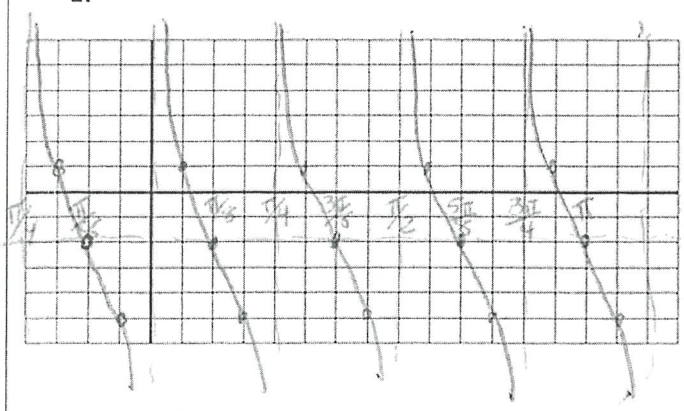
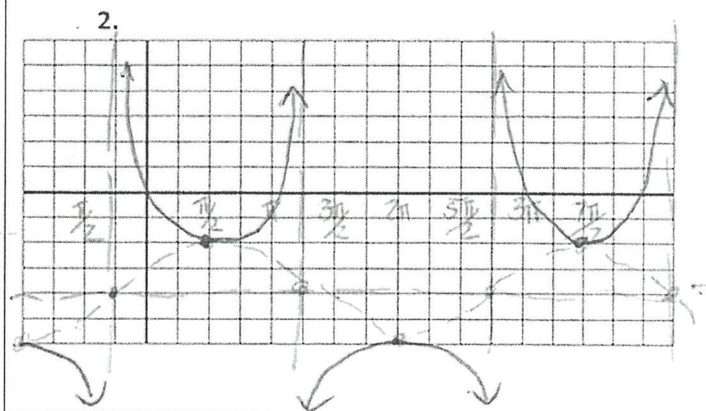
<u>IM3H More Final Review</u>

Module 4

1. $f(x) = 2 - 3\tan 4\left(x + \frac{\pi}{8}\right)$ Period $= \frac{\pi}{4}$



2. $y = 2\csc\frac{2}{3}\left(x + \frac{\pi}{4}\right) - 4$ Period: 3π



3. **Find all solutions in the equation in the interval [0, 2π).**

a. $\csc^2 x - \csc x - 2 = 0$

$(\csc x - 2)(\csc x + 1) = 0$

$\csc x = 2 \qquad \csc x = -1$

$\sin x = \frac{1}{2} \qquad \sin x = -1$

$x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{3\pi}{2}$

b. $3 - \sin^2 3x + 2\cos 3x = 5$

$3 - (1 - \cos^2 3x) + 2\cos 3x = 5$

$3 - 1 + \cos^2 3x + 2\cos 3x = 5$

$\cos^2 3x + 2\cos 3x - 3 = 0$

$(\cos 3x + 3)(\cos 3x - 1) = 0$

$\cancel{\cos 3x = -3} \qquad \cos 3x = 1$

$3x = 0 \qquad 3x = 2\pi \qquad x = 0, \frac{2\pi}{3}, \frac{4\pi}{3}$

$x = 0 \qquad x = 2\pi/3 \qquad \cancel{x = \frac{\pi}{3}, \frac{2\pi}{3}}$

c. $\sec x \sin x - 3\sin x = 0$

$\sin x(\sec x - 3) = 0$

$\sin x = 0 \qquad \sec x = 3$

$\boxed{x = 0, \pi} \qquad \cos x = 1/3$

$x =$ calculator

d. $3\cot^2 x - 1 = 0$

$\cot x = \pm\sqrt{\frac{1}{3}}$

$\cot x = \pm\frac{1}{\sqrt{3}}$

$\tan x = \pm\sqrt{3}$

$x = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$

e. $2\sin^2 3x + 5\sin 3x = 3$

$2\sin^2 3x + 5\sin 3x - 3 = 0$

$(2\sin 3x - 1)(\sin 3x + 3) = 0$

$2\sin 3x - 1 = 0 \qquad \cancel{\sin 3x = -3}$

$\sin 3x = \frac{1}{2}$

$3x = \frac{\pi}{6} \qquad 3x = \frac{5\pi}{6}$

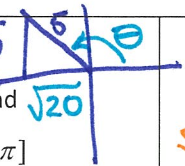
$x = \frac{\pi}{18} \qquad x = \frac{5\pi}{18}$

f. $2\tan^2\frac{x}{4} - \tan\frac{x}{4} - 6 = 0,$

$(2\tan\frac{x}{4} + 3)(\tan\frac{x}{4} - 2)$

calculator problem

4. Given $\sin\theta = \frac{\sqrt{5}}{5}$ and θ is in the inteval $[\frac{\pi}{2}, \pi]$. Find the exact values of cos2θ.



$\cos 2\theta = \cos^2\theta - \sin^2\theta$

$= \left(\frac{\sqrt{20}}{5}\right)^2 - \left(\frac{\sqrt{5}}{5}\right)^2$

$= \frac{20}{25} - \frac{5}{25} = \frac{15}{25} = \frac{3}{5}$

5. Use the half-angle formulas to find the exact value of sin105°

$\sin\left(\frac{210^\circ}{2}\right) = \sin 105^\circ$

$= \pm\sqrt{\frac{1-\cos 210}{2}}$

$= \pm\sqrt{\frac{1-\sqrt{3}/2}{2}} = \pm\sqrt{\frac{\frac{2+\sqrt{3}}{2}}{2}}$

$= \boxed{+\sqrt{2+\sqrt{3}}}$ positive only!

6. Use the sum formulas to find the exact value of tan255°.

$\tan(210^\circ + 45^\circ)$

$\frac{\tan 210 + \tan 45}{1 - \tan 210 \cdot \tan 45} = \frac{\frac{\sqrt{3}}{3} + 1}{1 - \frac{\sqrt{3}}{3} \cdot 1}$

$= \frac{\frac{\sqrt{3}}{3} + \frac{3}{3}}{\frac{3}{3} - \frac{\sqrt{3}}{3}} = \frac{\sqrt{3}+3}{\cancel{3}} \cdot \frac{\cancel{3}}{3-\sqrt{3}}$

$= \boxed{\frac{\sqrt{3}+3}{3-\sqrt{3}}}$

Verify the identities:

7. $\sin^2\alpha - \sin^4\alpha = \cos^2\alpha - \cos^4\alpha$

$\sin^2\alpha(1 - \sin^2\alpha) =$

$(1-\cos^2\alpha)(\cos^2\alpha) =$

$\cos^2\alpha - \cos^4\alpha =$ ✓

8. $\frac{1+\sin\theta}{\cos\theta} + \frac{\cos\theta}{1+\sin\theta} = 2\sec\theta$

$(1+\sin\theta)$ · cos

$(1+\sin\theta)$ · cos

$\frac{1 + 2\sin\theta + \underbrace{\sin^2\theta + \cos^2\theta}_{1}}{(1+\sin\theta)\cdot\cos\theta} =$

$\frac{2 + 2\sin\theta}{(1+\sin\theta)\cdot\cos\theta} \rightarrow \frac{2\cancel{(1+\sin\theta)}}{\cancel{(1+\sin\theta)}\cos\theta} =$

$\frac{2}{\cos\theta} = 2\sec\theta$ ✓

9. Find the exact value using a sum or difference formula.

a. $\sin\frac{11\pi}{12} = \sin\left(\frac{8\pi}{12} + \frac{3\pi}{12}\right)$

$\sin\left(\frac{2\pi}{3} + \frac{\pi}{4}\right)$

$\sin\frac{2\pi}{3}\cdot\cos\frac{\pi}{4} + \cos\frac{2\pi}{3}\cdot\sin\frac{\pi}{4}$

$\frac{\sqrt{3}}{2}\cdot\frac{\sqrt{2}}{2} + \frac{1}{2}\cdot\frac{\sqrt{2}}{2}$

$\frac{\sqrt{6}+\sqrt{2}}{4}$

b. $\cos\frac{11\pi}{12} = \cos\left(\frac{2\pi}{3} + \frac{\pi}{4}\right)$

$\cos\frac{2\pi}{3}\cdot\cos\frac{\pi}{4} - \sin\frac{2\pi}{3}\sin\frac{\pi}{4}$

$\frac{1}{2}\cdot\frac{\sqrt{2}}{2} - \frac{\sqrt{3}}{2}\cdot\frac{\sqrt{2}}{2}$

$\frac{\sqrt{2}-\sqrt{6}}{4}$

c. $\tan\frac{11\pi}{12} = \tan\left(\frac{2\pi}{3} + \frac{\pi}{4}\right)$

$= \frac{\tan\frac{2\pi}{3} + \tan\frac{\pi}{4}}{1 - \tan\frac{2\pi}{3}\cdot\tan\frac{\pi}{4}}$

$\frac{-\sqrt{3}+1}{1 - -\sqrt{3}\cdot 1}$

$= \frac{1-\sqrt{3}}{1+\sqrt{3}}$

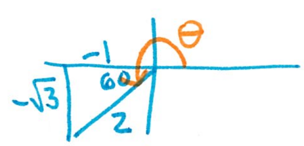
Evaluate:

