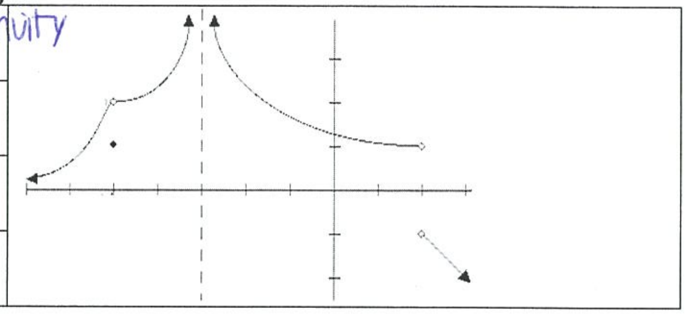
IM3H Module 8 Review Name: Key

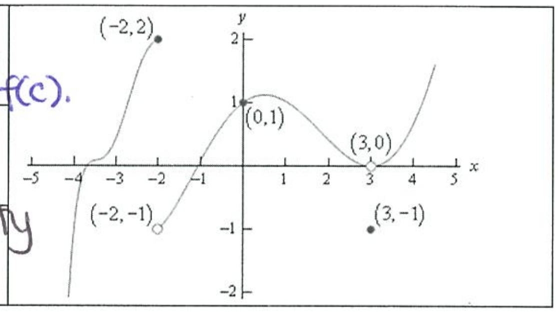
Limits and Derivatives

1. Find the values of the limits using the graph at the right.

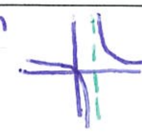
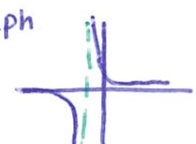
$\lim\limits_{x\to 2} f(x) =$ DNE non-removable discontinuity	
$\lim\limits_{x\to 2^+} f(x) =$ 1	
$\lim\limits_{x\to -3} f(x) = \infty$	
$\lim\limits_{x\to -5} f(x) =$ 2	

2. Describe the continuity of the graph at the following x-values.

If discontinuous, describe the condition of continuity that does not apply and if the discontinuity is removable or non-removable.

At $x = 3$, removable discontinuity The limit of f(x) exists at x=c, but its not equal to f(c).	
At $x = 0$, continuous	
At $x = -2$, non removable discontinuity The limit of f(x) does not exist at x=-2.	

3. Find the values of the following limits algebraically.

$\lim\limits_{x\to 5}\dfrac{x+1}{2x-5} = \dfrac{5+1}{2(5)-5} = \boxed{\dfrac{6}{5}}$	$\lim\limits_{x\to 5}\dfrac{x}{x-5}$ $\boxed{\text{DNE}}$ *graph  or *plug in values close to 5 on both sides
$\lim\limits_{x\to 1}\dfrac{\sqrt{x+3}-2}{x-1} \cdot \dfrac{\sqrt{x+3}+2}{\sqrt{x+3}+2} = \dfrac{x+3-4}{(x-1)(\sqrt{x+3}+2)}$ $= \dfrac{\cancel{x-1}}{(\cancel{x-1})(\sqrt{x+3}+2)} = \dfrac{1}{(\sqrt{x+3}+2)} = \dfrac{1}{(\sqrt{1+3}+2)} = \boxed{\dfrac{1}{4}}$	$\lim\limits_{x\to -2}\dfrac{3x+6}{(x+2)^2} = \dfrac{3(\cancel{x+2})}{(\cancel{x+2})(x+2)} = \dfrac{3}{x+2}$ $\boxed{\text{DNE}}$ *graph 
$\lim\limits_{x\to 7}\dfrac{3x^3-147x}{7-x} = \dfrac{3x(x^2-49)}{-(x-7)} = \dfrac{3x(x+7)(\cancel{x-7})}{-(\cancel{x-7})}$ $= -3x(x+7) = -3(7)(7+7) = \boxed{294}$	$\lim\limits_{x\to\infty}\dfrac{5x^2-9}{7x^2-4} = \boxed{\dfrac{5}{7}}$ *horizontal asymptote. or end behavior

4. **Use the function** $f(x) = \begin{cases} 2 & x \le -2 \\ x^2 - 2 & -2 < x < 3 \\ |x-4| & x \ge 3 \end{cases}$ **to find the values of the following limits:**

$\lim_{x \to -2^-} f(x) = 2$	$\lim_{x \to -2} f(x) = 2$	$\lim_{x \to 3^+} f(x) = 1$	$\lim_{x \to 3} f(x) =$ DNE

