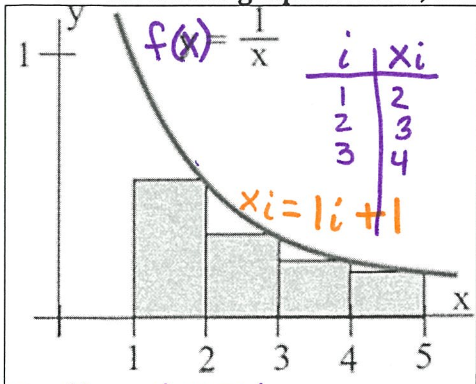
$\int_a^b f(x)\,dx = \sum_{i=1}^{n} f(x_i)\Delta x$

IM3H Module 8.10 Review Name:______________________________

Area under the curve and Riemann Sums $\Delta x = \frac{b-a}{n}$ ← total # of rectangles

1. Given the graphs below, write the summation notation for the Riemann Sum shown, and evaluate.



$f(x) = \frac{1}{x}$

i	x_i
1	2
2	3
3	4

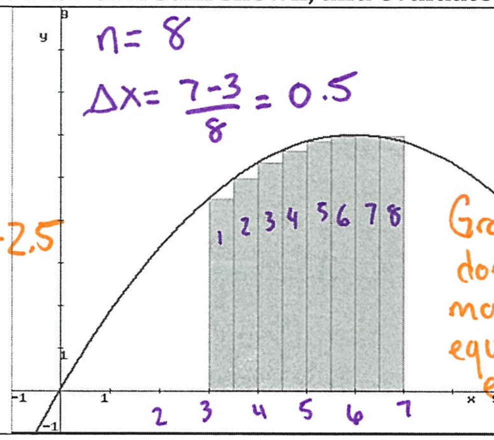
$x_i = 1i + 1$

$n = 4,\ \Delta x = 1$

$f(x) = -(x-6)^2 + 6$

i	x_i
1	3
2	3.5
3	4
4	4.5

$x_i = 0.5i + 2.5$



$n = 8$

$\Delta x = \frac{7-3}{8} = 0.5$

Graph does not match equation exact!

Area= $\sum_{i=1}^{4}\left(\frac{1}{i+1}\right)(1)$

$= (1)\left(\frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5}\right)$

$\int_1^5 f(x)\,dx = 1.28\overline{3}$

Area= $\sum_{i=1}^{8}\left(-(0.5i + 2.5 - 6)^2 + 6\right)(0.5)$

$= (0.5)\left(f(3) + f(3.5) + \ldots + f(6.5) + f(7)\right)$

$\int_3^7 f(x)\,dx =$

The rate that people are entering a local office is given below in people/hour. Use the table to answer questions 1-3.

Time (hours)	0	1	~~3~~ 2	~~5~~ 3	~~7~~ 4
~~f(t)~~ r(t) ppl/hr	12	7	3	5	8

2. Use a left Riemann sum with 4 subintervals to approximate the total number of people entering the office over the interval $0 \le t \le 4$.

$\Delta x = \frac{4-0}{4} = 1$

$n = 4$

i	x_i
1	0
2	1
3	2

$x_i = i - 1$

$= \boxed{\sum_{i=1}^{4} r(i-1)\cdot(1)}$

Area $= 1\left(r(0) + r(1) + r(2) + r(3)\right) = \boxed{27}$

3. Use a right Riemann sum with 4 subintervals to approximate the total number of people entering the office over the interval $0 \le t \le 4$.