10. $\cos^{-1}\left(\frac{1}{2}\right) =$ $\pi/3$	***11.*** $\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right) =$ $-\pi/3$	**12.** $\csc^{-1}(-\sqrt{2}) =$ $-\pi/4$

13. Find $\cos\theta$ if $\cot\theta = \frac{\sqrt{3}}{3}$ and $\csc\theta < 0$. → $\sin\theta < 0$

↓

$\tan\theta = \frac{3}{\sqrt{3}} = \sqrt{3}$

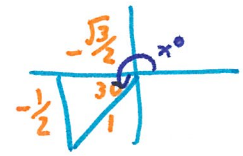


$\cos\theta = -1/2$

14. Find $sin2x$, if $cscx = -2, and\ \pi \le x \le \frac{3\pi}{2}$.

$\sin x = -\frac{1}{2}$

$\to$ Q III



$\sin 2x = 2\sin x \cos x$

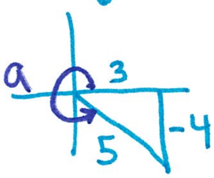
$\sin 2x = 2 \cdot \left(-\frac{1}{2}\right)\left(-\frac{\sqrt{3}}{2}\right) = \frac{\sqrt{3}}{2}$

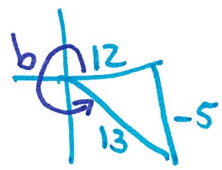
15. Find $\cos(a-b)$, if $\cos a = \frac{3}{5}$, $\sin b = -\frac{5}{13}$, and $angle\ a\ and\ b$ are in the same quadrant.

4th Quadrant

$\cos\theta > 0$

$\sin\theta < 0$





$\cos(a-b) = \cos a \cdot \cos b - \sin a \sin b$

$= \left(\frac{3}{5}\right)\left(\frac{12}{13}\right) - \left(\frac{-4}{5}\right)\left(\frac{-5}{13}\right)$

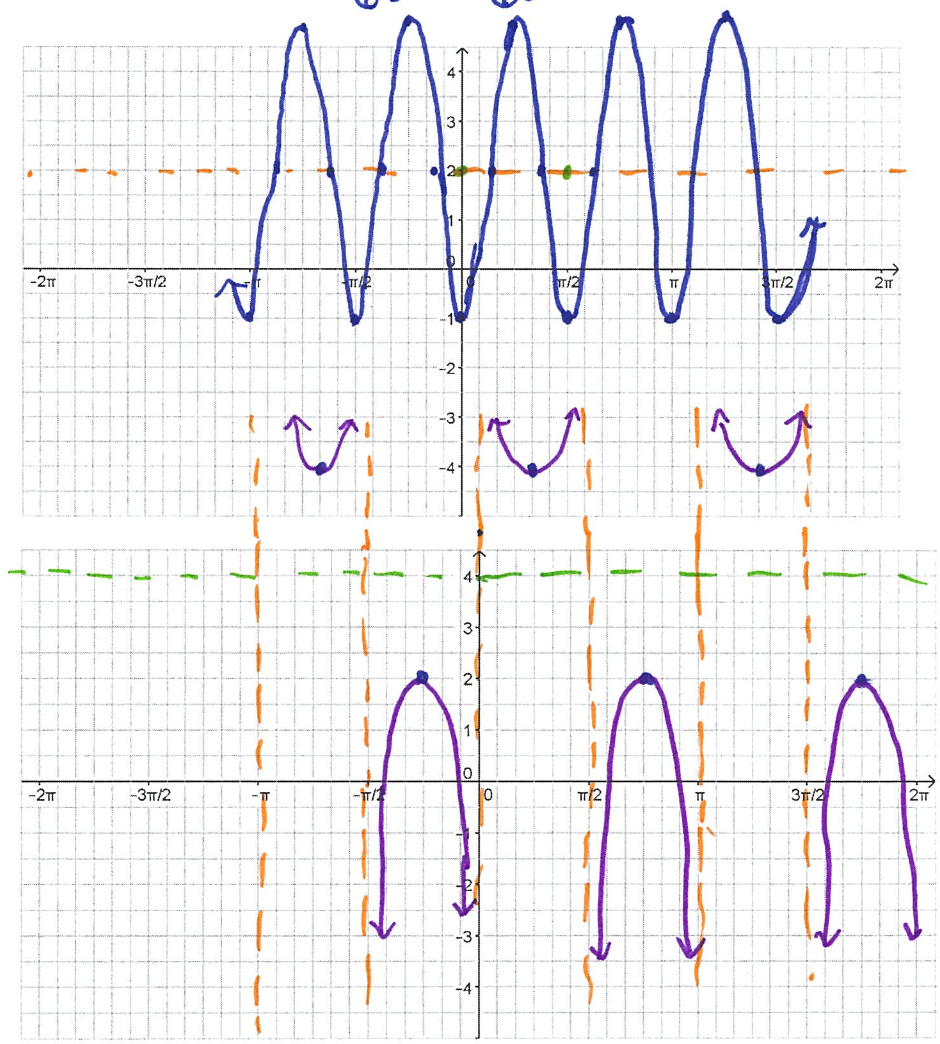
$\frac{36}{65} - \frac{20}{65} = \frac{16}{65}$

16. Graph the function:

$$f(x) = 2 - 3\sin 4\left(x + \frac{\pi}{8}\right)$$

Period $= \frac{2\pi}{4} = \pi/2$

Phase Shift $= -\pi/8$



17. Graph the function:

$$f(x) = 4 + 2\csc(2x - \pi)$$

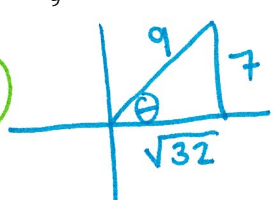
Period π

Phase shift $= +\pi/2$

18. Find the exact value.

a. $cos(arcsin(\frac{7}{9}))$

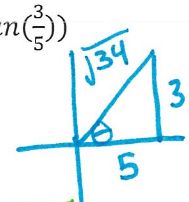
$\frac{\sqrt{32}}{9} = \frac{4\sqrt{2}}{9}$



b. $sin(arctan(\frac{3}{5}))$

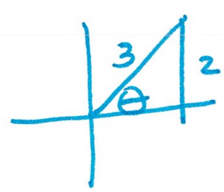
$\frac{3}{\sqrt{34}}$

$\boxed{\frac{3\sqrt{34}}{34}}$



c. $arcsin(sin(\frac{2}{3}))$

$\frac{2}{3}$



19. Find all solutions to the following trig equations.

a. $2sinx^2 - sinx - 2 = 1$

$2\sin^2 x - \sin x - 3 = 0$

$(2\sin x - 3)(\sin x + 1) = 0$

$\cancel{\sin x = \frac{3}{2}}$ $\quad \sin x = -1$

$\boxed{x = \frac{3\pi}{2} + 2\pi n}$

b. $cos^2x = 1 - sinx$

$1 - \sin^2 x = 1 - \sin x$

$0 = \sin^2 x - \sin x$

$0 = \sin x(\sin x - 1)$

$\sin x = 0 \quad \sin x = 1$

$\boxed{x = 0 + 2\pi n \quad x = \frac{\pi}{2} + 2\pi n \quad x = \pi + 2\pi n}$

c. $sinx - 2sinxcosx = 0$

$\sin x(1 - 2\cos x) = 0$

$\sin x = 0 \quad \cos x = \frac{1}{2}$

$\boxed{x = 0 + 2\pi n \quad x = \frac{\pi}{3} + 2\pi n \quad x = \pi + 2\pi n \quad x = \frac{5\pi}{3} + \frac{2\pi}{n}}$

Find the exact value without a calculator.

21. $\cos\left(\sin^{-1}\left(\frac{1}{2}\right)\right)$ $\quad \frac{\sqrt{3}}{2}$

22. $\sin\left(\cos^{-1}\left(\frac{\sqrt{2}}{2}\right)\right)$ $\quad \frac{\sqrt{2}}{2}$

23. $\sin^{-1}\left(\cos\left(\frac{\pi}{3}\right)\right)$ $\quad \frac{\pi}{6}$

24. $\cos^{-1}\left(\sin\left(\frac{\pi}{6}\right)\right)$ $\quad \pi/3$

25. $\sin^{-1}\left(\sin\left(\frac{7\pi}{4}\right)\right)$ $\quad -\frac{\sqrt{2}}{2}$ $\quad -\frac{\pi}{4}$

26. $\arccos\left(\sin\left(\frac{\pi}{3}\right)\right)$ $\quad \pi/6$

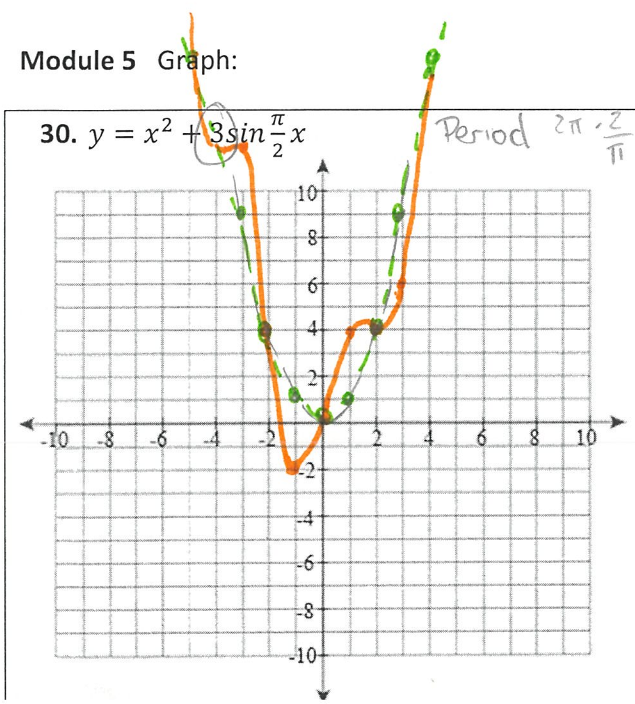
27. $\sin\left(\tan^{-1}\left(\sqrt{3}\right)\right)$ $\quad \frac{\sqrt{3}}{2}$

