 9 = 0$

$9(\cos x + 1) = 0$

$\cos x = -1$

$\boxed{x = \pi + 2\pi n}$

9. $(\tan x - 1)(\cos x + 1) = 0$

45 45

$\tan x = 1 \quad \cos x = -1$

$\boxed{x = \frac{\pi}{4} + 2\pi n \quad x = \pi + 2\pi n \quad \frac{5\pi}{4} + 2\pi n}$

10. $2\sin^2 x - \sin x - 1 = 0$

$(2\sin + 1)(\sin - 1) = 0$

30 30

$\sin x = -\frac{1}{2} \quad \sin x = 1$

$\boxed{x = \frac{7\pi}{6} + 2\pi n \quad x = \frac{11\pi}{6} + 2\pi n}$ $\boxed{x = \frac{\pi}{2} + 2\pi n}$

11. $\sec^2 x - 2 = 0$

$\sec^2 x = 2$

$\sec x = \sqrt{2} \quad \sec x = -\sqrt{2}$

$\cos x = \frac{\sqrt{2}}{2} \quad \cos x = \frac{-\sqrt{2}}{2}$

$\boxed{x = \frac{\pi}{4} + 2\pi n \quad x = \frac{3\pi}{4} + 2\pi n}$ $\boxed{x = \frac{5\pi}{4} + 2\pi n \quad x = \frac{7\pi}{4} + 2\pi n}$

12. $\cos 2x = \frac{\sqrt{3}}{2}$

30 30

$u = 2x$

$\cos u = \frac{\sqrt{3}}{2}$

$u = \frac{\pi}{6} \quad u = \frac{11\pi}{6}$

$2x = \frac{\pi}{6} + 2\pi n \quad 2x = \frac{11\pi}{6} + 2\pi n$

$\boxed{x = \frac{\pi}{12} + \pi n \quad x = \frac{11\pi}{12} + \pi n}$

Means it will repeat every π instead of 2π

13. $3\sin^2 x + 7\sin x + 4 = 0$

$(3\sin x + 4)(\sin x + 1) = 0$

$\sin x = -\frac{4}{3} \quad \sin x = -1$

not possible bigger than the unit circle

$\boxed{x = \frac{3\pi}{2} + 2\pi n}$

$2\sin x\cos x$

14. $\sin 2x = \sin x$

$2\sin x\cos x - \sin x = 0$

$\sin x(2\cos x - 1) = 0$

$\sin x = 0 \quad \cos x = \frac{1}{2}$

$$\boxed{\begin{array}{ll} x = 0 + 2\pi n & x = \frac{\pi}{3} + 2\pi n \\ x = \pi + 2\pi n & x = \frac{5\pi}{3} + 2\pi n \end{array}}$$

15. $\cos 2x + \cos x + 1 = 0$

$\cos^2 - \sin^2 + \cos + 1 = 0$

$\cos^2 - (1 - \cos^2) + \cos + 1 = 0$

$2\cos^2 + \cos = 0$

$\cos(2\cos + 1) = 0$

$\cos x = 0 \quad \cos x = -\frac{1}{2}$

$$\boxed{\begin{array}{ll} x = \frac{\pi}{2} + 2\pi n & x = \frac{2\pi}{3} + 2\pi n \\ x = \frac{3\pi}{2} + 2\pi n & x = \frac{4\pi}{3} + 2\pi n \end{array}}$$

16. $\sin\left(x + \frac{\pi}{4}\right) + \sin\left(x - \frac{\pi}{4}\right) = 1$

$\sin x\cos\frac{\pi}{4} + \cancel{\cos x\sin\frac{\pi}{4}}$

$+\left(\sin x\cos\frac{\pi}{4} - \cancel{\cos x\sin\frac{\pi}{4}}\right) = 1$

$2\left(\frac{\sqrt{2}}{2}\sin x\right) = 1$

$\sin x = \frac{\sqrt{2}}{2}$

$$\boxed{\begin{array}{l} x = \frac{\pi}{4} + 2\pi n \\ x = \frac{3\pi}{4} + 2\pi n \end{array}}$$

17. $\sin 2x\cos x - \cos 2x\sin x = \frac{\sqrt{2}}{2}$

$2\sin x\cos x\cos x - (\cos^2 - \sin^2)\sin = \frac{\sqrt{2}}{2}$

$2\sin x\cos^2 x - \overset{\sin}{\cos^2}{}_2 + \sin^3 = \frac{\sqrt{2}}{2}$

$\sin x(2\cos^2 x - \cos^2 x + \overset{(1-\cos^2)}{\sin^2}) = \frac{\sqrt{2}}{2}$

$\sin x(\cos^2 x + 1 - \cos^2) = \frac{\sqrt{2}}{2}$

$\sin x(1) = \frac{\sqrt{2}}{2}$

$\sin x = \frac{\sqrt{2}}{2}$

45 45

$$\boxed{\begin{array}{l} x = \frac{\pi}{4} + 2\pi n \\ x = \frac{3\pi}{4} + 2\pi n \end{array}}$$

