

# IM3H: Module 2 Rational Functions Review

Solve the following equations, state any extraneous solutions

1.  $\frac{y+7}{y-8} = \frac{y-4}{y+5}$

$$(y+5)(y+7) = (y-8)(y-4)$$

$$y^2 + 12y + 35 = y^2 - 12y + 32$$

$$0 = -24y - 3$$

$$y = \frac{3}{-24} = \boxed{-\frac{1}{8}}$$

2.  $\frac{3x^2+20}{5x} = \frac{4x+1}{5}$

$$5(3x^2+20) = 5x(4x+1)$$

$$15x^2 + 100 = 20x^2 + 5x$$

$$0 = 5x^2 + 5x + 100$$

$$0 = 5(x^2 + x + 20)$$

$$0 = 5(x+5)(x-4)$$

$$0 = x+5 \quad 0 = x-4$$

$$\boxed{x = -5} \quad \boxed{x = 4}$$

Identify the key features of each function. Graph each function

3.  $f(x) = \frac{3x^2+2x-8}{x^2-4} = \frac{(x+2)(3x-4)}{(x-2)(x+2)}$

$$2 = \frac{6}{3} \quad \frac{-4}{3}$$

$$(x+2)(3x-4)$$

holes:  $x = -2$

y-intercept:  $0, 2$

x-intercept(s):  $x = \frac{4}{3} = 1\frac{1}{3}$

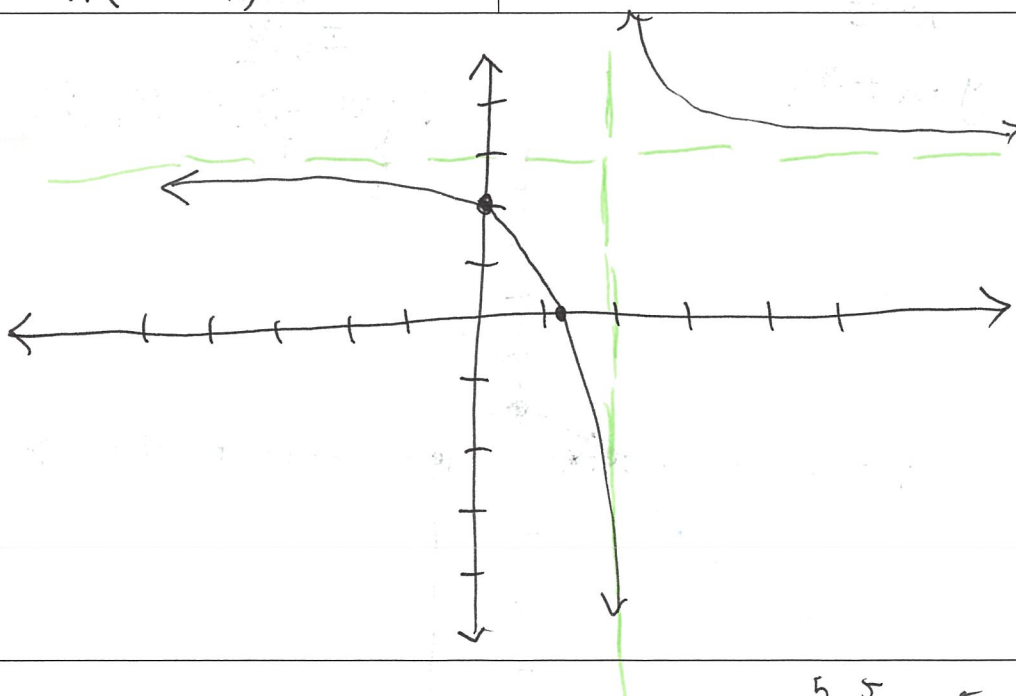
domain:  $(-\infty, -2) \cup (-2, 2) \cup (2, \infty)$

vertical asymptotes:  $x = 2$

horizontal asymptotes:  $y = 3$

slant asymptotes: NONE

Graph.



