

IM3H: Module 2 Rational Functions Review

Solve the following equations, state any extraneous solutions

$$1. \frac{y+7}{y-8} = \frac{y-4}{y+5}$$

$$(y+5)(y+7) = (y-8)(y-4)$$

$$y^2 + 12y + 35 = y^2 - 12y + 32$$

$$0 = -24y - 3$$

$$y = \frac{3}{-24} = -\frac{1}{8}$$

$$2. \frac{3x^2 + 20}{5x} = \frac{4x + 1}{5}$$

$$5(3x^2 + 20) = 5x(4x + 1)$$

$$15x^2 + 100 = 20x^2 + 5x$$

$$0 = 5x^2 + 5x + 100$$

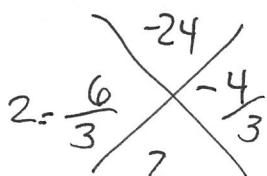
$$0 = 5(x^2 + x + 20)$$

$$0 = 5(x + 5)(x - 4)$$

$$\begin{array}{l|l} 0 = x + 5 & 0 = x - 4 \\ \hline x = -5 & x = 4 \end{array}$$

Identify the key features of each function. Graph each function

$$3. f(x) = \frac{3x^2 + 2x - 8}{x^2 - 4} = \frac{(x+2)(3x-4)}{(x-2)(x+2)}$$



$$(x+2)(3x-4)$$

holes: $x = -2$

y-intercept: $0, 2$

x-intercept(s): $x = 4/3 = 1\frac{1}{3}$

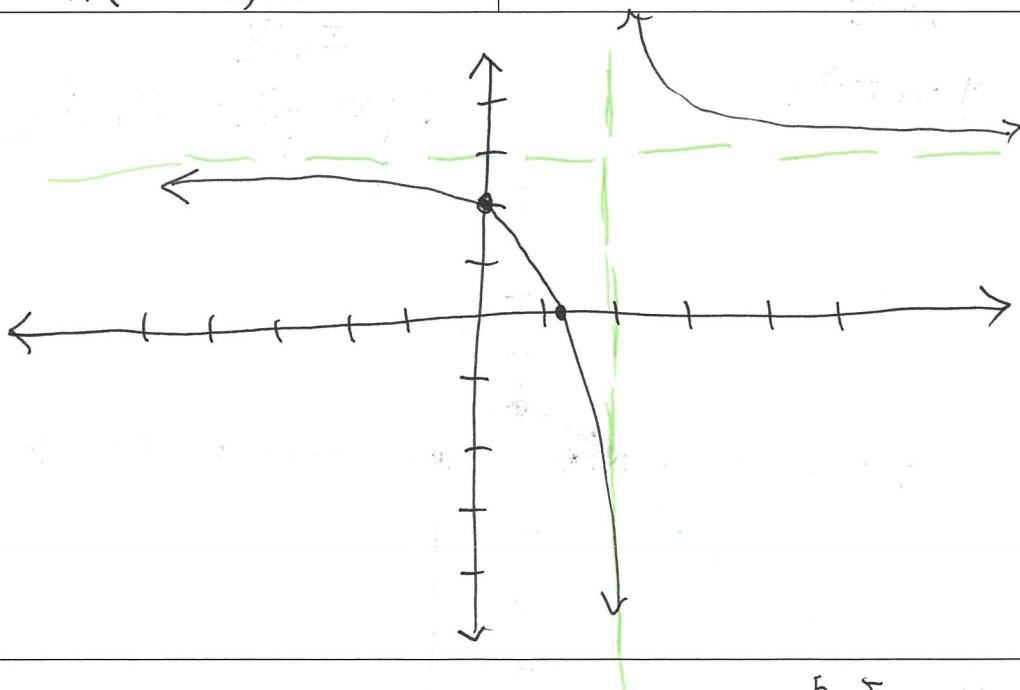
domain: $(-\infty, -2) (-2, 2) (2, \infty)$

vertical asymptotes: $x = 2$

horizontal asymptotes: $y = 3$

slant asymptotes: NONE

Graph.



$$\frac{5}{1} \cdot \frac{5}{5} = 5$$

-24

$$4. f(x) = \frac{6x^2 + 5x - 4}{12x^3 + 12x^2 - 9x} = \frac{(3x+4)(2x-1)}{3x(4x^2 + 4x - 3)}$$