4.4H Warm Up
Solving Trig equations

Solve the following equations and list the solutions in the interval $(0, 2\pi]$

$x = 60°$ or $\frac{\pi}{3}$

1. $3cot^2x - 1 = 0$

$\cot^2 x = \frac{1}{3}$

$\cot x = \pm\frac{\sqrt{3}}{3}$

$\boxed{x = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}}$

2. $2sin^2x + 5sin\,x = 3$

$2\sin^2 x + 5\sin x - 3 = 0$

30 30

$(2\sin x - 1)(\sin x + 3) = 0$

$\sin x = \frac{1}{2}$ $\quad$ $\sin x = -3$

$x = \frac{\pi}{6}, \frac{5\pi}{6}$ $\quad$ not possible

Solve the following equations and list the general solutions.

3. $2tan^2\frac{x}{2} - tan\frac{x}{2} - 6 = 0$

$(2\tan(\frac{x}{2}) + 3)(\tan(\frac{x}{2}) - 2)$

STOP HERE!

$\tan(\frac{x}{2}) = -\frac{3}{2}$ $\quad$ $\tan(\frac{x}{2}) = 2$

✱ Need to use calculator

$u = \frac{x}{2}$

$\tan u = -\frac{3}{2}$ $\quad$ $\tan u = 2$

5

$u \approx -56.3$

$\frac{x}{2} \approx -56.3$

4. $sec3x\ sin3x - 3sin3x = 0$

STOP HERE

$\sin 3x(\sec 3x - 3) = 0$

✱ calculator

$u = 3x$

$\sin 3x = 0$ $\quad$ $\sec 3x = 3$

$\sin u = 0$ $\quad$ $\frac{1}{\sin u} = \frac{1}{3}$

$u = 0 + 2\pi n$

$3x = 0 + 2\pi n$

$\boxed{x = 0 + \frac{2\pi}{3}n}$

$u = \pi + 2\pi n$

$3x = \pi + 2\pi n$

$\boxed{x = \frac{\pi}{3} + \frac{2\pi n}{3}}$

4.4H "Sine" Language

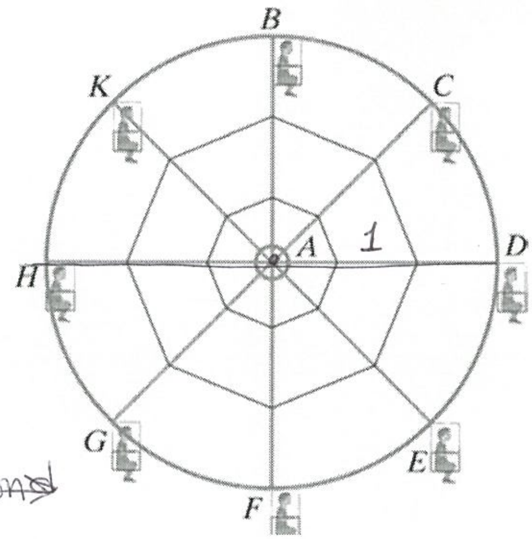
A Solidify Understanding Task

Part 1

Carlos and Clarita are participating in an afterschool enrichment class where they have to build a model replica of a Ferris wheel.

They are required to use the following measurements:

- The Ferris wheel has a radius of 1 meter.
- The center of the Ferris wheel is at ground level



underground

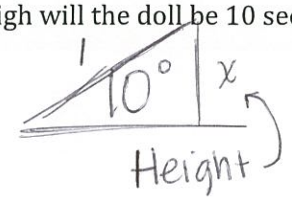
Carlos has also been carefully timing the rotation of the wheel by placing a doll in the cart in position D and has observed the following additional fact:

- The Ferris wheel makes one complete rotation counterclockwise every 360 seconds

1. Using this new information, how many degrees does the Ferris wheel rotate per second (angular speed)?

1 angle per second $\frac{360 \text{ angle}}{360 \text{ sec}}$

2. How high will the doll be 10 seconds after passing position A on the diagram?



$\sin 10 = \frac{x}{1}$

$x \approx .174 \text{ m}$

3. Calculate the height of the doll at each of the following times t, where t represents the number of seconds since the doll passed position A on the diagram. Keep track of any patterns you notice in the ways you calculate the height. As you calculate each height, record the time and height on the diagram as the coordinates (time, height).

Elapsed time, t, since passing position A	Calculations	Height of the doll
10 sec	$\sin(10)$	0.174m
20 sec	$\sin(20)$	0.342m
30 sec	$\sin(30)$	$\frac{1}{2}$ m
45 sec	$\sin(45)$	$\frac{\sqrt{2}}{2}$ m
60 sec	$\sin(60)$	$\frac{\sqrt{3}}{2}$ m
90 sec	$\sin(90)$	1m

4. Examine your calculations for finding the height of the doll during the first 90 seconds after passing position A. During this time, the angle of rotation of the doll is somewhere between 0° and 90°. Write a general formula for finding the height of the doll during this time interval.