$\Delta x = 1$

$n = 4$

i	x_i
1	1
2	2
3	3

$x_i = i$

$\boxed{\sum_{i=1}^{4} r(i)(1)}$

Area $= 1\left(r(1) + r(2) + r(3) + r(4)\right) = \boxed{23}$

4. The graph of a function g is given. Estimate $\int_{-3}^{3} g(x)dx$ with 6 equal subintervals using

(a) right endpoints

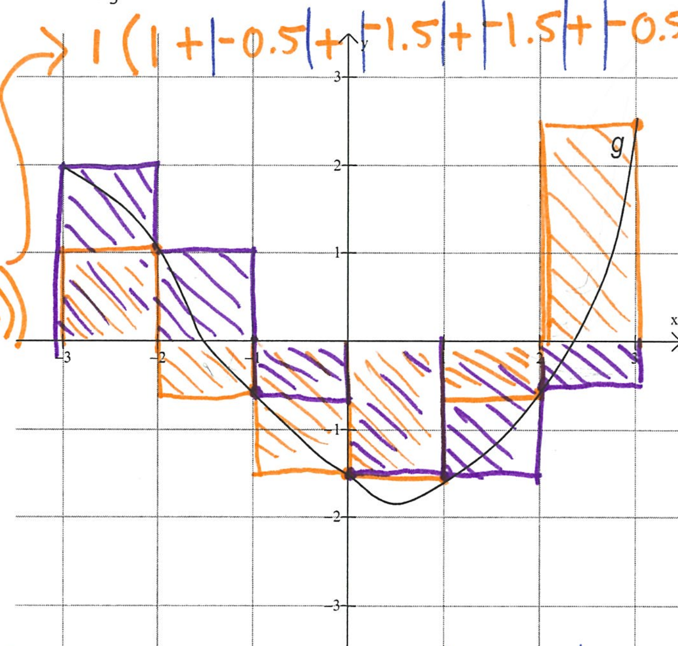
$$\sum_{i=1}^{6} g(i-3)(1) = 8.5$$

$= 1\left(g(-2)+g(-1)+g(0)+\ldots+g(3)\right)$

$\rightarrow 1(1+|-0.5|+|-1.5|+|-1.5|+|-0.5|+3)$

i	x_i
1	-2
2	-1
3	0

$x_i = i-3$



(b) left endpoints

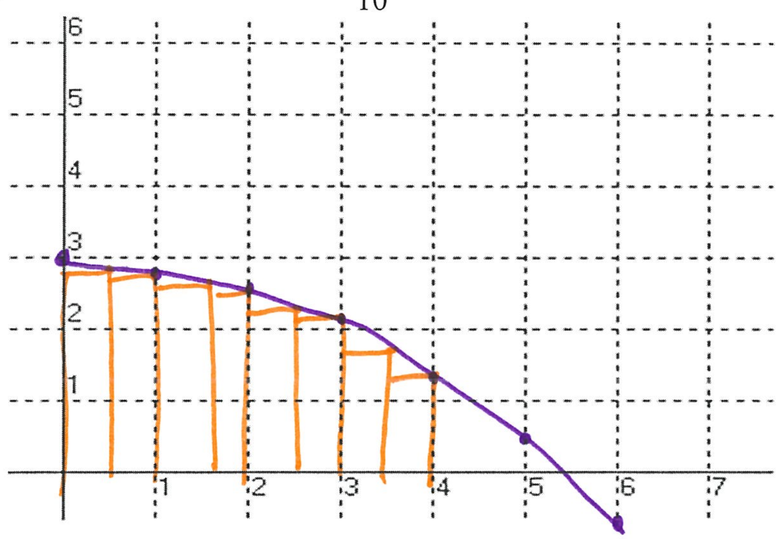
$$\sum_{i=1}^{6} g(i-4)(1) = 7.$$

i	x_i
1	-3
2	-2
3	-1

$x_i = i + -4$

$= 1\left(g(-3)+g(-2)+\ldots+g(2)\right) = 1\left(2+1+|-0.5|+|-1.5|+|-1.5|+|-0.5|\right) = -1$

5. Draw the graph of $f(x) = -\frac{1}{10}x^2 + 3$ below.



x	$f(x)$
0	3
1	2.9
2	2.6
3	2.1
4	1.4
5	0.5

★ Absolute value of the negative areas!

Calculate the Area under the curve between x=0 and x=4 using 8 -right hand rectangles.

$n=8$

$\Delta x = \frac{4-0}{8} = 0.5$

$= 0.5\left(f(0.5)+f(1)+f(1.5)+\ldots+f(3.5)+f(4)\right)$

$\boxed{= 9.45}$?

Write your answer using summation notation

i	x_i
1	0.5
2	1
3	1.5

$x_i = 0.5i$

$$= \sum_{i=1}^{8} \left(-\frac{1}{10}(0.5i)^2 + 3\right)(0.5)$$