28. $\cos\left(\tan^{-1}(-1)\right)$ $\quad \frac{\sqrt{2}}{2}$

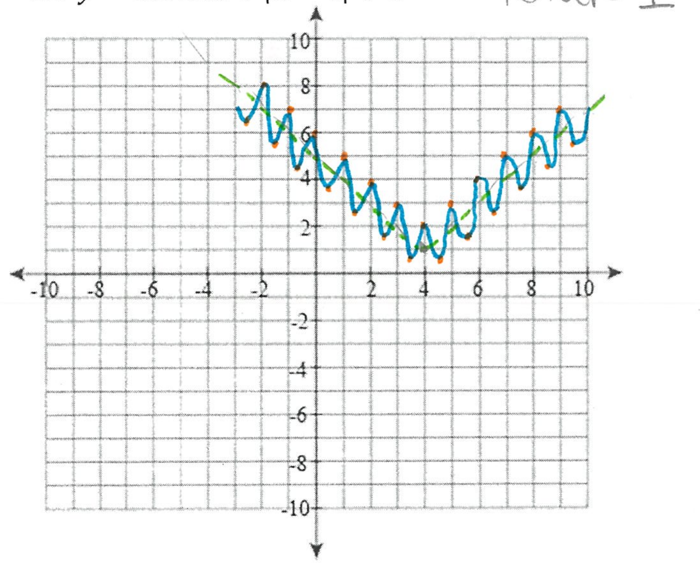
29. $\tan^{-1}(\cos(\pi))$ $\quad -\frac{\pi}{4}$

Module 5 Graph:

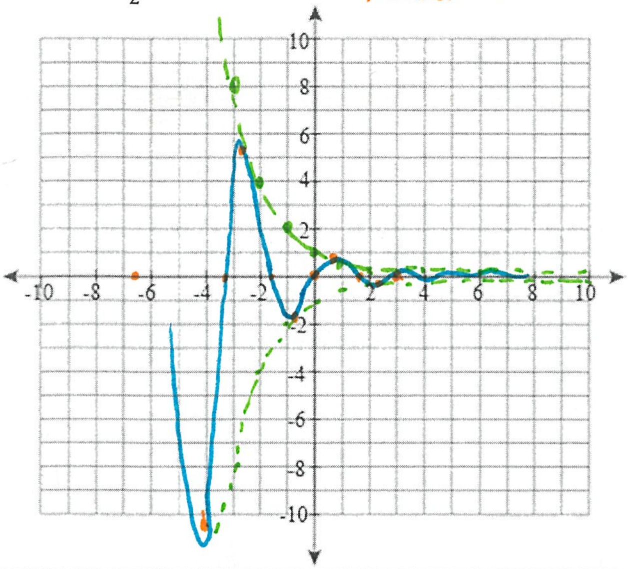
30. $y = x^2 + 3\sin\frac{\pi}{2}x$ $\quad$ Period $2\pi \cdot \frac{2}{\pi} = 4$



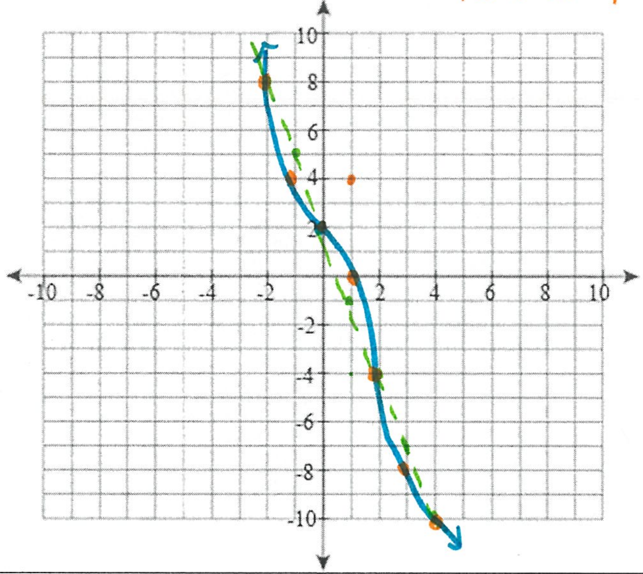
31. $y = \cos 2\pi x + |x - 4| + 1$ $\quad$ Period = 1



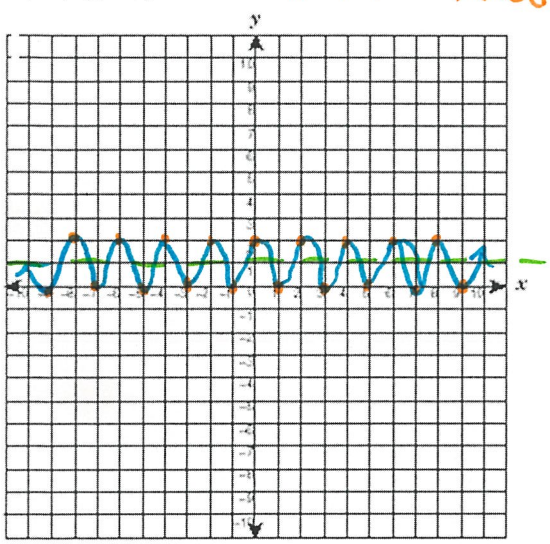
32. $y = \frac{1}{2}^{x} \cdot sin2x$ Period = $\pi \sim 3.14$



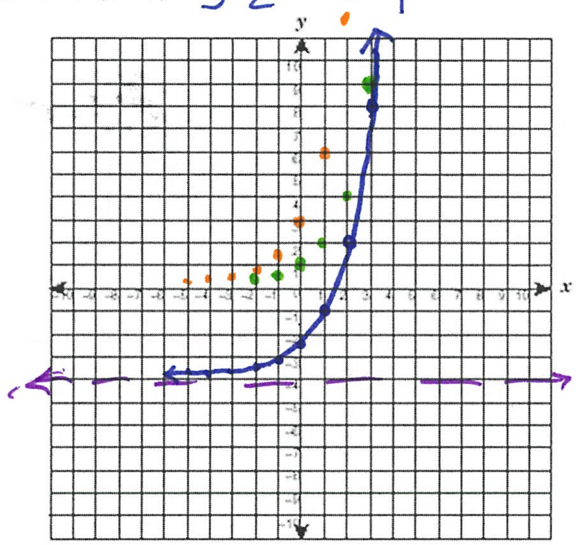
33. $y = -3x + 2 + sin\frac{\pi}{2}x$ Period = 4



34. Given that $f(x) = x + 1$ *and* $g(x) = \cos\pi x$, graph $f(g(x))$ $\cos \pi x + 1$ Period=2

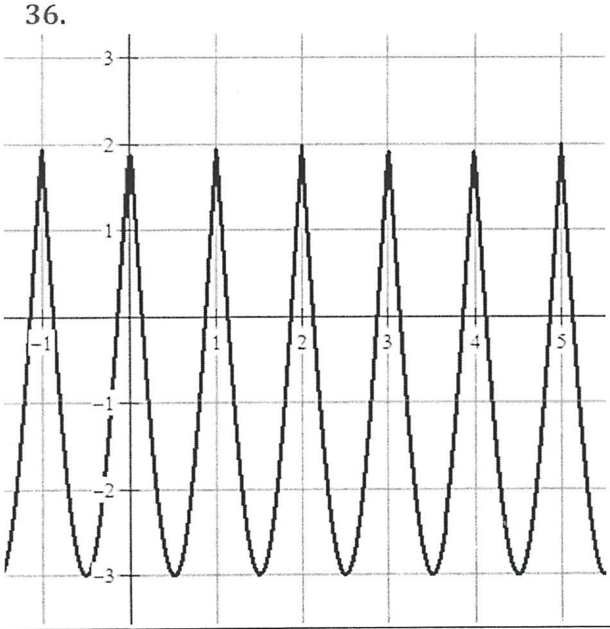


35. Given that $f(x) = 3x - 4$ *and* $g(x) = 2^{x-1}$ graph $f(g(x))$ $3 \cdot 2^{x-1} - 4$



For each graph below, write the function graphed and them write the function as a composition of two functions.

36.