$y = \sin x$

5. Calculate the height of the doll at each of the following times t, where t represents the number of seconds since the doll passed position A on the diagram. Keep track of any patterns you notice in the ways you calculate the height. As you calculate each height, record the time and height on the diagram as the coordinates (time, height).

120 sec	$\alpha = 60°$ $+\sin 60$	$\frac{\sqrt{3}}{2}$ m
150 sec	$\alpha = 30°$ $+\sin 30$	$\frac{1}{2}$ m
180 sec	$\sin(180) = 0$	0 m
225 sec	$-\sin(45)$	$-\frac{\sqrt{2}}{2}$ m
270 sec	$\sin(270)$	-1 m
315 sec	$\sin(315) = -\sin(45)$	$-\frac{\sqrt{2}}{2}$ m
330 sec	$-\sin(30)$	$-\frac{1}{2}$ m
360 sec	$\sin(360)$	0 m
400 sec	$\sin(40)$	0.64 m
420 sec	$= \sin(60)$	$\frac{\sqrt{3}}{2}$ m
630 sec	$= \sin(270)$	-1 m
660 sec	$\sin(300) = -\sin(60)$	$-\frac{\sqrt{3}}{2}$ m
720 sec	$= \sin(360)$	0 m

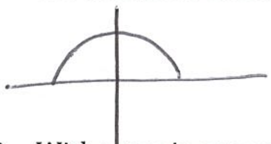
6. How might you find the height of the doll in other "quadrants" of the Ferris wheel, when the angle of rotation is greater than 90°?

Part 2

Carlos and Clarita are making notes of what they have observed about this new way of defining $\sin\theta$.

Carlos: "For some angles the calculator gives me positive values for $\sin\theta$, and for other angles it gives me negative values."

1. Without using your calculator, list at least five angles of rotation for which the value of $\sin\theta$ produced by the calculator should be **positive**.



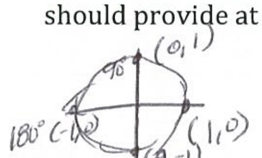
$\sin\theta$ is positive when y is positive so any θ where $\pi > \theta > 0$

2. Without using your calculator, list at least five angles of rotation for which the value of $\sin\theta$ produced by the calculator should be **negative**.

$\sin\theta$ is negative when the values of y are negative $\pi < \theta < 2\pi$

Clarita: "Yeah, and sometimes we can't even draw a triangle at certain positions on the Ferris wheel, but the calculator still gives us values for the sine at those angles of rotation."

3. List possible angles of rotation that Clarita is talking about - positions for which you can't draw a triangle to use as a reference. Then, without using your calculator, give the value of the sine that the calculator should provide at those positions.



← use unit circle, $y = \sin\theta$

$x = \cos\theta$

Carlos: "And, because of the symmetry of the circle, some angles of rotation should have the same values for the sine."

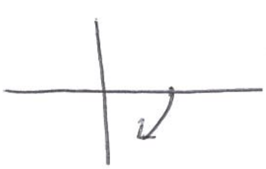
4. Without using your calculator, list at least five **pairs** of angles that should have the same sine value.

any coterminal angles.

Clarita: "Right! And if we go around the circle more than once, the calculator still gives us values for the sine of the angle of rotation, and multiple angles have the same value of the sine."

Carlos: "Can you have a sine value for an angle less than zero?"

5. List some angles that satisfy Clarita's statement. Explain why her statement is true.



yes angles are negative when drawn ← like this. Find reference angle and solve normally.

6. a. For which angles of rotation are the values of sine positive?

$0 < \theta < \pi$ (Quadrants I & II)

b. For which angles of rotation are the values of sine negative?

$\pi < \theta < 2\pi$ (Quadrants III & IV)

7. Explain how you find the angle of rotation in **quadrant II, III, and IV** when the reference angle has a measurement of θ.