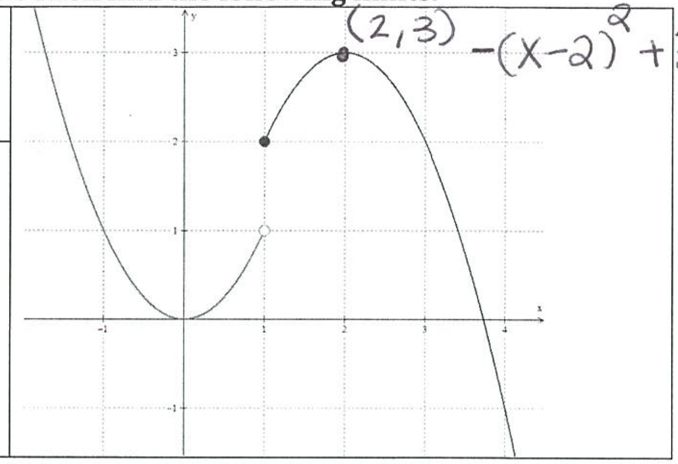
5. **Write the equation of the piece-wise function below. Then find the following limits.**

$$g(x) = \begin{cases} x^2 & x < 1 \\ -(x-2)^2 + 3 & x \ge 1 \end{cases}$$

$\lim_{x \to 1^-} g(x) = 1$

$\lim_{x \to 1^+} g(x) = 2$

$\lim_{x \to 1} g(x) =$ DNE



6. **Find the derivative of each function. That is, find:** $f'(x) = \lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$ *(This equation will NOT be given on test.)*

$f(x) = -\frac{3}{2x-1}$

$$f'(x) = \lim_{h \to 0} \frac{\frac{-3}{2x+2h-1} + \frac{3}{2x-1}}{h}$$

$$\cancel{-6x} \cancel{+3} + \cancel{6x} + 6h \cancel{-3}$$

$$\frac{-3(2x-1) + 3(2x+2h-1)}{(2x+2h-1)(2x-1)} \cdot \frac{1}{h}$$

$$\frac{6\cancel{h}}{(2x+2h-1)(2x-1)} \cdot \frac{1}{\cancel{h}}$$

$$\lim_{h \to 0} \frac{6}{(2x+2h-1)(2x-1)} = \boxed{\frac{6}{(2x-1)^2}}$$

$k(x) = \sqrt{x-5}$

$$f'(x) = \lim_{h \to 0} \frac{\sqrt{x+h-5} - \sqrt{x-5}}{h} \cdot \frac{\sqrt{x+h-5} + \sqrt{x-5}}{\sqrt{x+h-5} + \sqrt{x-5}}$$

$$\cancel{x} + h \cancel{-5} \cancel{-x} \cancel{+5}$$

$$= \frac{x+h-5-(x-5)}{h(\sqrt{x+h-5} + \sqrt{x-5})}$$

$$= \lim_{h \to 0} \frac{\cancel{h}}{\cancel{h}(\sqrt{x+h-5} + \sqrt{x-5})}$$

$$= \boxed{\frac{1}{2\sqrt{x-5}}}$$

7. **Calculate the average rate of change between** $x = 5$ & $x = 8$ **on the function,** $f(x) = -16x^2 - 112x + 1920$.

slope

$$\frac{f(8) - f(5)}{8 - 5} = \frac{0 - 960}{3} = \boxed{-320}$$

8. **Calculate the instantaneous rate of change at** $x = 2$ **on the function** $g(x) = -x^2 + 3$.

① $g'(x) = \lim_{h \to 0} \frac{-(x+h)^2 + 3 - (-x^2+3)}{h}$

$$= \frac{\cancel{-x^2} - 2x\cancel{h} - h^{\cancel{2}} \cancel{+3} \cancel{+x^2} \cancel{-3}}{\cancel{h}}$$