Function graphed: $-|\sin \pi x| + 2$

Composed functions;

$k(x) = f(g(x))$

$f(x) =$ $-|x| + 2$

$g(x) =$ $\sin \pi x$

37. Given the following functions, find a composition of functions with each feature listed below.

$a(x) = \frac{x-1}{x-2},\ b(x) = \log_2 x,\ c(x) = |3x|,\ d(x) = 2x^2 + 4,\ e(x) = x - 4,\ f(x) = x^2 + 5x - 4$

a. A composition of functions with a ~~range of [32, ∞)~~ y-int of 32 $f(d(x))$

b. A composition of functions with no roots $c(d(x))$

c. A composition of functions with an asymptote at $x = 4$ $b(e(x))$

d. A composition of functions with end behavior: As $x \to \infty, y \to 1$ $a(e(x))$

38. Given $f(x) = 2x - 3,\quad g(x) = x^2 - 2x,\quad and \quad h(x) = -5x$. Find $g\big(f(h(x))\big)$.

$f(h(x)) = 2(-5x) - 3$
$= -10x - 3$

$g(f(h(x))) = (-10x-3)^2 - 2(-10x-3)$

39. Given $f(x) = 2x^3 - 9x^2 + x + 12,\quad g(x) = 2x - 3,\quad and \quad h(x) = x + 1$.

e. Find $g(x) - f(x)$

$2x - 3 - (2x^3 - 9x^2 + x + 12) = -2x^3 + 9x^2 + x - 15$

f. Find $g(x) \cdot h(x)$

$(2x-3)(x+1) = 2x^2 - x - 3$

	$\cancel{x^3}\ x$	-4
$2x^2$	$2x^3$	$-8x^2$
$-x$	$-x^2$	$4x$
-3	$-3x$	12

g. Find $\frac{f(x)}{g(x) \cdot h(x)}$

$\frac{2x^3 - 9x^2 + x + 12}{(2x-3)(x+1)} = x - 4$

$\hookrightarrow (2x^2 - x - 3)$

40. Given:

$f(x) = 2x - 4,$
$g(x) = \cos\frac{\pi}{4}x,\quad and$
$h(x) = -5x$

Period $= 2\pi \cdot \frac{4}{\pi} = 8$

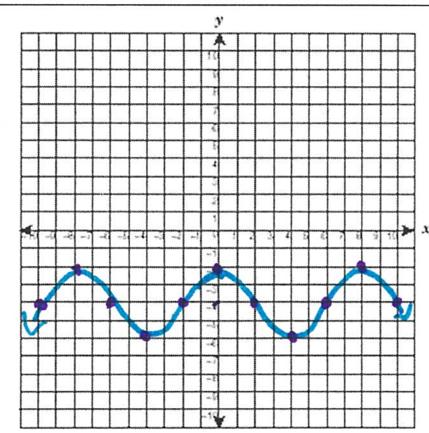
Graph the following:

a. $f(g(x))$
b. $(f + g)(x)$
c. $f \cdot g(x)$

~~h.~~ a

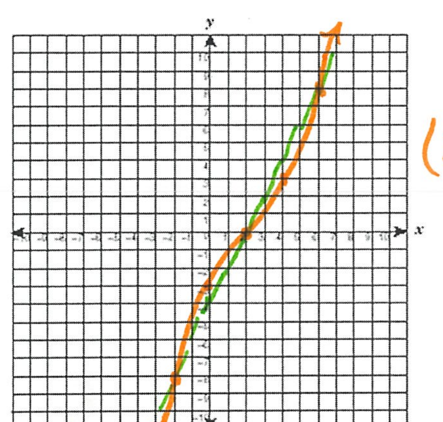
$f(g(x))$

$2\cos\frac{\pi}{4}x - 4$



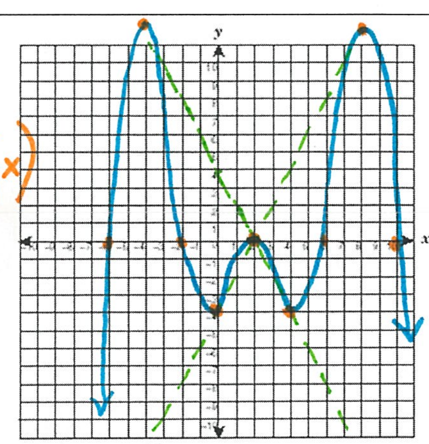
~~i.~~ b

$2x - 4 + \cos\frac{\pi}{4}x$



~~j.~~ c

$(2x-4)(\cos\frac{\pi}{4}x)$



$\star \quad \sin^2 x + \cos^2 x = 1, \quad 1 + \cot^2 x = \csc^2 x, \quad \tan^2 x + 1 = \sec^2 x$

Module 6

Eliminate the parameter to write the parametric equations as a rectangular equation.

41. $x = 3 \csc t \rightarrow \frac{x}{3}$

$y = 3 \cot^2 t \rightarrow \sqrt{\frac{y}{3}}$

$1 + \left(\sqrt{\frac{y}{3}}\right)^2 = \left(\frac{x}{3}\right)^2$

$1 = \frac{x^2}{9} - \frac{y}{3}$

42. $x = 4 \sin(2t) \rightarrow \frac{x}{4}$

$y = 2 \cos(2t) \rightarrow \frac{y}{2}$

$\left(\frac{x}{4}\right)^2 + \left(\frac{y}{2}\right)^2 = 1$

$\frac{x^2}{16} + \frac{y^2}{4} = 1$

43. $x = \cos t \rightarrow x$

$y = 2 \sin^2 t \rightarrow \sqrt{\frac{y}{2}}$

$x^2 + \left(\sqrt{\frac{y}{2}}\right)^2 = 1$

$x^2 + \frac{y^2}{4} = 1$

44. $x = 4 \sec t \rightarrow \frac{x}{4}$

$y = 3 \tan t \rightarrow \frac{y}{3}$

$\left(\frac{y}{3}\right)^2 + 1 = \left(\frac{x}{4}\right)^2$

$1 = \frac{x^2}{16} - \frac{y^2}{9}$

45. $x = 4 + 2 \cos t \rightarrow \frac{x-4}{2}$

$y = -1 + 4 \sin t \rightarrow \frac{y+1}{4}$

$\left(\frac{x-4}{2}\right)^2 + \left(\frac{y+1}{4}\right)^2 = 1$

$\frac{(x-4)^2}{4} + \frac{(y+1)^2}{16} = 1$

46. $x = -4 + 3\tan^2 t \rightarrow \sqrt{\frac{x+4}{3}}$

$y = 7 - 2 \sec t \rightarrow \frac{y-7}{-2}$

$\left(\sqrt{\frac{x+4}{3}}\right)^2 + 1 = \left(\frac{y-7}{-2}\right)^2$

$1 = \frac{(y-7)^2}{4} - \frac{(x+4)}{3}$

Problems 11 and 12: Write two new sets of parametric equations for the following rectangular equations.

47. $y = (x + 2)^3 - 4$

let $t = x + 1 \rightarrow x = t - 1$

$y = (t+1)^3 - 4$

OR

$x = 2t$

$y = (2t+2)^3 - 4$

48. $x = \sqrt{y^2 - 3}$

$x = t$

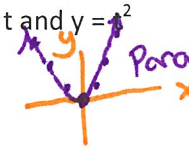
$y = \sqrt{t^2 + 3}$

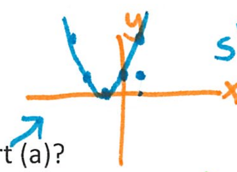
OR

$x = \sqrt{9t^2 - 3}$

$y = 3t$

49. For the parametric equations $x = t$ and $y = t^2$





shift left 1

a) Sketch the graph.
b) Graph $x = t - 1$ and $y = t^2$. How does this compare to the graph in part (a)?
c) Graph $x = t$ and $y = t^2 - 3$. How does this compare to the graph in part (a)?

shift down 3

d) Write parametric equations which will give the graph in part (a) a vertical stretch by a factor of 2 and move the graph 5 units to the right. (Hint: Verify on your calculator!)

$x = t + 5 \quad y = 2t^2$

50. Do the following sets of parametric equations cross at the same time so they collide or do their paths just intersect? Justify your answer.

c) $x_1 = 4t$ and $\quad x_2 = 5t - 6$
$\quad y_1 = ½t + 5 \quad y_2 = t + 2$

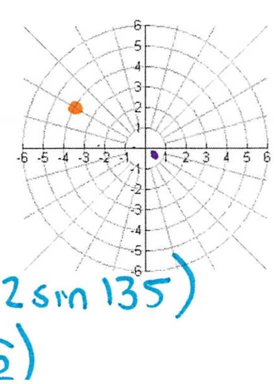
$x \rightarrow 4t = 5t - 6$

$t = 6$

$y_1 = 8, \ y_2 = 8$

collide at $t = 6$ $(24, 8)$

Plot the point given in polar coordinates and find two additional polar representations of the point, using -360º < θ < 360º.



51. (4, 150º)

$(-4, -30)$

$(4, -210)$ $(-4, 330)$

52. (- ½ , -210º)

$(\frac{1}{2}, -30)$

$(-½, 150)$ $(\frac{1}{2}, 330)$

Find the corresponding rectangular coordinates for the point given in polar coordinates.

$x = r\cos\theta \quad y = r\sin\theta$

53. $(5, -\frac{\pi}{6})$

$x = 5\cos\left(-\frac{\pi}{6}\right), \ y = 5\sin\left(-\frac{\pi}{6}\right)$

$\left(\frac{5\sqrt{3}}{2}, -\frac{5}{2}\right)$

54. (-2, 135º)

$(-2\cos 135, -2\sin 135)$

$(-\sqrt{2}, \sqrt{2})$

Find the polar coordinates for 0 < θ < 360º. Pay attention to the quadrant!