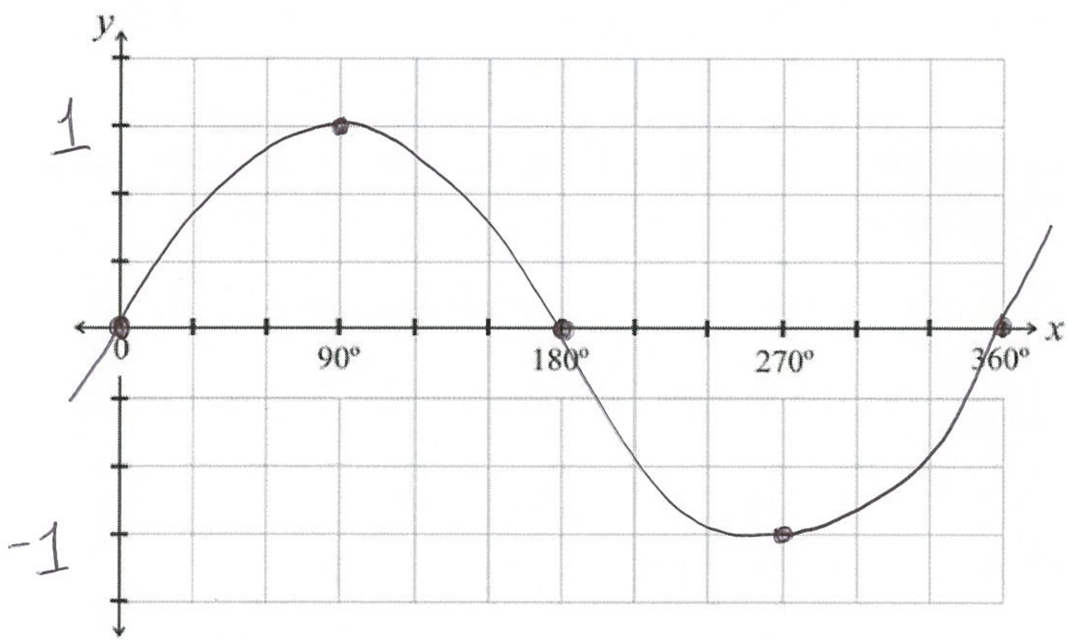
α = reference angle

QII = $+\sin\alpha$

QIII = $-\sin\alpha$

QIV = $-\sin\alpha$

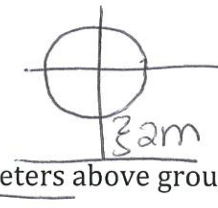
8. Based on the data you calculated, as well as any additional insights you might have about riding on Ferris wheels, sketch a graph of the height of the doll on this Ferris wheel as a function of the elapsed time since the rider passed the position farthest to the right on the Ferris wheel. We can consider this position as the rider's starting position at time $t = 0$. Be sure to include the starting position.



9. Write the equation of the graph you sketched in question 8.

$y = \sin x$

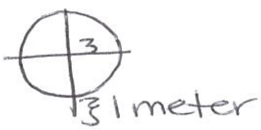
10. Carlos realizes that a Ferris wheel at ground level does not make sense.



a. How would the graph and equation change if he built the Ferris wheel 2 meters above ground?

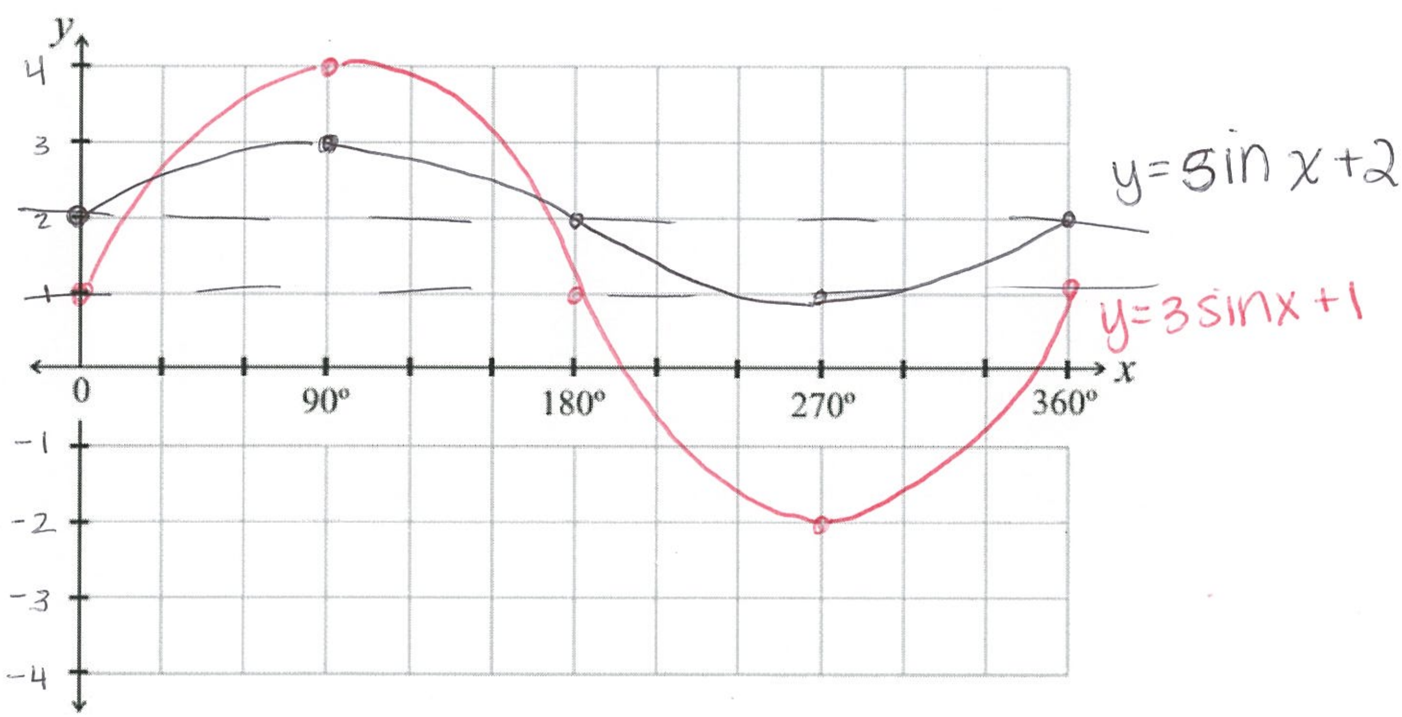
New Equation: $y = \sin x + 2$

b. How would the graph and equation change if he built the Ferris wheel 1 meter above the ground with a radius of 3?



