Module 1 Polynomials Review

Identify the choice that best completes the statement or answers the question.

c 1. Classify $-3x^5 + 4x^3 + x^2 + 9$ by degree and by number of terms. degree = 5

a. quadratic binomial
b. quartic polynomial of 4 terms
c. quintic polynomial of 4 terms
d. cubic binomial

c 2. Write the expression $(x + 5)(x + 2)$ as a polynomial in standard form.

a. $x^2 + 3x + 10$
b. $x^2 - 3x + 7$
c. $x^2 + 7x + 10$
d. $x^2 + 3x - 3$

$x^2 + 5x + 2x + 10$

ad 3. Write $2x^3 + 14x^2 + 20x$ in factored form.

a. $2x(x + 5)(x + 2)$
b. $2x(x + 5)(x - 2)$
c. $5x(x + 2)(x + 2)$
d. $2x(x + 2)(x + 5)$

$x(2x^2 + 14x + 20)$

$2x(x^2 + 7x + 10)$

d 4. Write a polynomial function in standard form with zeros at 5, –4, and –3.

a. $f(x) = x^3 - 60x^2 + 2x - 23$
b. $f(x) = x^3 + 2x^2 - 23x + 7$
c. $f(x) = x^3 - 17x^2 - 420x + 7$
d. $f(x) = x^3 + 2x^2 - 23x - 60$

$(x-5)(x+4)(x+3)$

$(x-5)(x^2+7x+12)$

$x^3+7x^2+12x-5x^2-35x-60$

$x^3+2x^2-23x-60$

b 5. Find the zeros of $f(x) = (x + 2)^6(x + 3)^4$ and state the multiplicity.

a. –2, multiplicity 6; ~~4, multiplicity –3~~
b. –2, multiplicity 6; –3, multiplicity 4
c. 6, multiplicity –2; –3, multiplicity 4
d. 6, multiplicity –2; 4, multiplicity –3

d 6. Divide $-x^3 + 4x^2 - x - 3$ by $x + 2$. → Plug in –2 and look at remainder.

$= -(-2)^3 + 4(-2)^2 - (-2) - 3$

$= 23$

a. $-x^2 + 6x - 13$
b. $-x^2 + 2x + 11$, R –29
c. $-x^2 + 2x + 11$
d. $-x^2 + 6x - 13$, R 23

Divide using division.

a 7. $(x^4 + 12x^3 - 91x^2 + 26x + 20) \div (x - 5)$

a. $x^3 + 17x^2 - 6x - 4$
b. $x^3 - 22x^2 - 79x + 34$
c. $x^3 + 12x^2 - 22x + 34$
d. $x^3 - 6x^2 - 4x + 17$

	x^3	$17x^2$	$-6x$	-4
x	x^4	$17x^3$	$-6x^2$	$-4x$
-5	$-5x^3$	$-85x^2$	$30x$	20

No Remainder

Factor the expression.

b 8. $x^3 - 125 = (a-b)(a^2+ab+b^2)$

a. $(x + 5)(x^2 + 5x + 25)$
b. $(x - 5)(x^2 + 5x + 25)$
c. $(x + 5)(x^2 - 5x + 25)$
d. $(x - 5)(x^2 - 5x + 50)$

b 9. $c^3 - 512$

a. $-(c - 8)(c^2 + 8c + 64)$
b. $(c - 8)(c^2 + 8c + 64)$
c. $(c + 8)(c^2 + 8c + 64)$
d. $(c - 8)(c^2 - 8c - 64)$

b 10. Solve $27x^3 + 125 = 0$. Find all complex roots. → $(3x+5)(9x^2-15x+25)=0$

$x = -\frac{5}{3}$ Quadratic Formula

a. no solution
b. $-\frac{5}{3}, \frac{15 \pm 15i\sqrt{3}}{18}$
c. $-\frac{5}{3}, \frac{5}{3}$
d. $\frac{5}{3}, \frac{15 \pm 15\sqrt{3}}{18}$

a 11. Solve $x^4 - 40x^2 = -144$.