55. (-4, -4)

3rd Q

$r = \sqrt{32} = 4\sqrt{2}$

$\tan\theta = 1 \to 45°$

$225°$

$(4\sqrt{2}, \frac{5\pi}{4})$

56. $(2, -2\sqrt{3})$

$r = \sqrt{4+12} = 4$

$\tan\theta = -\sqrt{3} \to \theta = -\frac{\pi}{3}$ or $-60°$

$(4, -\frac{\pi}{3})$

Convert the rectangular equation to polar form. (solve for r)

57. $x^2 + y^2 - 6y = 0$

$r = 6\sin\theta$

$r^2 - 6r\sin\theta = 0$

$r(r - 6\sin\theta) = 0$

58. $5x + 7y = 12$

$5r\sin\theta + 7r\cos\theta = 12$

$r = \dfrac{12}{5\sin\theta + 7\cos\theta}$

Convert the polar equation to rectangular form.

59. r = 4 sinθ

$x = r\sin\theta$

$r = 4\frac{x}{r}$

$y^2 = -x^2 + 4x$

$r^2 = 4x$

$x^2 + y^2 = 4x$

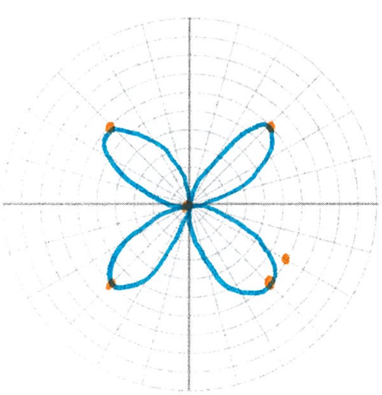
60. $r = \frac{4}{1-\cos\theta}$

$\sqrt{x^2+y^2} = \dfrac{4}{1 - \frac{y}{r}}$

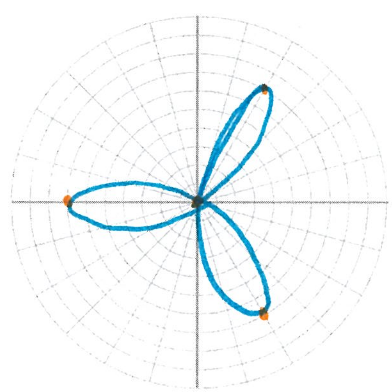
$x^2 + y^2 = \dfrac{16}{\left(1 - \frac{y}{\sqrt{x^2+y^2}}\right)^2}$

Graph

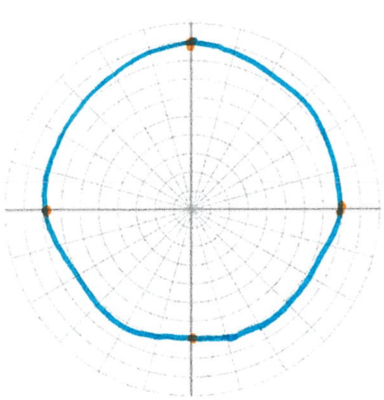
61. $r = 6\sin 2\theta$



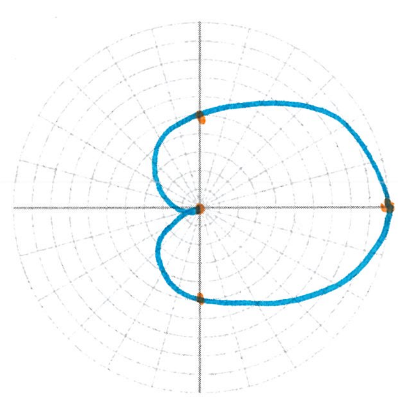
62 $r = -7\cos 3\theta$



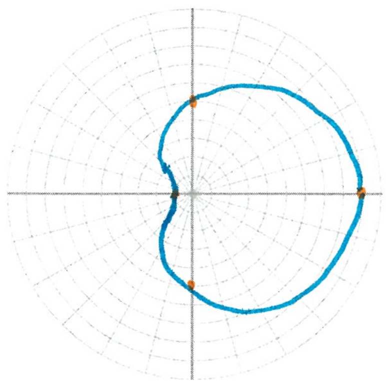
63. $r = 8 + \sin\theta$



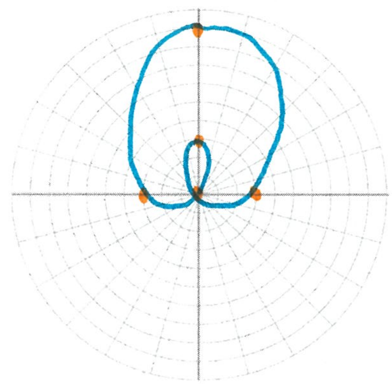
64. $r = 5 + 5\cos\theta$



65. $r = 5 + 4\cos\theta$



66. $r = 3 + 6\sin\theta$



COMPLEX NUMBER PRACTICE

67. Write the complex numbers in polar form (trigonometric form) $r(\cos\theta + i\sin\theta)$

(a) $z = 2 - 2i$ $2\sqrt{2}(\cos(-45) + i\sin(-45))$

(b) $w = -1 - \sqrt{3}i$ $2(\cos(240) + i\sin(240))$

(c) $y = 4\sqrt{3} + 4i$ $8(\cos(30) + i\sin(30))$

(d) $x = -\sqrt{5} + \sqrt{5}i$ $\sqrt{10}(\cos(135) + i\sin(135))$

$r = \sqrt{a^2 + b^2}$ $a + bi$

$\tan\theta = \left(\frac{b}{a}\right)$

68. Using the complex numbers w-z above, simplify the following using polar form.

a. $z \cdot w \rightarrow 4\sqrt{2}(\cos 195 + i\sin 195)$

b. $x \div w \rightarrow \frac{\sqrt{10}}{2}(\cos(-105) + i\sin(-105))$

c. $y \cdot x \rightarrow 8\sqrt{10}(\cos(165) + i\sin(165))$

d. $z^7 \rightarrow (2\sqrt{2})^7(\cos(-315) + i\sin(-315))$

e. w^4

e) $2^4(\cos(960) + i\sin(960))$

69. Write in simplified polar form.

a. $(3 + 2i)^{30}$

$\left(\sqrt{13}(\cos(33.7) + i\sin(33.7))\right)^{30}$

$\sqrt{13}^{30}(\cos(30 \cdot 33.7) + i\sin(30 \cdot 33.7))$

b. $(2 - 6i)^{21} \rightarrow \left(2\sqrt{10}(\cos(-71.6) + i\sin(-71.6))\right)^{21}$

$(2\sqrt{10})^{21}(\cos(21 \cdot -71.6) + i\sin(21 \cdot -71.6))$

70. ECCENTRICITY – Find the eccentricity and identify the conic section

a. $r = \frac{7}{3 - \frac{2}{5}\cos\theta}$ $\cdot \frac{1}{3}$ / $\cdot \frac{1}{3}$

$\frac{7/3}{1 - \frac{2}{15}\cos\theta}$

$e = \frac{2}{15}$, Ellipse

b. $r = \frac{4}{4 + \frac{1}{4}\sin\theta}$ $\cdot \frac{1}{4}$ / $\cdot \frac{1}{4}$

$= \frac{1}{1 + \frac{1}{16}\sin\theta}$

$e = \frac{1}{16}$ Ellipse

Module 7

71. $A = \{11, 12.5, 13, 9, 12.5\}$ $B = \{1.2, 2.1, 1.8, 1.7, 1.9\}$

a. Find the standard deviation of each set.

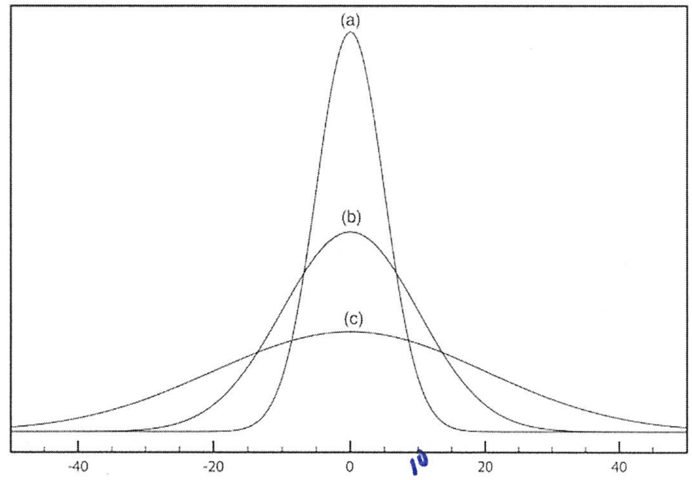
$\sigma_A = 1.46$ $\sigma_B = .301$

b. What is one standard deviation above A? Two below B?

72. Estimate the standard deviation for each

$\sigma_a = 5$ $\sigma_c = 20$ or 15

$\sigma_B = 10$



73. Does a larger standard deviation make the curve more wide or more narrow?

More wide

74. Explain the difference between a standard deviation of 1.2 versus a standard deviation of 34.

the average distance that data is from the mean is lower when $\sigma = 1.2$ versus $\sigma = 34$

75. Identify each situation as a survey, observational study, or an experiment.

a) Stark Industries wants to know what their customer satisfaction is. They randomly select 123 customers and ask them. survey

b) To determine if the new Nike Frees make you run faster, the Nike team randomly assign people into two groups: Group 1 receives Nike Frees and group 2 receives a placebo (look-alike shoe). Both groups are timed and the results are compared. experiment

c) To determine whether exercise raises test scores, researchers randomly selected a group of participants and recorded the number of hours each participant exercised and the rise or fall of their test scores. observational study

76. Provide an example for each of the following

a. Simple random sample

random generator

b. Cluster random sample

Pick 10 2nd period classes to survey

c. Systematic random sample

Pick every 5th person

d. Stratified random sample

Pick 3 people from every 2nd period class to survey.