$$\lim_{h \to 0} -2x - h$$

$$g'(x) = -2x \quad \boxed{g'(2) = -4}$$

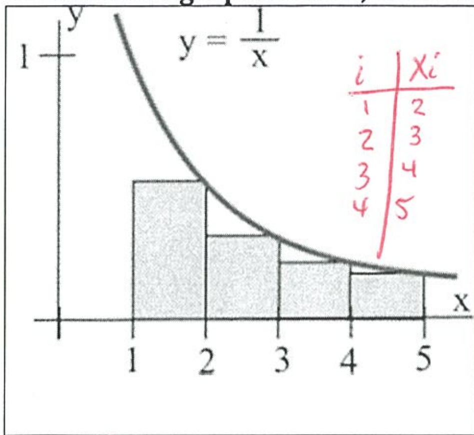
OR

② SLOPE

$$\frac{g(1.9999) - g(2)}{1.999 - 2}$$

$$= \boxed{-4}$$

9. Given the graphs below, write the summation notation for the Riemann Sum shown.

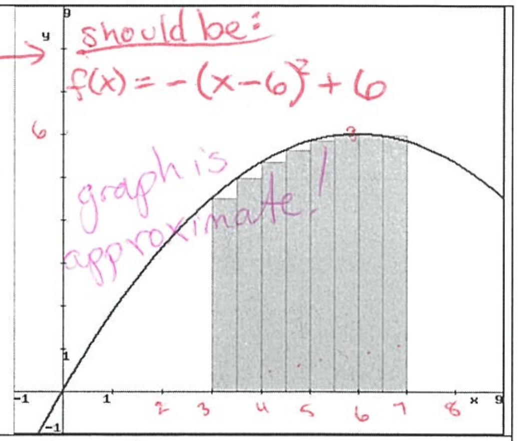


i	x_i
1	2
2	3
3	4
4	5

$f(x) = -(x-6)^{2} + 6$ → should be: $f(x) = -(x-6)^2 + 6$

i	x_i
1	3
2	3.5
3	4
4	4.5
5	5
⋮	

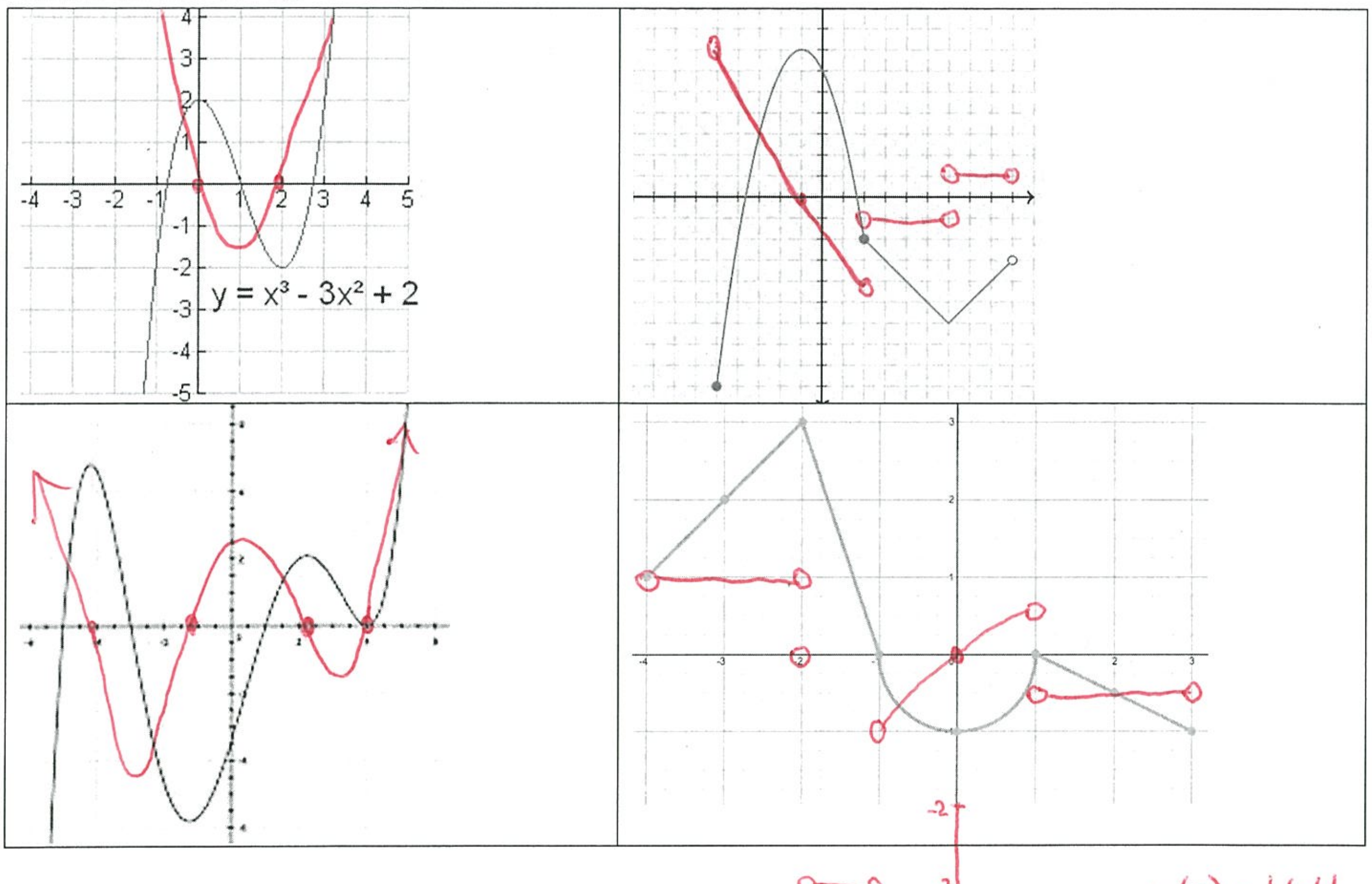
+0.5



Area= $\sum_{i=1}^{4} 1\left(\frac{1}{i+1}\right)$

Area= $\sum_{i=1}^{8} 0.5\left(-((0.5i+2.5)-6)^2+6\right)$

10. Sketch the derivative given function.



11. Given the equation for $g(x) = 3x^2 - 9x + 2$, find the equation of the tangent line at x=9 $g(9) = 164$

$\lim_{h\to 0} \frac{3(x+h)^2 - 9(x+h) + 2 - (3x^2 - 9x + 2)}{h} \rightarrow \lim_{h\to 0} \frac{6xh + 3h^2 - 9h}{h}$

$g'(x) = 6x - 9$

$g'(9) = 45$

$\boxed{y = 45(x-9) + 164}$

12. At what x-values, does the function $f(x) = \frac{2}{x}$ have a slope of -1/3.

$\lim_{h\to 0} \frac{\frac{2}{x+h} - \frac{2}{x}}{h}$

$\lim_{h\to 0} \frac{-2}{(x+h)(x)}$

$\lim_{h\to 0} \frac{2x - 2x - 2h}{(x+h)(x)} \cdot \frac{1}{h}$

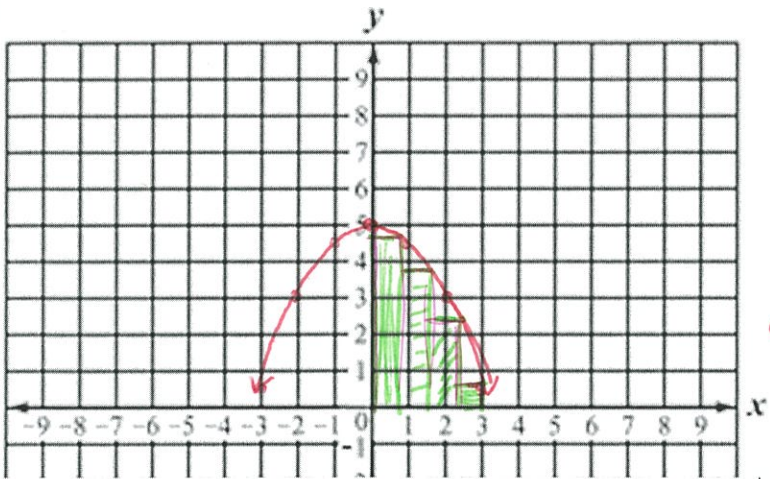
$f'(x) = \frac{-2}{x^2}$