a. 2, –2, 6, –6
b. no solution
c. 2, –2
d. 2, –6

$x^4 - 40x^2 + 144 = 0$

$(x^2 - 4)(x^2 - 36) = 0$

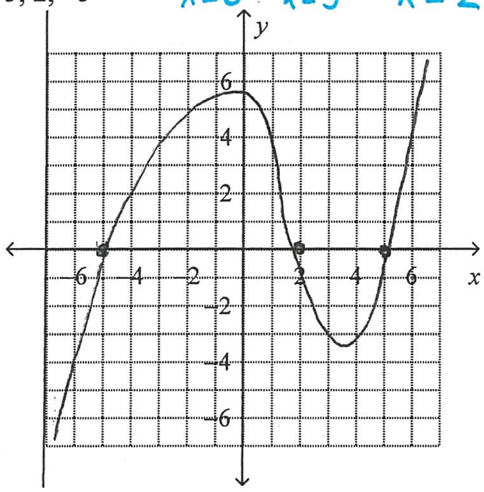
$x = \pm 2$

$x = \pm 6$

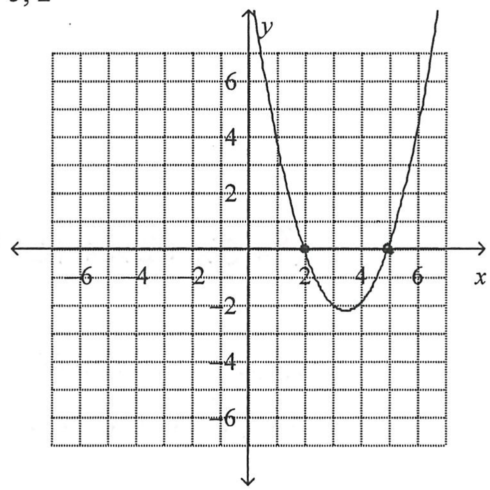
x^3 end behavior ↗ ↙

b 12. Find the zeros of $y = x(x-5)(x-2)$. Then graph the equation.

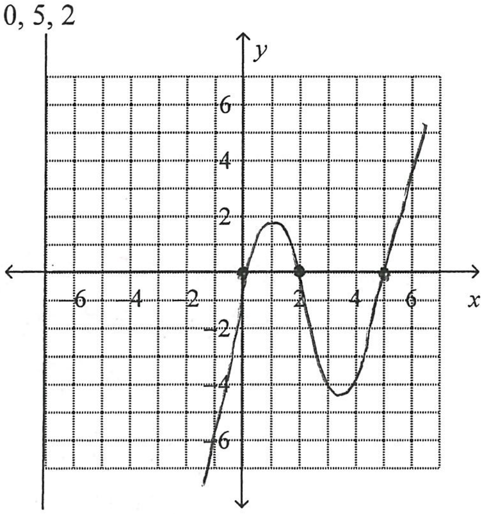
a. 5, 2, –5 $x=0$ $x=5$ $x=2$



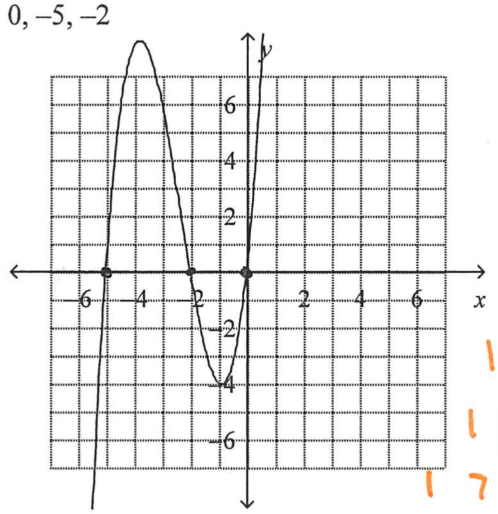
c. 5, 2



b. 0, 5, 2



d. 0, –5, –2



Pascals Δ

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6
1 7 21 35 35 21 7

Use Pascal's Triangle to expand the binomial.