77. A normal distribution of scores has a standard deviation of 10. Find the z-scores corresponding to each of the following values:

a) A score that is 20 points above the mean.

$z = \frac{20}{10} = 2 \rightarrow$ 97.72%

b) A score that is 10 points below the mean.

$z = \frac{-10}{10} = -1 \rightarrow$ 15.87%

c) A score that is 15 points above the mean

$z = \frac{15}{10} = 1.5 \rightarrow$ 93.32%

d) A score that is 30 points below the mean.

$z = \frac{-30}{10} = -3 \rightarrow$ 0.13%

78. The Welcher Adult Intelligence Test Scale is composed of a number of subtests. On one subtest, the raw scores have a mean of 35 and a standard deviation of 6. Assuming these raw scores form a normal distribution:

e) What number represents the 65th percentile (what number separates the lower 65% of the distribution)?

$z = .385 = \frac{x - 35}{6} = 37.31$

f) What number represents the 90th percentile?

$1.28 = \frac{x-35}{6} \Rightarrow 42.68$

g) What is the probability of getting a raw score between 28 and 38?

$\frac{28-35}{6} = -1.17 \qquad \frac{38-35}{6} = .5 \qquad .6915 - .1210$

57.05%

h) What is the probability of getting a raw score between 41 and 44?

$\frac{41-35}{6} = 1 \qquad \frac{44-35}{6} = 1.5 \qquad .9332 - .8413$

9.19%

79. Scores on the SAT form a normal distribution with $\mu = 500$ and $\sigma = 100$.

i) What is the minimum score necessary to be in the top 15% of the SAT distribution?

$1.04 = \frac{x-500}{100}$ 604

j) Find the range of values that defines the middle 80% of the distribution of SAT scores (372 and 628).

90% $\rightarrow z = 1.28 = \frac{x-500}{100} \rightarrow 628$

10% $\rightarrow z = -1.28 = \frac{x-500}{100} \rightarrow 372$

80. For a normal distribution, find the z-score that separates the distribution as follows:

k) Separate the highest 30% from the rest of the distribution.

.525

l) Separate the lowest 40% from the rest of the distribution.

-.25

m) Separate the highest 75% from the rest of the distribution.

-.675

Module 8

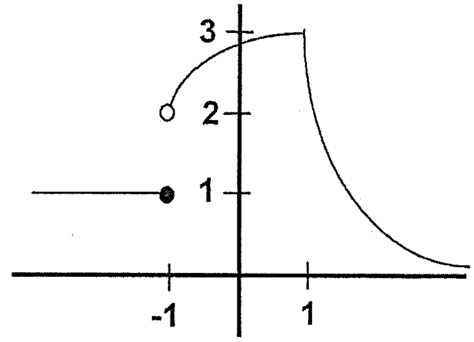
81. The picture on the left shows the graph of a certain function.
Based on that graph, answer the questions:

a) $\lim_{x\to -1^-} f(x) =$ ~~3~~ 1

b) $\lim_{x\to -1^+} f(x) = 2$

c) $\lim_{x\to 1} f(x) = 3$

d) $\lim_{x\to 0} f(x) = 2.8$



e) Is the function continuous at x = -1?
NO

f) Is the function continuous at x = 1?
Yes

g) Is the function differentiable at x = -1?
Skip

h) Is the function differentiable at x = 1?
skip

i) Is f'(0) positive, negative, or zero?
(slope) Positive

k) What is f'(-2) ?
0 (slope)

Skip g, h

82. Find each of the following limits (show your work):

a) $\lim_{x\to 3} 4\pi = 4\pi$

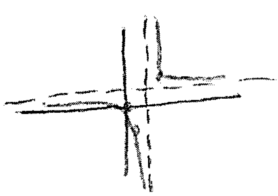
b) $\lim_{x\to 3} \frac{x^2-2x}{x+3} = \frac{1}{2}$

$\frac{9-6}{6}$

c) $\lim_{x\to 3} \frac{3-x}{x^2+2x-15} = \frac{-(x-3)}{(x+5)(x-3)}$

$\frac{0}{9+6-15} = \frac{0}{0}$ $\lim_{x\to 3} \frac{-1}{x+5} = -\frac{1}{8}$

d) $\lim_{x\to 1^+} \frac{x}{x-1} = \infty$



e) $\lim_{x\to 1^-} \frac{x}{x-1} = -\infty$

f) $\lim_{x\to 1} \frac{x}{x-1} =$ DNE

l) $\lim_{x\to -\infty} \frac{3x^2-1}{2-3x-4x^2} = -3/4$

HA ⇒ $y = -3/4$

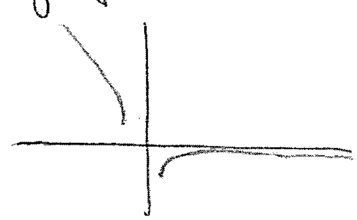
m) $\lim_{x\to -\infty} \frac{3x^2-1}{2-3x} = -\infty$

	x	$2/3$	
$-3x$	$3x^2$	$-2x$	$-1/3$
2	$2x$	$4/3$	

Slant : $y = x + 2/3$
Asymptote

n) $\lim_{x\to\infty} \sqrt{x^2-1} - x =$ ~~[illegible]~~ 0

graph



83. Consider the following function: $f(x) = \begin{cases} x^2, & \text{if } x \ge 0 \\ x-2, & \text{if } x < 0 \end{cases}$

a) Find $\lim_{x \to 0^-} f(x)$ $= -2$

b) Find $\lim_{x \to 0^+} f(x)$ $= 0$

c) Find $\lim_{x \to 2} f(x)$ (note that x approaches *two*, not *zero*) $= 4$

d) Is the function continuous at x = 0, No, $\lim_{x \to 0} f(x) = DNE$

f) Is $f(x) = \begin{cases} \dfrac{x^2-1}{x+1}, & \text{if } x \ne -1 \\ 17 & \text{if } x = -1 \end{cases}$ continuous at -1 ? If not, is the discontinuity removable? NOT continuous

$\frac{(-1)^2-1}{-1+1} = \frac{0}{0}$ $\quad \frac{\cancel{(x+1)}(x-1)}{\cancel{x+1}} = 0$ Yes, it is removable.

g) Is there a value of *k* that makes the function *g* continuous at *x* = *0*? If so, what is that value?

$g(x) = \begin{cases} x-2, & \text{if } x \le 0 \\ k(3-2x) & \text{if } x > 0 \end{cases}$

$x - 2 = k(3-2x)$ when $x = 0$

$-2 = k(3)$

$k = -2/3$

84. Find the value of *k*, if any, that would make the following function continuous at ~~x=4~~. $x = 2$

$f(x) = \begin{cases} \dfrac{x^2-4}{x-2} & \text{if } x \ne 2 \\ k & \text{if } x = 2 \end{cases}$

$\frac{x^2-4}{x-2} = k$ when $x = 2$

$\frac{\cancel{(x-2)}(x+2)}{\cancel{x-2}} = k$

$x + 2 = k$, $\boxed{k = 4}$

85. Use the *definition* of derivative to find the derivative of the function

a. $f(x) = 3x^2 + 2$. $\lim_{h \to 0} \frac{3(x+h)^2 + 2 - 3x^2 - 2}{h} \to \frac{\cancel{3x^2} + 6x\cancel{h} + 3h^{\cancel{2}} + \cancel{2} - \cancel{3x^2} - \cancel{2}}{\cancel{h}}$

$f'(x) = 6x$

$\lim_{h \to 0} 6x + 3h = 6x$

b. $f(x) = \dfrac{1}{1-x}$

$\lim_{h \to 0} \frac{\frac{1}{1-x-h} - \frac{1}{1-x}}{h} = \frac{\frac{\cancel{1} - \cancel{x} - \cancel{1} + \cancel{x} + h}{(1-x-h)(1-x)}}{h} = \frac{\cancel{h}}{(1-x-h)(1-x)} \cdot \frac{1}{\cancel{h}} = \lim_{h \to 0} \frac{1}{(1-x-h)(1-x)}$

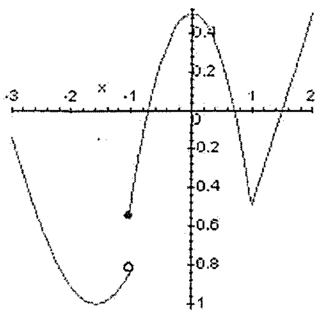
$f'(x) = \frac{1}{2(1-x)}$

c. $f(x) = \sqrt{x}$

$\lim_{h \to 0} \frac{\sqrt{x+h} - \sqrt{x}}{h} = \frac{\cancel{x} + h - \cancel{x}}{h(\sqrt{x+h} + \sqrt{x})} = \lim_{h \to 0} \frac{1}{\sqrt{x+h} + \sqrt{x}} = \frac{1}{2\sqrt{x}}$

$f'(x) = \frac{1}{2\sqrt{x}}$

86. Consider the function whose graph you see below, and find a number $x = c$ such that



a) f is not continuous at $x = a$ $X = -1$

b) f is continuous but not differentiable at $x = b$, $X = 1$

c) f' is positive at $x = c$, $X = 2$ (anywhere the slope is positive)

d) f' is negative at $x = d$ $X = -3$ (decreasing slope)

e) f' is zero at $x = e$ $X = 0$ (max or min)

f) f' does not exist at $x = f$

$X = -1$, $X = 1$

87. Please find the derivative for each of the following functions (do not simplify unless you think it is helpful).

a. $f(x) = \pi^2 + x^2 + \sin(x) + \sqrt{x}$

skip

b. $f(x) = x^2(x^4 - 2x)$

$f'(x) = 6x^5 - 6x^2$

work on next page

c. $f(x) = x^2(x^3 - \frac{1}{x})$

$f(x) = x^5 - X$

$f'(x) = 5x^4 - 1$

d. $f(x) = 3x^5 - 2x^3 + 5x - \sqrt{2}$

$f'(x) = 15x^4 - 6x^2 + 5$

e. $f(x) = \dfrac{x^4 - 2x + 3}{x^2}$

$f'(x) = \dfrac{3x^4 + 2x - 6}{x^2}$

f. $f(x) = \sin^2(x)$

skip.

see next few pages for work!