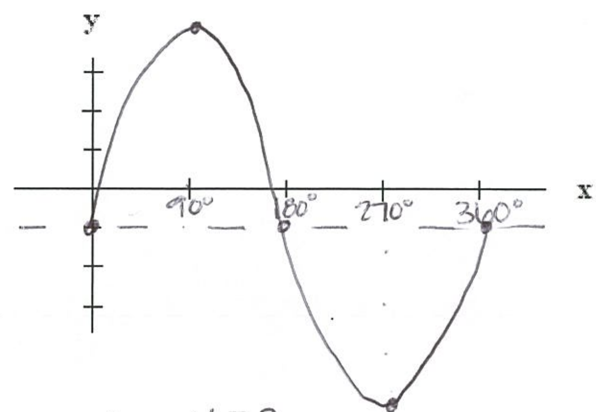
New Equation: $y = 3\sin x + 1$

Graph both equations from part a and b on the graph below.



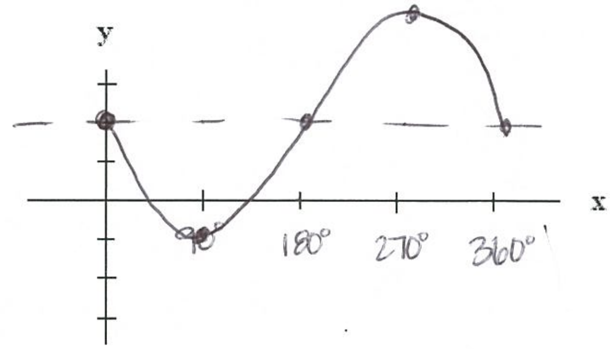
Practice

$y = 5sinx - 1$ Amplitude 5 Midline $y = -1$



$y = -3\, sinx + 2$ Amplitude 3 Midline $y = 2$

ALWAYS positive



$y = a\sin(bx) + c$

4.5H Warm-Up
More Ferris Wheels
A Solidify Understanding Task

In a previous task, *"Sine" Language*, you calculated the height of a doll on a model Ferris wheel at different times *t*, where *t* represented the elapsed time after the rider passed the position farthest to the right on the Ferris wheel.

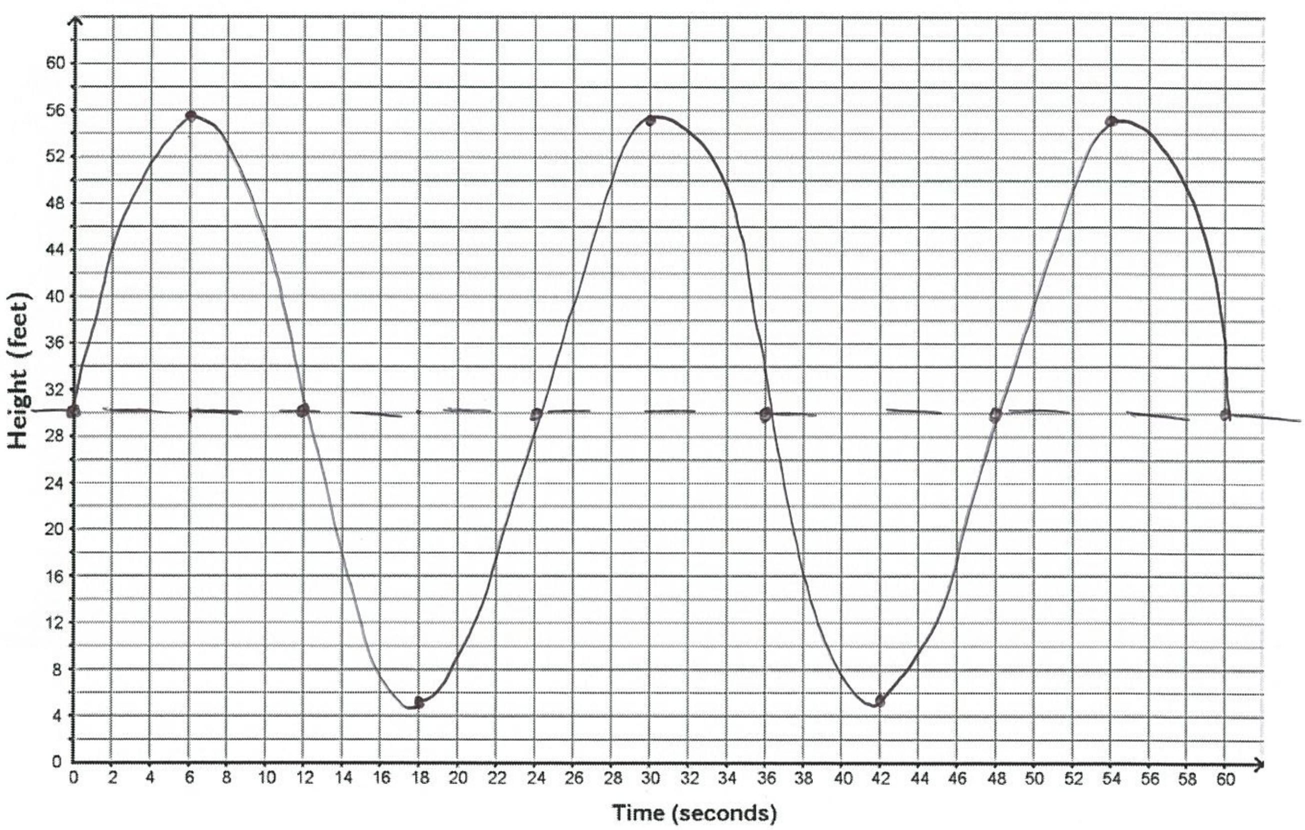
Use the following facts for a new Ferris wheel :