c 13. $(s-5v)^5 = 1s^5 + 5s^4(-5v) + 10s^3(-5v)^2 + 10s^2(-5v)^3 + 5s(-5v)^4 + 1(-5v)^4$

a. $s^5 - 5s^4v + 10s^3v^2 - 10s^2v^3 + 5sv^4 - v^5$

b. $s^5 + 125s^4v - 1250s^3v^2 + 6250s^2v^3 - 15625sv^4 + 15625v^5$

c. $s^5 - 25s^4v + 250s^3v^2 - 1250s^2v^3 + 3125sv^4 - 3125v^5$

d. $s^5 - 25s^4 + 250s^3 - 1250s^2 + 3125s - 3125$

d 14. $(d-5)^6$ $1d^6 + 6d^5(-5) + 15d^4(-5)^2 + 20(d^3)(-5)^3 + 15d^2(-5)^4 + 6d(-5)^5 + 1(-5)^6$

a. $d^6 + 6d^5 + 15d^4 + 20d^3 + 15d^2 + 6d + 1$

b. $d^6 - 6d^5 + 15d^4 - 20d^3 + 15d^2 - 6d + 1$

c. $d^6 + 30d^5 + 375d^4 + 2500d^3 + 9375d^2 + 18750d + + 15625$

d. $d^6 - 30d^5 + 375d^4 - 2500d^3 + 9375d^2 - 18750d + 15625$

a 15. $(d+3)^7$ $1d^7 + 7d^6 \cdot 3 + 21d^5 3^2 + 35d^4 3^3 + 35d^3 3^4 + 21d^2 3^5 + 7d\, 3^6 + 1 \cdot 3^7$

a. $d^7 + 21d^6 + 189d^5 + 945d^4 + 2835d^3 + 5103d^2 + 5103d + 2187$

b. $d^7 - 7d^6 + 21d^5 - 35d^4 + 35d^3 - 20d^2 + 7d - 1$

c. $d^7 + 7d^6 + 21d^5 + 35d^4 + 35d^3 + 20d^2 + 7d + 1$

d. $d^7 - 21d^6 + 189d^5 - 945d^4 + 2835d^3 - 5103d^2 + 5103d - 2187$

$-8x^4 + 20x^5 \rightarrow 20x^5 - 8x^4$

b 16. Write $4x^2(-2x^2 + 5x^3)$ in standard form. Then classify it by degree and number of terms.

a. $2x + 9x^4$; quintic binomial
c. $2x^5 - 8x^4$; quintic trinomial
b. $20x^5 - 8x^4$; quintic binomial
d. $20x^5 - 10x^4$; quartic binomial

5 2 term

d 17. Determine which binomial is a factor of $-2x^3 + 14x^2 - 24x + 20$.

a. $x + 5 \rightarrow x = -5 \rightarrow 740$
b. $x + 20$ $x = -20 \rightarrow 22100$
c. $x - 24$ $x = 24 \rightarrow -20140$
d. $x - 5$ $x = 5 \rightarrow 0$

Plug in each root and try to get 0!

a 18. Use the Rational Root Theorem to list all possible rational roots of the polynomial equation $x^3 + x^2 - 7x - 4 = 0$. Do not find the actual roots.

PRZs: $\pm 1, 2, 4$

a. $-4, -2, -1, 1, 2, 4$
c. $1, 2, 4$
b. no roots
d. $-4, -1, 1, 4$

b 19. Find the rational roots of $x^4 + 8x^3 + 7x^2 - 40x - 60 = 0$.

a. $2, 6$ → -32, 2976
b. $-6, -2$ → 0, 0
c. $-2, 6$
d. $-6, 2$

Plug in!

Find the roots of the polynomial equation.

b 20. $x^3 - 2x^2 + 10x + 136 = 0$

Check $x = 4 \rightarrow 208$

$x = -4 \rightarrow 0$

a. $-3 \pm 5i, -4$
c. $\cancel{-3 \pm i}, 4$
b. $3 \pm 5i, -4$
d. $\cancel{3 \pm i}, 4$

$x^2 \quad -6x \quad +34 \rightarrow$

x	x^3	$-6x^2$	$34x$
+4	$4x^2$	$-24x$	136 ✓

$\dfrac{6 \pm \sqrt{36-136}}{2}$

$\dfrac{6 \pm \sqrt{-100}}{2}$

$\dfrac{6 \pm 10i}{2}$

a 21. $2x^3 + 2x^2 - 19x + 20 = 0$ $x = -4 \rightarrow 0$

a. $\dfrac{3+i}{2}, \dfrac{3-i}{2}, -4$
c. $\dfrac{-3+i}{2}, \dfrac{-3-i}{2}, -4$
b. $\dfrac{-3+2i}{2}, \dfrac{-3-2i}{2}, 4$
d. $\dfrac{3+2i}{2}, \dfrac{3-2i}{2}, 4$