87 a

$$\lim_{h\to 0} \frac{\pi^2 + (x+h)^2 + \sin(x+h) + \sqrt{x+h} - \pi^2 - x^2 - \sin x - \sqrt{x}}{h}$$

$$\cancel{\pi^2} + \cancel{x^2} + 2xh + h^2 + \sin(x+h) + \sqrt{x+h} - \cancel{\pi^2} - \cancel{x^2} - \sin x - \sqrt{x}$$

$$\frac{2xh + h^2 + \sin x \cosh + \cos x \sin h + \sqrt{x+h} - \sqrt{x} - \sin x}{h}$$

87b

$$\lim_{h\to 0} \frac{(x+h)^2\left((x+h)^4 - 2(x+h)\right) - x^2(x^4 + 2x)}{h}$$

$$\lim_{h\to 0} \frac{(x+h)^6 - 2(x+h)^3 - x^6 + 2x^3}{h}$$

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1

$$\lim_{h\to 0} \frac{\cancel{x^6} + 6x^5h + 15x^4h^2 + 20x^3h^3 + 15x^2h^4 + 6xh^5 + h^6 - \cancel{2x^3} - 6x^2h - 6xh^2 - 2h^3 - \cancel{x^6} + \cancel{2x^3}}{\cancel{h}}$$

$$\lim_{h\to 0} \frac{6x^5 + 15x^4h + 20x^3h^2 + 15x^2h^3 + 6xh^4 + h^5 - 6x^2 - 6xh - 2h^2}{h}$$

$$\boxed{f'(x) = 6x^5 - 6x^2}$$

87c) $$\lim_{h\to 0} \frac{(x+h)^5 - (x+h) \overset{-(x^5-x)}{- x^5 + x}}{h}$$ 1 5 10 10 5 1

$$\lim_{h\to 0} \frac{\cancel{x^5} + 5x^4\cancel{h} + 10x^3h^{\cancel{2}\,1} + 10x^2h^{\cancel{3}\,2} + 5xh^{\cancel{4}\,3} + h^{\cancel{5}\,4} \cancel{-x} - \cancel{h}^{\,1} - \cancel{x^5} + \cancel{x}}{\cancel{h}}$$

$$\lim_{h\to 0} 5x^4 + 10x^3h + 10h^2x^2 + 5xh^3 + h^4 - 1$$

$$f'(x) = 5x^4 - 1$$

87d) $$\lim_{h\to 0} \frac{3(x+h)^5 - 2(x+h)^3 + 5(x+h) \cancel{-\sqrt{2}} - 3x^5 + 2x^3 - 5x \cancel{+\sqrt{2}}}{h}$$

$$\lim_{h\to 0} \frac{3(x^5+5x^4h+10x^3h^2+10x^2h^3+5xh^4+h^5) - 2(x^3+3x^2h+3xh^2+h^3) + 5x + 5h - 3x^5 + 2x^3 - 5x}{h}$$

$$\lim_{h\to 0} \frac{\cancel{3x^5} + 15x^4\cancel{h} + 30x^3h^{\cancel{2}\,1} + 30x^2h^{\cancel{3}\,2} + 15xh^{\cancel{4}\,3} + h^{\cancel{5}\,4} - \cancel{2x^3} - 6x^2\cancel{h} - 6xh^{\cancel{2}} - 2h^{\cancel{3}} + \cancel{5x} + 5\cancel{h} - \cancel{3x^5} + \cancel{2x^3} - \cancel{5x}}{\cancel{h}}$$

$$\lim_{h\to 0} \underline{15x^4} + 30x^5h + 30x^2h^2 + 15xh^3 + h^4 \underline{-6x^2} - 6xh - 2h^2 + \underline{5}$$

$$f'(x) = 15x^4 - 6x^2 + 5$$

87e

$$\lim_{h\to 0} \frac{\dfrac{x^2\left((x+h)^4-2(x+h)+3\right)}{x^2\left((x+h)^2\right)} - \dfrac{(x^4-2x+3)(x^2+2xh+h^2)}{(x^2)(x+h)^2}}{h}$$

$$\lim_{h\to 0} \frac{\dfrac{\overbrace{x^2(x^4+4x^3h+6x^2h^2+4xh^3+h^4-2x-2h+3)}^{\cancel{x^6}+4x^5h+6x^4h^2+4x^3h^3+x^2h^4-\cancel{2x^3}-2x^2h+\cancel{3x^2}} - \cancel{x^6}+\cancel{2x^3}-\cancel{3x^2}-2x^5h+4x^2h-6xh-x^4h^2+2xh^2-3h^2}{(x)(x+h)^2}}{h}$$

$$\lim_{h\to 0} \frac{\dfrac{4x^5h+6x^4h^2+4x^3h^3+x^2h^4-2x^2h-2x^5h+4x^2h-6xh-x^4h^2+2xh^2-3h^2}{(x)(x+h)^2}}{\cancel{h}}$$

$$\lim_{h\to 0} \frac{4x^5+6x^4h+4x^3h^2+x^2h^3-2x^2-2x^5+4x^2-6x-x^4h+2xh-3h}{(x)(x+h)^2}$$

$$\frac{4x^5-2x^2-2x^5+4x^2-6x}{x^3}$$

$$f'(x)=\frac{3x^5+2x^2-6x}{x^3}$$

$$\boxed{f'(x)=\frac{3x^4+2x-6}{x^2}}$$

88. Find the equation of the tangent line to the function at the given point:

a) $f(x) = x^2 - x + 1$, at x = 0

$\boxed{\text{① Find derivative}}$

$f'(x) = \lim_{h \to 0} \frac{(x+h)^2 - (x+h) + 1 - (x^2 - x + 1)}{h}$

$\frac{x^2 + 2xh + h^2 - x - h + 1 - x^2 - x - 1}{h}$

$\boxed{\text{② use point slope formula}}$

$f'(x) = 2x - 1$ $\quad f(0) = 1$ $\quad$ point $(0, 1)$

$f'(0) = -1$ $\quad$ slope = −1

$y - 1 = -1(x - 0)$

$\boxed{y = -x + 1}$

b) $f(x) = x^3 - 2x$, at x = 1

$\boxed{\text{① Find derivative}}$

$f'(x) = \lim_{h \to 0} \frac{(x+h)^3 - 2(x+h) - (x^3 - 2x)}{h}$

$= \frac{x^3 + 3x^2h + 3xh^2 + h^3 - 2x - 2h - x^3 + 2x}{h}$

$f'(x) = 3x^2 - 2$

$\boxed{\text{② Point-Slope formula at } x = 1}$

$f'(1) = 1 \leftarrow$ slope $\quad$ point $\rightarrow f(1) = -1 \quad (1, -1)$

$y + 1 = 1(x - 1)$

$\boxed{y = x - 2}$

89. Suppose the function $f(x) = \dfrac{x^4 - 2x + 3}{x^2}$ indicates the position of a particle.

① Simplify: $x^2 - \frac{2}{x} + \frac{3}{x^2}$

a) Find the velocity after 10 seconds First derivative

$f'(x) = 2x + \frac{2}{x^2} - \frac{6}{x^3}$ $\quad f'(10) = 20 + \frac{2}{100} - \frac{6}{1000} = 20.014$

b) Find the acceleration after 10 seconds Second derivative (Derivative of the first derivative)

$f''(x) = 2 - \frac{4}{x^3} + \frac{18}{x^4}$ $\quad f''(10) = 2 - \frac{4}{1000} + \frac{18}{10000} = 1.9978$

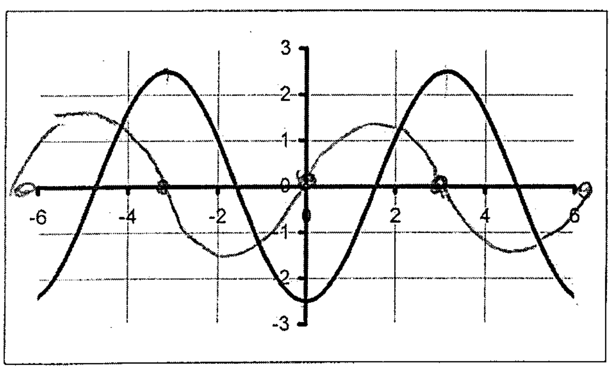
c) When is the particle at rest (other than for t = 0)