~~$\frac{8}{6}$~~ vertical asymptotes: $x = -\frac{3}{2}$ ~~$x = 0$~~

horizontal asymptotes: $y = 0$

slant asymptotes: NONE

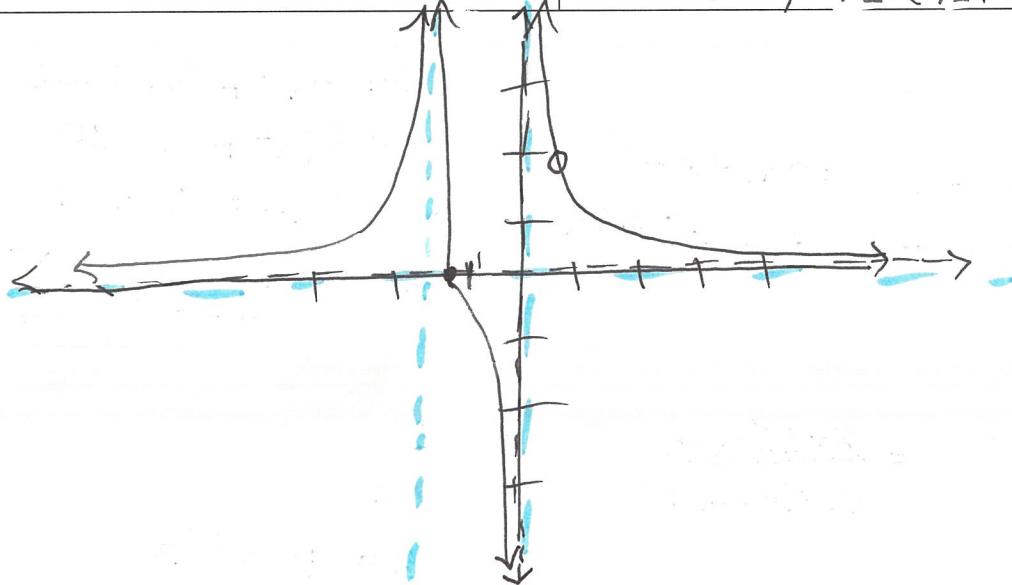
holes: $x = \frac{1}{2}$

y-intercept: none

x-intercept(s): $x = -\frac{4}{3} = -1\frac{1}{3}$

domain: $(-\infty, -\frac{3}{2}) \cup (-\frac{3}{2}, 0) \cup (0, \frac{1}{2}) \cup (\frac{1}{2}, \infty)$

Graph.



$$5. f(x) = \frac{2x^3 + 2x^2 - 3x - 3}{2x^2 - x - 3} = \frac{(2x^2 - 3)(x + 1)}{(2x - 3)(x + 1)}$$

vertical asymptotes: $x = \frac{3}{2}$

horizontal asymptotes: NONE

slant asymptotes: $y = x + \frac{3}{2}$

holes: $x = -1$

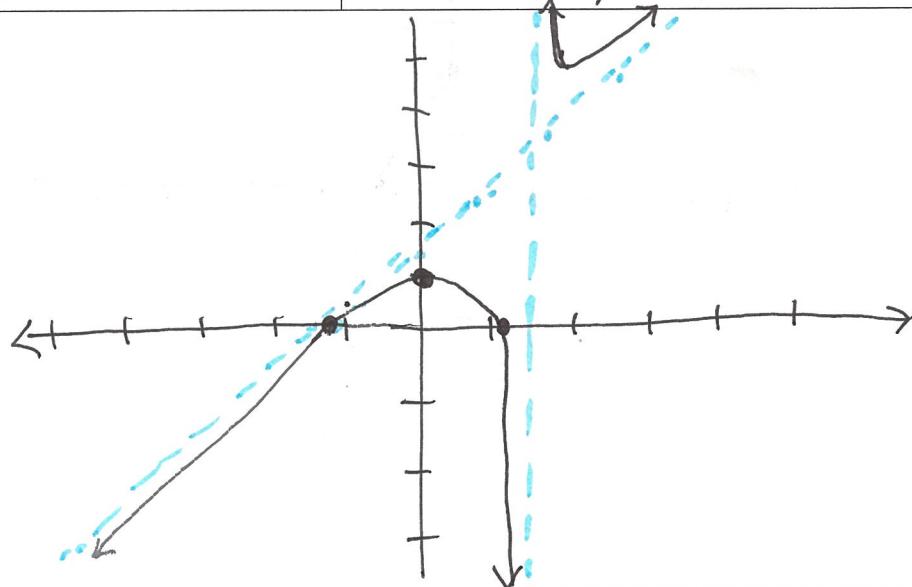
y-intercept: 0, 1

x-intercept(s): $\pm \sqrt{\frac{3}{2}}$

domain: $(-\infty, -1) \cup (-1, -\frac{3}{2}) \cup (\frac{3}{2}, \infty)$

Graph.

$$\begin{array}{c} x + \frac{3}{2} \\ \hline 2x^2 & | & 2x^3 & | & 3x^2 \\ -x & | & -x^2 & | & \\ -3 & | & -3x & | & \end{array}$$