$$\frac{5}{1.5} = 5$$

4.  $f(x) = \frac{6x^2 + 5x - 4}{12x^3 + 12x^2 - 9x} = \frac{(3x+4)(2x-1)}{3x(4x^2+4x-3)}$

vertical asymptotes:  $x = -3/2$   ~~$x = 0$~~

horizontal asymptotes:  $y = 0$

slant asymptotes: NONE

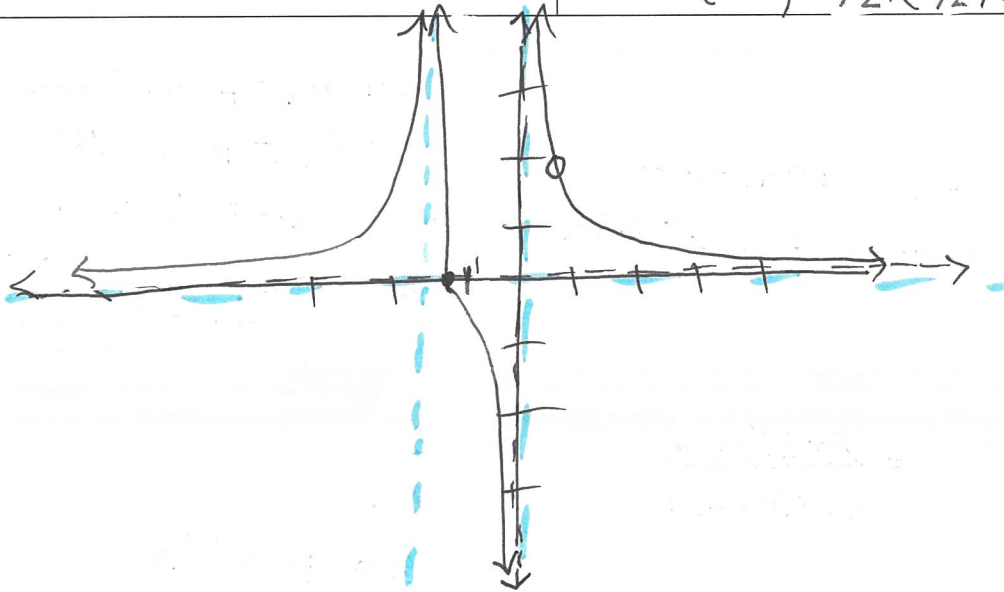
holes:  $x = 1/2$

y-intercept: none

x-intercept(s):  $x = -4/3 = -1 1/3$

domain:  $(-\infty, -3/2) \cup (-3/2, 0) \cup (0, 1/2) \cup (1/2, \infty)$

Graph.



5.  $f(x) = \frac{2x^3 + 2x^2 - 3x - 3}{2x^2 - x - 3} = \frac{(2x^2 - 3)(x + 1)}{(2x - 3)(x + 1)}$