$\frac{-2}{x^2} = -\frac{1}{3}$

$x^2 = 6 \rightarrow \boxed{x = \pm\sqrt{6}}$

13. Consider the function, $g(x) = -\frac{1}{2}x^2 + 5$ for $0 \le x \le 3$. Estimate $\int_0^3 g(x)\,dx$ using right-endpoint rectangles of width 0.75 unit. Follow the steps below as necessary to complete the problem:

a. Sketch a graph showing the curve over the indicated domain. Draw in right-endpoint rectangles from the x-axis to the curve showing a width of 0.75 unit for each rectangle. The rectangles should be below the curve.



b. Find the height of each rectangle by using the y-values of $g(x)$.

4.71875, 3.875, 2.46875

0.5

c. Write an expression for the sum of the areas of the rectangles.

$0.75(4.71875 + 3.875 + 2.46875 + 0.5)$

d. Estimate $\int_0^4 g(x)\,dx$. 3 typo?

8.671875

e. What could you do to make the estimate of $\int_0^4 g(x)\,dx$ more accurate?

Use more rectangles.

14. Sketch the graph of f(x) given the following features.

• $f'(x)$ is positive for all values of x on $(-3, 2)$ • $f'(x)$ is negative if $x < -3$ or $x > 2$. • $f(1) = 5$ • $f(x) = 0$ *only at* $x = 8$	• $f'(x) = 0$ *when* $x = -6, 3, 5,$ • $f(x) > 0$ for all real numbers above x-axis • $f'(x) > 0$ *on two different intervals* • $f(0) = 4$