- The Ferris wheel has a radius of 25 feet.
- The center of the Ferris wheel is 30 feet above the ground.
- The Ferris wheel makes one complete rotation counterclockwise every 24 seconds.

1. Using what you learned in the previous task, as well as any additional insights you might have about riding on Ferris wheels, sketch a graph of the height of a rider on this Ferris wheel as a function of the elapsed time since the rider passed the position farthest to the right on the Ferris wheel. We can consider this position as the rider's starting position at time $t = 0$. Be sure to include the starting position.

t	3	4	6	8	9	12	16	20	24
f(t)	$25(\frac{\sqrt{2}}{2})+30$								

47.7



2. Write the equation of the graph you sketched in question 1.

$y = 25\sin(15x) + 30$

3. We began this task by considering the graph of the height of a rider on a Ferris wheel with a radius of 25 feet and center 30 feet off the ground, which makes one revolution counterclockwise every 24 seconds. How would your **graph** change if:

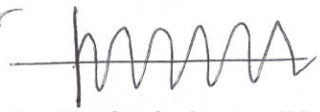
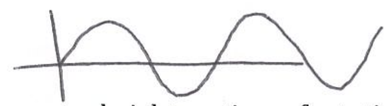
   a. the radius of the wheel was larger? or smaller?

   the graph would go higher & lower on the y-axis

   b. the height of the center of the wheel was greater? or smaller?

   the midline would shift, the entire graph would move up or down

   c. the wheel rotates faster? or slower?

   faster  slower 

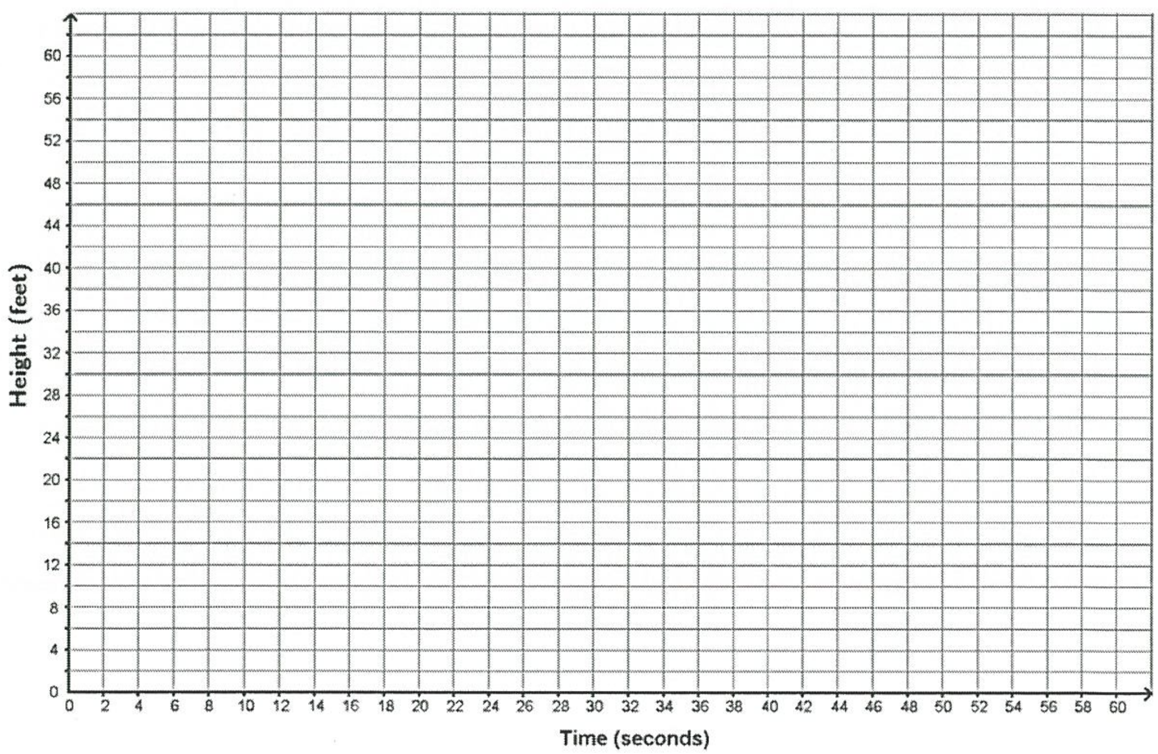
4. Of course, Ferris wheels do not all have this same radius, center height, or time of rotation. Describe a different Ferris wheel by changing at least one of the facts listed above.