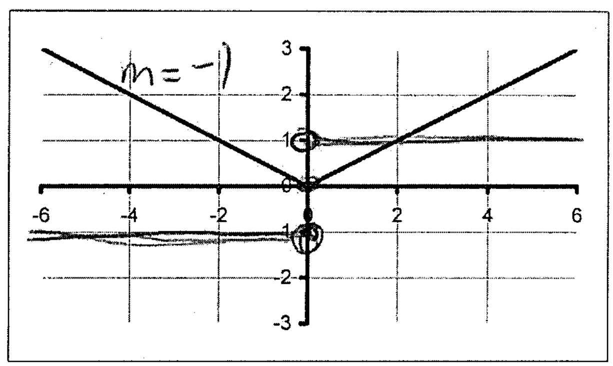
$f'(x) = 2x + \frac{2}{x^2} - \frac{6}{x^3}$ $\quad$ (when derivative is zero)

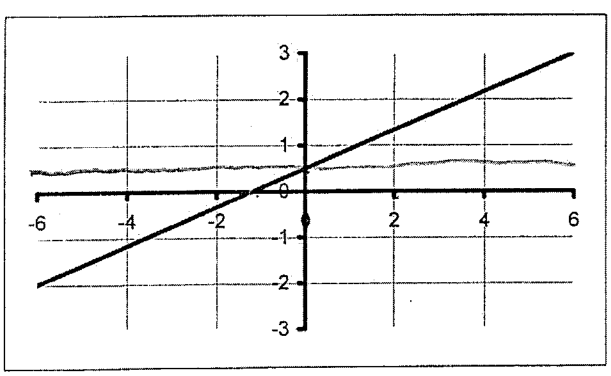
d) When is the particle moving forward and when backward

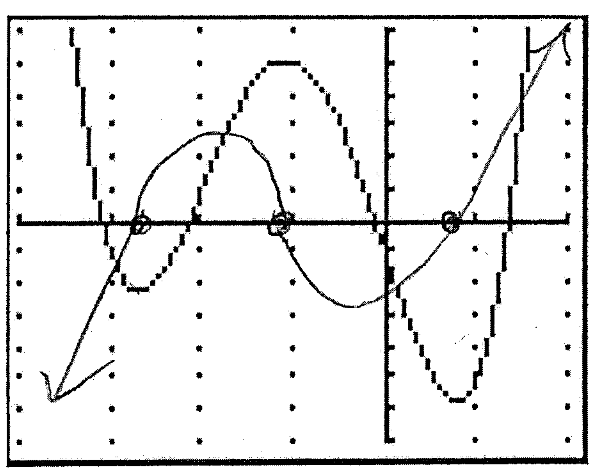
(when derivative is neg/pos)

90. Sketch the graph of the derivative of each of the following functions on the same graph.





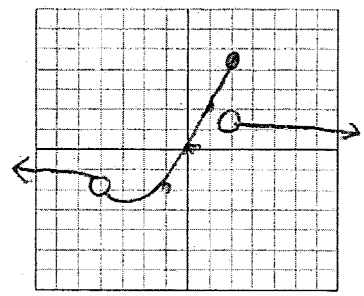




91. Draw a graph with the following conditions. answers may vary.

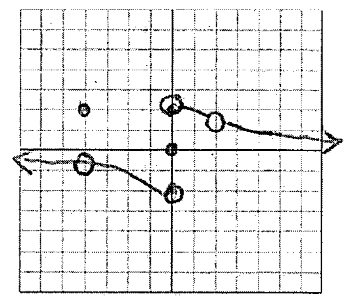
Function #1

- f(0) = 0
- f(1) = 2
- f(-1) = -2
- at f(3) there is a non-removable discontinuity
- at f(-4) there is a removable discontinuity
- $\lim_{x\to-\infty} f(x) = -1$
- $\lim_{x\to\infty} f(x) = 1$



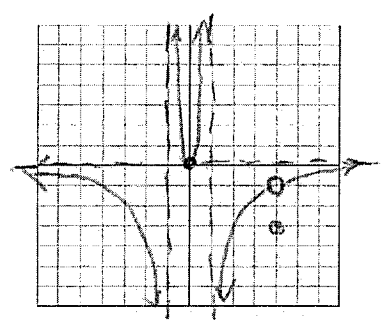
Function #2

- $\lim_{x\to\infty} f(x) = 0$
- $\lim_{x\to-\infty} f(x) = 0$
- f(0) = 0
- at f(-4) there is a removable discontinuity
- $\lim_{x\to 2} f(x)$ exists, but the graph is discontinuous
- $\lim_{x\to 0^+} f(x) = 2$
- $\lim_{x\to 0^-} f(x) = -2$



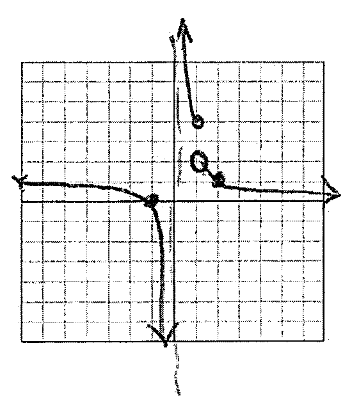
Function #3

- $\lim_{x\to\infty} f(x) = 0$
- $\lim_{x\to-\infty} f(x) = 0$
- f(0) = 0
- $\lim_{x\to 1^+} f(x) = -\infty$
- $\lim_{x\to -1^-} f(x) = -\infty$
- $\lim_{x\to 1^-} f(x) = \lim_{x\to -1^+} f(x) = \infty$
- $\lim_{x\to 4} f(x)$ exists, but the graph is discontinuous

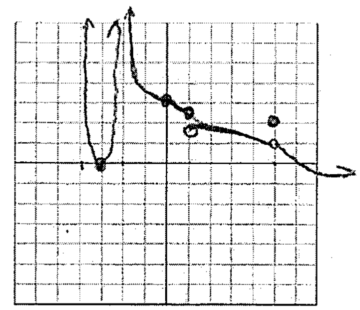


Function #4

- $\lim_{x\to\infty} f(x) = 0$
- $\lim_{x\to-\infty} f(x) = 1$
- f(-1) = 0
- $\lim_{x\to 1} f(x)$ does not exist
- $\lim_{x\to 0^+} f(x) = \infty$
- $\lim_{x\to 0^-} f(x) = -\infty$
- f(2) = 1

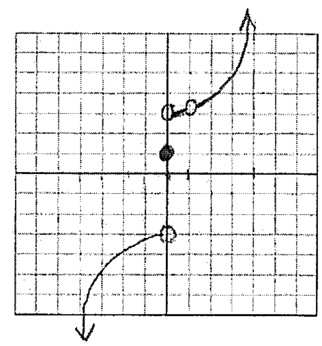


Function #5



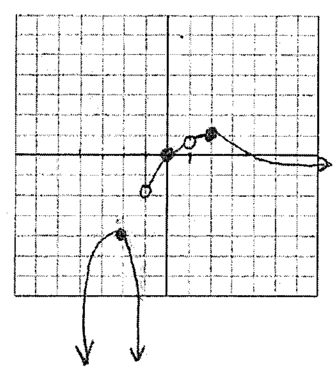
- f(-3) = 0
- $\lim_{x\to\infty} f(x) = -1$
- $\lim_{x\to -2^+} f(x) = \lim_{x\to -2^-} f(x) = \infty$
- at f(5) there is a removable discontinuity
- $\lim_{x\to 1} f(x)$ does not exist
- f(0) = 3
- $\lim_{x\to -4^+} f(x) = \infty$

Function #6



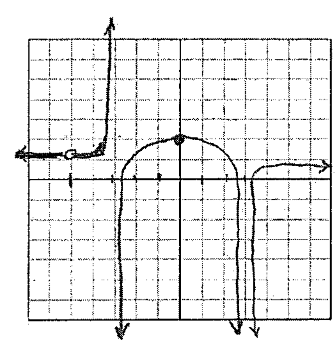
- $\lim_{x\to 0^+} f(x) = 3$
- $\lim_{x\to 0^-} f(x) = -3$
- f(0) = 1
- $\lim_{x\to 1} f(x)$ exists, but the graph is discontinuous
- $\lim_{x\to -4^+} f(x) = -\infty$
- $\lim_{x\to 4^-} f(x) = \infty$

Function #7



- f(0) = 0
- f(2) = 1
- f(-2) = -4
- $\lim_{x\to -1} f(x)$ does not exist
- $\lim_{x\to 1} f(x)$ exists, but the graph is discontinuous
- $\lim_{x\to\infty} f(x) = -1$
- $\lim_{x\to -4^+} f(x) = -\infty$

Function #8



- $\lim_{x\to\infty} f(x) = 1$
- $\lim_{x\to 3^+} f(x) = -\infty$
- $\lim_{x\to -3^+} f(x) = -\infty$
- f(0) = 2
- at f(-5) there is a non-removable discontinuity
- $\lim_{x\to -\infty} f(x) = 1$
- $\lim_{x\to 3^-} f(x) = -\infty$
- $\lim_{x\to -3^-} f(x) = \infty$