$2x^2 \quad -6x \quad +5$

x	$2x^3$	$-6x^2$	$5x$
+4	$8x^2$	$-24x$	20

$x = \dfrac{6 \pm \sqrt{36-40}}{4}$

$x = \dfrac{3 \pm i}{2}$

d 22. $x^4 - 5x^3 + 11x^2 - 25x + 30 = 0$

$x = 2 \rightarrow 0$
$x = 3 \rightarrow 0$

a. $\cancel{-2}, -3, \pm i\sqrt{5}$
c. $\cancel{-2}, 3, \pm\sqrt{5}$
b. $2, \cancel{-3}, \pm\sqrt{5}$
d. $2, 3, \pm i\sqrt{5}$

c 23. A polynomial equation with rational coefficients has the roots $3 + \sqrt{6}, 2 - \sqrt{5}$. Find two additional roots.

a. $6 - \sqrt{3}, 5 + \sqrt{2}$
c. $3 - \sqrt{6}, 2 + \sqrt{5}$
b. $3 + \sqrt{6}, 2 - \sqrt{5}$
d. $6 + \sqrt{3}, 5 - \sqrt{2}$

conjugate

a 24. For the equation $2x^4 - 5x^3 + 10 = 0$, find the number of complex roots and the possible number of real roots.

possible

a. 4 complex roots; 0, 2 or 4 real roots
b. 4 complex roots; 1 or 3 real roots
c. 3 complex roots; 1 or 3 real roots
d. 3 complex roots; 0, 2 or 4 real roots

Degree 4: 4 complex, 0 real
2 complex, 2 real
0 complex, 4 real

For the equation, find the number of complex roots, the possible number of real roots, and the possible rational roots.

PRZs → $\pm 1, \pm 5$

____ 25. $x^7 - 2x^6 + 3x^2 - 2x + 5 = 0$

a. 7 complex roots; 1, 3, 5, or 7 real roots; possible rational roots: $\pm 1, \pm 5$
b. 7 complex roots; 2, 4, or 6 real roots; possible rational roots: $\pm 1, \pm 5$
c. 5 complex roots; 1, 3, or 5 real roots; possible rational roots: $\cancel{\pm\frac{5}{2}}, \pm 1, \pm 5$
d. 5 complex roots; 1, 3, or 5 real roots; possible rational roots: $\pm 1, \pm 5$

Not possible
Must have an even number of complex roots

$4x^6 - 4x^3 + 8 = 0$ $\qquad \dfrac{\pm 1, 2, 4, 8}{1, 2, 4}$

d 26. $8 - 4x^3 + 4x^6 = 0$

a. 6 complex roots; 2, 4, or 6 real roots; possible rational roots: $\pm\cancel{\frac{1}{8}}, \pm\frac{1}{4}, \pm\frac{1}{2}, \pm 1, \pm 2, \pm 4$

b. 6 complex roots; 2, 4, or 6 real roots; possible rational roots: $\pm\frac{1}{4}, \pm\frac{1}{2}, \pm 1, \pm 2, \pm 4, \pm 8$ ✓

c. 6 complex roots; 0, 2, 4, or 6 real roots; possible rational roots: $\pm\cancel{\frac{1}{8}}, \pm\frac{1}{4}, \pm\frac{1}{2}, \pm 1, \pm 2, \pm 4$

d. 6 complex roots; 0, 2, 4, or 6 real roots; possible rational roots: $\pm\frac{1}{4}, \pm\frac{1}{2}, \pm 1, \pm 2, \pm 4, \pm 8$ ✓

b 27. Find all zeros of $2x^4 - 5x^3 + 53x^2 - 125x + 75 = 0$.

