Find all zeros, vertical asymptotes, + domain.

① $y = \dfrac{-3x^2 - 30x - 63}{2x^2 + 7x - 4}$

② $\dfrac{x^2 - 16}{x^3 - 8}$

Simplify

③ $\dfrac{x+7}{2x} - \dfrac{3x+1}{x+5}$

④ $\dfrac{2x^2 + 5x - 3}{3x^2 + 5x - 28} \div \dfrac{x^2 + 5x + 6}{3x^2 - x - 14}$

Identify as proper or improper and find the end behavior.

⑤ $\dfrac{3x+5}{x^2 - 1}$

⑥ $\dfrac{3x+5}{2x-1}$

⑦ $\dfrac{2x^2 + 7x + 5}{x+3}$

Graph using asymptotes + transformations.

⑧ $y = -2 + \dfrac{3}{x-3}$

① $\dfrac{-3x^2-30x-63}{2x^2+7x-4} = \dfrac{-3(x^2+10x+21)}{(2x+1)(x+4)} = \dfrac{-3(x+3)(x+7)}{(2x-1)(x+4)}$

zeros: $(-3,0)\ (-7,0)$ * use numerator

VA: vertical asymptote: $x=\frac{1}{2}$, $x=-4$ * use denominator

Domain: $(-\infty,-4)\cup(-4,\frac{1}{2})\cup(\frac{1}{2},\infty)$

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② $\dfrac{x^2-16}{x^3-8} = \dfrac{(x-4)(x+4)}{(x-2)(x^2+2x+4)}$ → imaginary roots!

zeros: $(4,0)$, $(-4,0)$

VA: $x=2$

Domain: $(-\infty,2)\cup(2,\infty)$

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③ $\dfrac{x+7}{(x+5)\,2x}^{(x+5)} - \dfrac{3x+1}{x+5\,(2x)}^{(2x)}$

$\dfrac{x^2+12x+35}{2x(x+5)} - \dfrac{6x^2+2x}{2x(x+5)}$

$= \dfrac{-5x^2+10x+35}{2x(x+5)}$

$= \boxed{\dfrac{-5(x^2-2x-7)}{2x(x+5)}}$

④ $\dfrac{2x^2+5x-3}{3x^2+5x-28} \div \dfrac{x^2+5x+6}{3x^2-x-14}$

$\dfrac{(2x-1)(x+3)}{(3x-7)(x+4)} \cdot \dfrac{3x^2-x-14}{x^2+5x+6}$

$\dfrac{(2x-1)\cancel{(x+3)}}{\cancel{(3x-7)}(x+4)} \cdot \dfrac{\cancel{(3x-7)}(x+2)}{(x+2)\cancel{(x+3)}}$

$= \boxed{\dfrac{2x-1}{x+4}}$

(5) $\dfrac{3x+5}{x^2-1}$

Proper, so Horizontal Asymptote is $y=0$.

End Behavior:

as $x \to \infty$, $f(x) \to 0$

as $x \to -\infty$, $f(x) \to 0$

(6) $\dfrac{3x+5}{2x-1} = \dfrac{3}{2} + \dfrac{6.5}{2x-1}$

$\frac{3}{2}$ ← Horizontal Asymptote

Improper

$$2x-1 \overline{)\,3x+5}$$

$3/2$

$-(3x - 3/2)$

$6.5$

End Behavior:

As $x \to \infty$, $f(x) \to 3/2$

$x \to -\infty$, $f(x) \to 3/2$

(7) $\dfrac{2x^2+7x+5}{x+3}$

Improper

| | $2x$ | $1$ |
|---|---|---|
| $x$ | $2x^2$ | $x$ | $2$ |
| $+3$ | $6x$ | $3$ | |

$= 2x+1 + \dfrac{2}{x+3}$

↑ Horizontal or slant Asymptote

End Behavior

As $x \to \infty$, $f(x) \to \infty$

$x \to -\infty$, $f(x) \to -\infty$

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Graph

$y = -2 + \dfrac{3}{x-3}$

↑ Horizontal Asymptote $y=-2$

↑ Vertical Asymptote $x=3$

Table

| $x$ | $y$ |
|---|---|
| $0$ | $-2+1=-3$ |
| $4$ | $-2+3=1$ |