   Description of my Ferris wheel:

   radius

   $y = 15\sin x + 20$ height above ground

5. Sketch a graph of the height of a rider on your Ferris wheel from question 4 as a function of the elapsed time since the rider passed the position farthest to the right on the Ferris wheel.



6. Write the equation of the graph you sketched in question 5.

7. How does the **equation** of the rider's height change if:

   a. the radius of the wheel is larger? or smaller?

   b. the height of the center of the wheel is greater? or smaller?

   c. the wheel rotates faster? or slower?

*look at answers on page 34 #3

8. Write the equation of the height of a rider on each of the following Ferris wheels t seconds after the rider passes the farthest right position.

   a. The radius of the wheel is 30 feet, the center of the wheel is 45 feet above the ground, and the angular speed of the wheel is 15 degrees per second counterclockwise.

   $y = 30\sin 15x + 45$

   b. The radius of the wheel is 50 feet, the center of the wheel is at ground level (you spend half of your time below ground), and the wheel makes one revolution ***clockwise*** every 15 seconds.

   $y = -50\sin 24x$

   $\frac{360}{15} = 24$

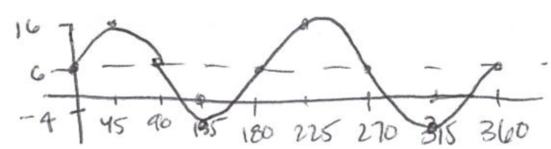
9. Explain how each part of the functions below changes the original $y = sinx$ graph.

   a. $y = 10\sin(2x) + 6$

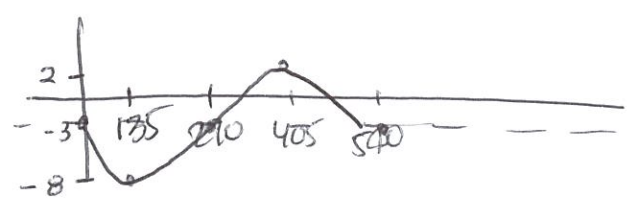
   up and down 10 from midline

   see the sin graph 2 times

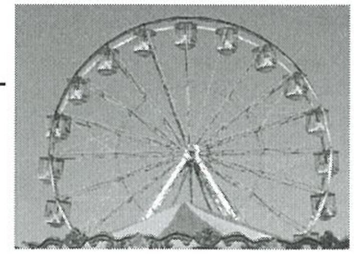
   midline



   b. $y = -5\sin\left(\frac{2}{3}x\right) - 3$



4.5H Moving Shadows
A Practice Understanding Task



In spite of his nervousness, Carlos enjoys his first ride on the Ferris wheel. He does, however, spend much of his time with his eyes fixed on the ground below him. After a while, he becomes fascinated with the fact that, since the sun is directly overhead, his shadow moves back and forth across the ground beneath him as he rides around on the Ferris wheel.

Recall the following facts for the model replica Ferris wheel Carlos is building:

- The Ferris wheel has a radius of 1 meter.
- The center of the Ferris wheel is 1 meter above the ground.
- The Ferris wheel makes one complete rotation counterclockwise every 360 seconds

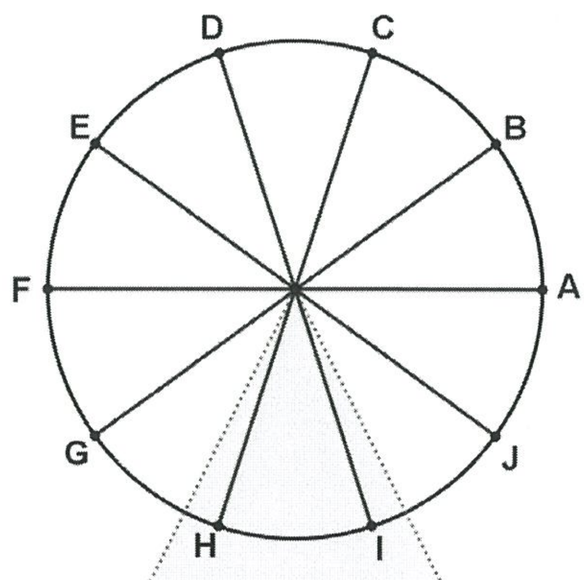
To describe the location of the dolls' shadow as it moves back and forth on the ground beneath, we could measure the shadow's **horizontal distance** (in feet) to the right or left of the point directly beneath the center of the model Ferris wheel, with locations to the right of the center having positive value and locations to the left of the center having negative values. For instance, in this system, the doll's shadow's location will have a value of 1 meter when he is at the position farthest to the right of the center on the Ferris wheel, and a value of -1 when it is at a position farthest to the left of the center.