a. ~~$-1, -\frac{3}{2}, \pm 5i$~~ $\qquad$ c. $1, \frac{3}{2}, \pm 5$

b. $1, \frac{3}{2}, \pm 5i$ $\qquad$ d. ~~$-1, -\frac{3}{2}, \pm 5$~~

$PRZ = \dfrac{\pm 1, 3, 25, 5, 15, 75}{\pm 1, \pm 2}$

$x = 1$ root ✓

	$2x^3$	$-3x^2$	$50x$	-75
x	$2x^4$	$-3x^3$	$50x^2$	$-75x$
-1	$-2x^3$	$3x^2$	$-50x$	75

$x^2(2x-3) + 25(2x-3)$

$(x^2+25)(2x-3)$

$x = \pm 5i \qquad x = 3/2$

d 28. Use the Binomial Theorem to expand $(d - 3b)^3$.

a. $d^3 - \cancel{3d^2b} + 3db^2 - b^3$

b. $d^3 + \cancel{3d^2b} + 3db^2 + b^3$

c. $d^3 + \cancel{9d^2b} + 27db^2 + 27b^3$

d. $d^3 - 9d^2b + 27db^2 - 27b^3$

$d^3 + 3d^2(-3b) + 3d(-3b)^2 + 1(-3b)^3$

29. What is the area of a square whose perimeter is 16x + 28?

$4 \overline{)16x + 28}$ $= 4x + 7$

$(4x+7)(4x+7) = \boxed{16x^2 + 56x + 49}$

30. Bradley is extending his rectangular living room. The original dimensions are 6 feet by 11 feet. If he extended his room by x + 8 feet, what is the new area? How many square feet of area did he add? Multiple answers!

New Areas $\qquad$ Added area

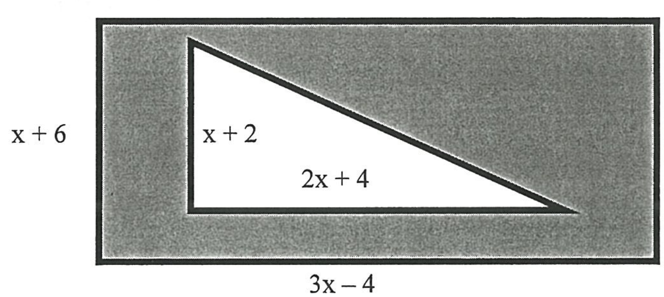
$(6+x+8)(11) = 11x + 48$ $\qquad$ $11x - 18$

$6(11+x+8) = 6x + 114$ $\qquad$ $6x + 48$

$(6+x+8)(11+x+8) =$

$(x+14)(x+19) = x^2 + 33x + 266$ $\qquad$ $x^2 + 33x + 200$

31. What is the area of the shaded region below? **(Area of rectangle = LW** $\qquad$ **Area of triangle = ½ bh)**



$(x+6)(3x-4)$

$3x^2 + 14x - 24 -$

$x^2 - 4x - 4$

$= \boxed{2x^2 + 10x - 28}$

$\frac{1}{2}(x+2)(2x+4) = \frac{1}{2}(2x^2 + 8x + 8) = x^2 + 4x + 4$

32. What is the volume of a rectangular prism with a length of 8, a width of 3x + 7, and a height of 2x – 5?
(V = Lwh)

$(8)(3x+7)(2x-5)$

$8(6x^2-15x+14x-35)$

$\boxed{48x^2-8x-280}$

33. Let $g(x)=x^3-5x^2-23x-8$.

a. List all the possible rational roots. (*p/q*'s)

$\frac{8}{1}$ $\frac{1,2,4,8}{1}$ = $\boxed{\pm1, \pm2, \pm4, \pm8}$

b. Use a calculator to help determine which values are the roots and perform division with those roots.