vertical asymptotes:  $x = 3/2$

horizontal asymptotes: NONE

slant asymptotes:  $y = x + 3/2$

holes:  $x = -1$

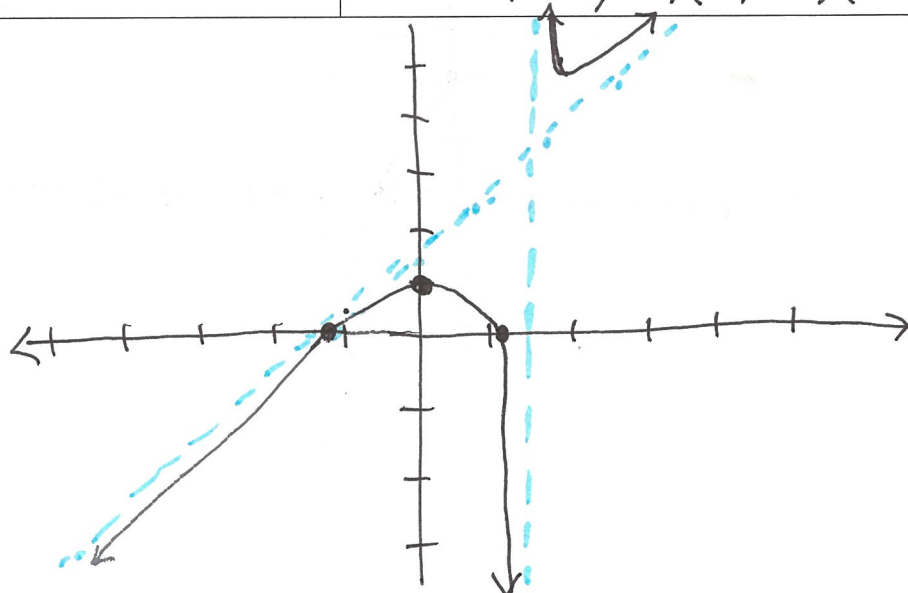
y-intercept: 0, 1

x-intercept(s):  $\pm \sqrt{3/2}$

domain:  $(-\infty, -1) \cup (-1, 3/2) \cup (3/2, \infty)$

Graph.

$$\begin{array}{r|l} & x + 3/2 \\ \hline 2x^2 & 2x^3 \quad 3x^2 \\ -x & -x^2 \\ -3 & -3x \end{array}$$



Simplify each expression, and state restrictions.

$$6. \frac{x+1}{x-2} + \frac{x}{2x-4} = \frac{2x+2}{2(x-2)} + \frac{x}{2(x-2)} = \frac{3x+2}{2(x-2)}$$

$$7. \frac{x+3}{x^2+x-2} - \frac{4x^2}{2x^4-2x^2} = \frac{2x^2(x+3) - 4(x^2)(x+2)}{2x^2(x+2)(x-1)} = \frac{2x^3+6x^2-4x^3-8x^2}{2x^2(x+2)(x-1)} = \frac{-2x^3-2x^2}{2x^2(x+2)(x-1)}$$

$$8. \frac{3x-21}{x^2-3x+2} \cdot \frac{x^2-1}{x-7} = \frac{3(x-7)}{(x-2)(x-1)} \cdot \frac{(x+1)(x-1)}{x-7} = \frac{3(x+1)}{x-2} = \frac{-(x+1)}{(x+2)(x-1)}$$

$$9. \frac{6x^2-54}{x^7-x^6-6x^5} \div \frac{24x-8}{3x^5+5x^4-2x^3} = \frac{6(x-3)(x+3)}{x^6(x-3)(x+2)} \cdot \frac{x^3(3x-1)(x+2)}{8(3x-1)} = \frac{3(x+3)}{4x^2}$$

$$10. \frac{\frac{x}{x-2}+1}{\frac{3}{x^2-4}+1} = \frac{\frac{x+x-2}{x-2}}{\frac{3+x^2-4}{x^2-4}} = \frac{(x+2)(x-1)}{x-2} \cdot \frac{(x+2)(x-2)}{(x+1)(x-1)} = \frac{(x+2)^2}{x+1}$$

Solve.

$$11. \left(\frac{1}{x} + \frac{1}{3}\right) \left(\frac{6}{x^2}\right) 3x^2 = 3x + x^2 = 18$$

$$x^2 + 3x - 18 = 0$$

$$(x+6)(x-3) = 0$$

$$x = -6$$

$$x = 3$$

$$12. \left(\frac{1}{x} + \frac{x}{x+2}\right) x(x+2) = x+2 + x^2 = x(x+2)$$

$$x^2 - 2x = 0$$

$$3x + 2 = 0$$

$$x = -2/3$$

$$13. \frac{1}{x-3} \leq \frac{9}{4x+3}$$

$$4x+3 \leq 9x-27$$

$$30 \leq 5x$$

$$x \geq 6$$

and  $-\frac{3}{4} < x < 3$

VA  $x=3$  VA  $x=-3/4$

Look at the graphs.