Simplify each expression, and state restrictions.

$$6. \frac{x+1}{x-2} + \frac{x}{2(x-2)} \quad \frac{2x+2}{2(x-2)} + \frac{x}{2(x-2)} = \frac{3x+2}{2(x-2)}$$

$$7. \frac{x+3}{x^2+x-2} - \frac{4x^2}{2x^4-2x^2} = \frac{2x^2(x+3) - 4(x^2)(x+2)}{2x^2(x+2)(x-1)} = \frac{2x^3+6x^2-4x^3-8x^2}{2x^2(x+2)(x-1)} = \frac{-2x^3-2x^2}{2x^2(x+2)(x-1)}$$

$$8. \frac{3x-21}{x^2-3x+2} \cdot \frac{x^2-1}{x-7} = \frac{3(x+1)(x-7)}{(x-2)(x-1)} \cdot \frac{(x+1)(x-1)}{x-7} = \boxed{\frac{3(x+1)}{x-2}}$$

$$9. \frac{6x^2-54}{x^7-x^6-6x^5} \div \frac{24x-8}{3x^5+5x^4-2x^3} = \frac{6(x-3)(x+3)}{x^5(x-3)(x+2)} \cdot \frac{x^3(3x-1)(x+2)}{8(3x-1)} = \boxed{\frac{3(x+3)}{4x^2}}$$

$$10. \frac{\frac{x}{x-2}+1}{\frac{3}{x^2-4}+1} = \frac{\frac{x+x-2}{x-2}}{\frac{3+x^2-4}{x^2-4}} = \frac{(x+2)(x-1)}{x-2} \cdot \frac{(x+2)(x-2)}{(x+1)(x-1)} = \boxed{\frac{(x+2)^2}{x+1}}$$

Solve.

$$11. \frac{3x^2}{11} \left(\frac{1}{x} + \frac{1}{3} \right) \left(\frac{6}{x^2} \right) 3x^2 \quad 3x + x^2 = 18 \quad \boxed{x = -6} \\ x^2 + 3x - 18 = 0 \quad x = 3 \\ (x+6)(x-3) = 0$$

$$12. \left(\frac{1}{x} + \frac{x}{x+2} = 1 \right) \times (x+2) \quad x+2 + x^2 = x(x+2) \quad \boxed{x = -\frac{2}{3}} \\ x^2 - 2x = 0$$

$$13. \frac{1}{x-3} \leq \frac{9}{4x+3} \quad 4x+3 \leq 9x-27 \quad \boxed{x \geq 6}$$

$$\text{VA } x=3 \quad \text{VA } x = -\frac{3}{4}$$

and $\boxed{-\frac{3}{4} < x < 3}$

Look at
the graphs.

14. Write a rational function with the following features.

- Domain: $(-\infty, -5) \cup (-5, 8) \cup (8, \infty)$ \rightarrow VA $\Rightarrow x = -5, x = 8$

- HA at $y = 3$, some degree

- This function has no real roots

imaginary roots!

$$f(x) = \frac{3(x^2 - 6x + 10)}{(x+5)(x-8)}$$

roots: $x = 3 \pm i$

* Answers may vary. Numerator must be quadratic.

15. Write a rational function with the following features.

- $(5, 27)$ is on the function

- HA at $y = x + 1$

- VA at $x = 3$

Plug in $(5, 27)$

$$27 = 5 + 1 + \frac{r}{5-3}$$

$$f(x) = x + 1 + \frac{r}{x-3}$$

↑
slant ↑
VA

$$21 = \frac{r}{2}$$

$r = 42$

$$f(x) = x + 1 + \frac{42}{x-3}$$

16. Write a rational function with the following features.

- $(12, 0)$ is on the function

- $(0, 4)$ is on the function

- HA at $y = 0$ (Proper)

- VA at $x = -3$ & $x = 1$

$$4 = \frac{0-12}{(0+3)(0-1)(0-p)} \rightarrow 4 = \frac{4}{-p}$$

$p = -1$

$$f(x) = \frac{x-12}{(x+3)(x-1)(x-p)}$$

To get $(0, 4)$ add a factor
must add it to the denominator
to stay Proper!

$$f(x) = \frac{x-12}{(x+3)(x-1)(x+1)}$$

17. Write a rational function with the following features.

- Slant asymptote at $y = 4x + 9$

- VA at $x = 2$

- $(1, -2)$ is on the function

$$f(x) = 4x+9 + \frac{r}{x-2}$$

$$-2 = 4(1) + 9 + \frac{r}{1-2}$$

$$-15 = \frac{r}{-1} \quad r = 15$$

$$f(x) = 4x+9 + \frac{15}{x-2}$$