$(1) = 1-5-23-8 = -35$ ×

$(-1) = -1-5+23-8 = 9$ ×

$(-2) = -8-20+46-8$ ×

$(-3) = -27-45+69-8 = -11$ ×

$(8) = 512-320-184-8 = 0$ ✓

	x^2	$+3x$	$+1$
x	x^3	$3x^2$	x
-8	$-8x^2$	$-24x$	-8

✓

$x^2+3x+1 \rightarrow x = \frac{-3\pm\sqrt{9-4}}{2}$

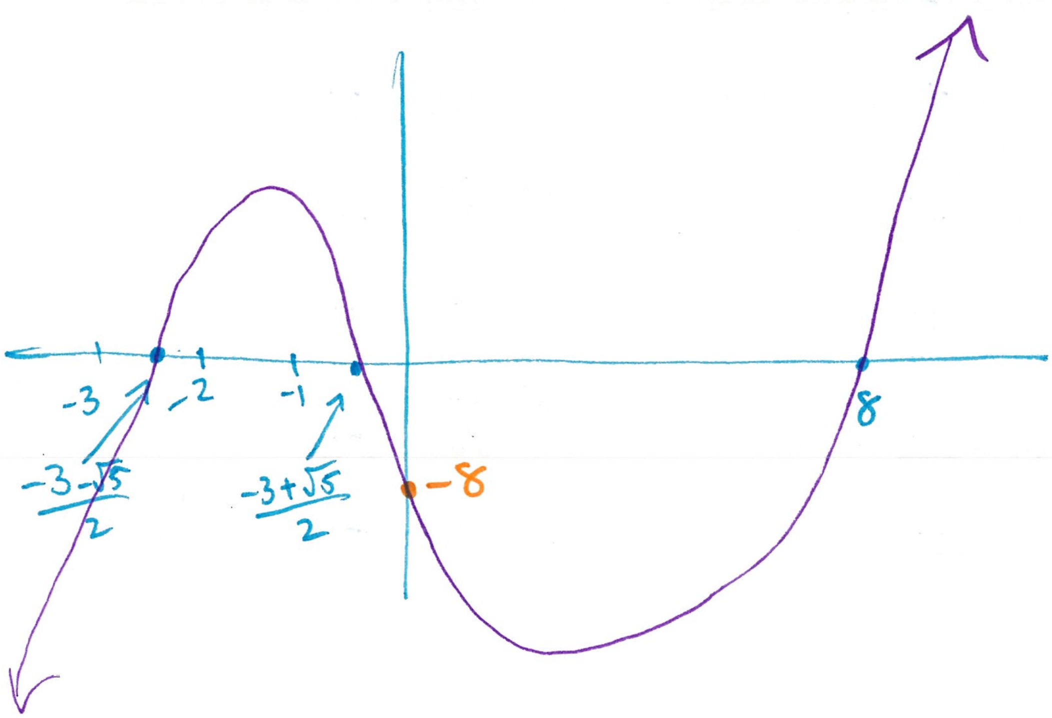
$x = \frac{-3\pm\sqrt{5}}{2}$

c. Write the polynomial in factored form and determine the zeros of the function. List the multiplicity of each zero.

$g(x) = (x-8)(x^2+3x+1)$ $x=8,\ m=1$

$x = \frac{-3\pm\sqrt{5}}{2},\ m=1$

d. Sketch the graph of the polynomial.



34. Let $f(x) = 2x^3 - 5x^2 + x + 2$

a. List all the possible rational roots. (p/q's) $\pm 1, \pm 2, \pm \frac{1}{2}$

b. Use a calculator to help determine which values are the roots and perform division with those roots.

$f(1) = 2 - 5 + 1 + 2 = 0$ ✓

$2x^2 \quad -3x \quad -2 \longrightarrow (2x+1)(x-2)$

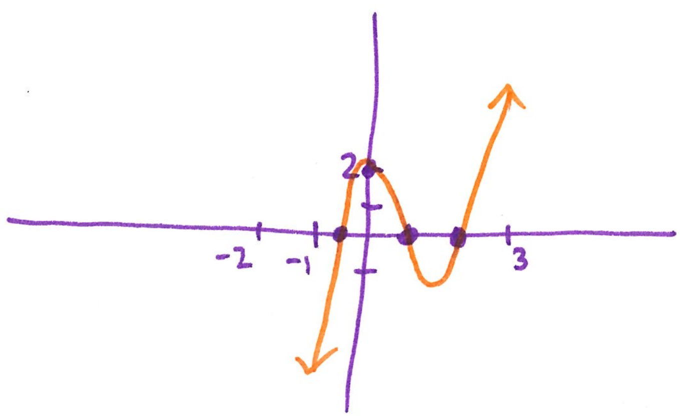
x	$2x^3$	$-3x^2$	$-2x$	
-1	$-2x^2$	$3x$	2	✓

c. Write the polynomial in factored form and determine the zeros of the function. List the multiplicity of each zero.