1. In this new measurement system, if the doll's shadow is at 0 feet, where is the doll sitting on the Ferris wheel?

   A

2. In our previous work with the Ferris wheel, t represents the number of seconds since the doll passed the position farthest to the right on the Ferris wheel. How long will it take the doll to be at the position described in question 1?

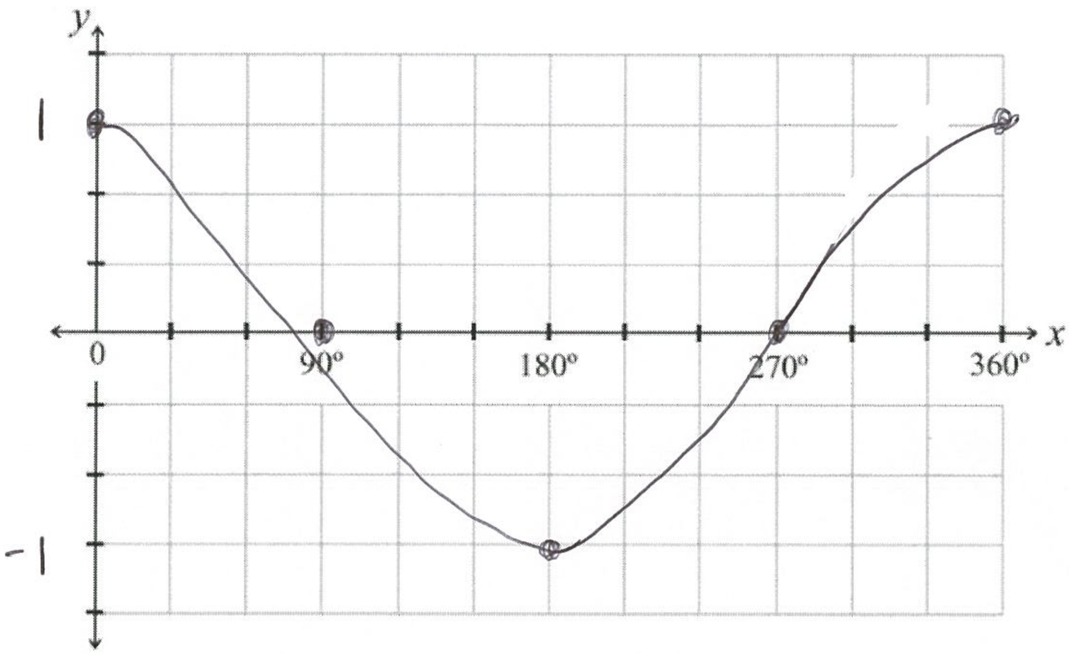
   360 seconds



3. Calculate the location of the doll's shadow from center at the times t given in the following table, where t represents the number of seconds since Carlos passed the position farthest to the right on the Ferris wheel. **Keep track of any patterns you notice in the ways you calculate the location of the shadow.** As you calculate each location, plot the location of the shadow from center on the graph on the following page.

Elapsed time since passing position A	Location of the shadow from center
30 sec	cos (30)
60 sec	sin (60)
90 sec	sin (90)
135 sec	sin (135)
150 sec	
180 sec	
210 sec	
225 sec	
240 sec	
300 sec	
315 sec	
330 sec	
360 sec	
390 sec	
410 sec	
450 sec	
480 sec	
495 sec	
510 sec	

4. Sketch a graph of the horizontal location of the doll's shadow from center as a function of time t, where t represents the elapsed time after the doll passes position A, the farthest right position on the Ferris wheel.

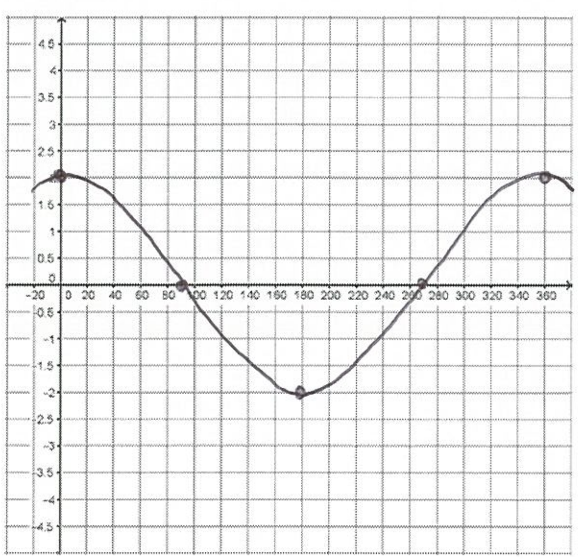


5. Write a general equation for finding the location of the shadow at any instant in time.

$y = \cos x$

Graph the following functions.

6. $y = 2\cos x$



7. $y = -3\cos x + 2$