$f(x) = (x-1)(x-2)(2x+1)$

$x = 1, \ m = 1$
$x = 2, \ m = 1$
$x = -\frac{1}{2}, \ m = 1$

d. Sketch the graph of the polynomial.



35. Fill in the blanks for the end behavior for each of the following functions.

a. $f(x) = x^3 - 5x$
$f(x) \to \underline{-}\infty$ as $x \to -\infty$
and
$f(x) \to \underline{+}\infty$ as $x \to +\infty$

b. $f(x) = -x^5 - 3x^3 + 2$
$f(x) \to \underline{+}\infty$ as $x \to -\infty$
and
$f(x) \to \underline{-}\infty$ as $x \to +\infty$

c. $f(x) = x^4 - 4x^2 + x$
$f(x) \to \underline{+}\infty$ as $x \to -\infty$
and
$f(x) \to \underline{+}\infty$ as $x \to +\infty$

d. $f(x) = x + 12$
$f(x) \to \underline{-}\infty$ as $x \to -\infty$
and
$f(x) \to \underline{+}\infty$ as $x \to +\infty$

e. $f(x) = -x^2 + 3x + 1$
$f(x) \to \underline{-}\infty$ as $x \to -\infty$
and
$f(x) \to \underline{-}\infty$ as $x \to +\infty$

f. $f(x) = -x^8 + 9x^5 - 2x^4$
$f(x) \to \underline{-}\infty$ as $x \to -\infty$
and
$f(x) \to \underline{-}\infty$ as $x \to +\infty$

36. For each of the following, use the end behavior and x-intercepts to match the equation to its graph.

I 1. $f(x) = x^3 - 3x^2$ $x^2(x-3)$

B 2. $f(x) = x$

K 3. $f(x) = -3(x-1)(x-2)^2(x-3)$

NONE 4. $f(x) = 4x^2 - 9$ $(2x-3)(2x+3)$

J 5. $f(x) = x^2(x-3)^3$
0 3

C 6. $f(x) = -2x^3 + 8x$ $-2x(x^2-8)$

A 7. $f(x) = (x-1)(x-3)(x-5)$

F 8. $f(x) = -2x^2 + 16x - 24$
$-2(x-2)(x-6)$

O 9. $f(x) = -(x-4)(x-3)(x-1)^2$

H 10. $f(x) = x^4 - 3x^3$
$x^3(x-3)$

M 11. $f(x) = -2(x+3)^2(x+1)^2$

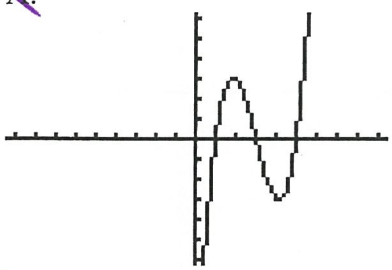
E 12. $f(x) = -x^3 + 9x$ $-x(x^2-9)$

D 13. $f(x) = 3x^4 - 3x^3 - 3x^2 + 3x$
$3x(x^3-x^2-x+1)$

G 14. $f(x) = -5$ $3x(x^2-1)(x-1)$

I 15. $f(x) = x^3 - 3x^2$
$x^2(x-3)$

A.



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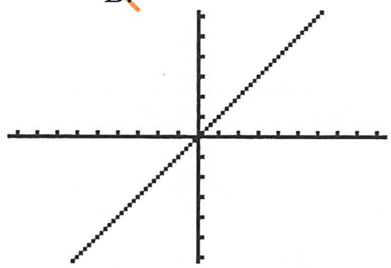

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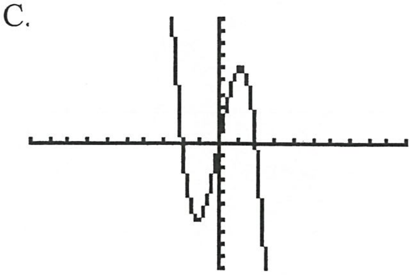

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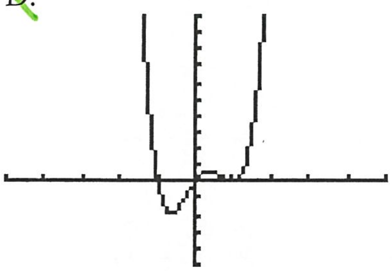

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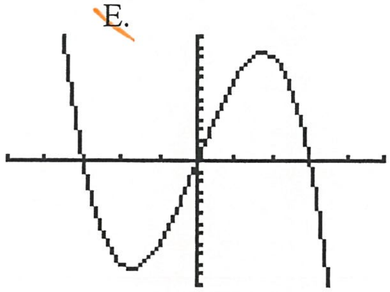

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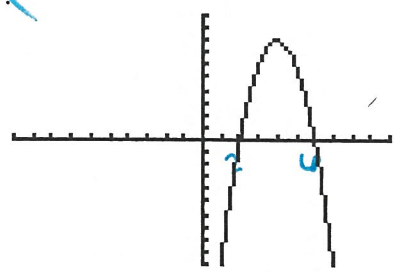
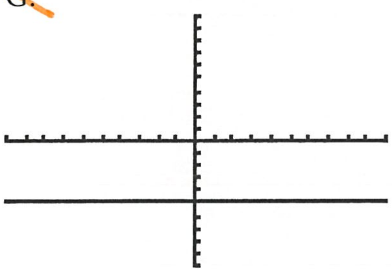
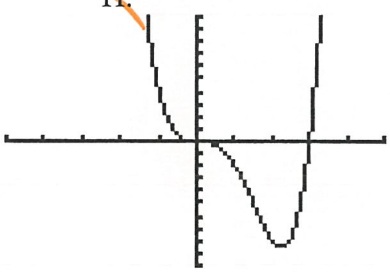


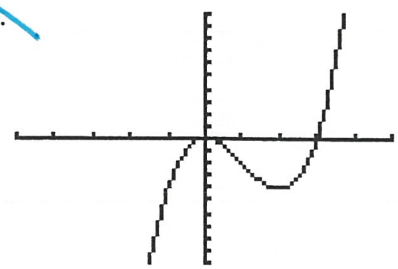
G.



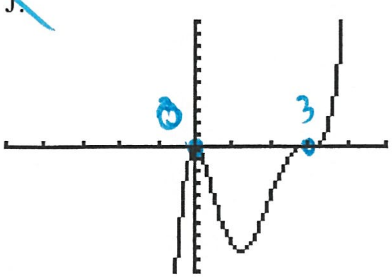
H.



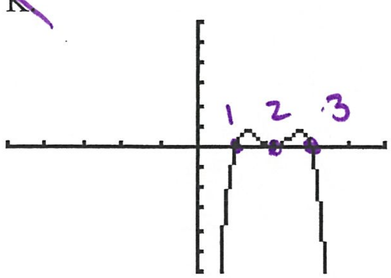
I.



J.



K.



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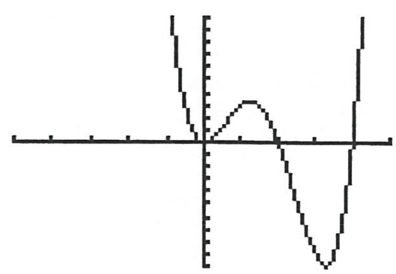

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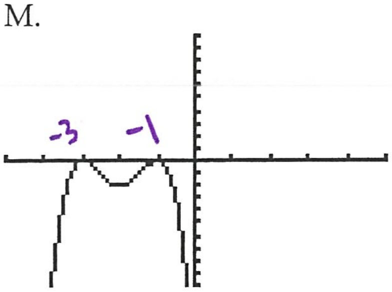

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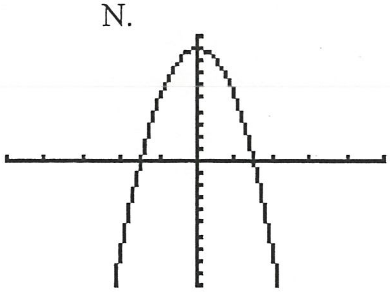

O.