

Int Math 3 Honors
First Day Review of IM2H

Name: Key
Period: _____

**Please make sure you can successfully complete the following problems in order to be successful in Module 1 of Int Math 3 Honors.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Solve the quadratics below by factoring.

1. $x^3 + 5x^2 + 6x = 0$
 $x(x^2 + 5x + 6) = 0$
 $x(x+2)(x+3) = 0$

$$x = 0, -2, -3$$

2. $10x^2 - x + 9 = 0$
 $x = \frac{1 \pm \sqrt{1 - 4(9)(10)}}{20}$

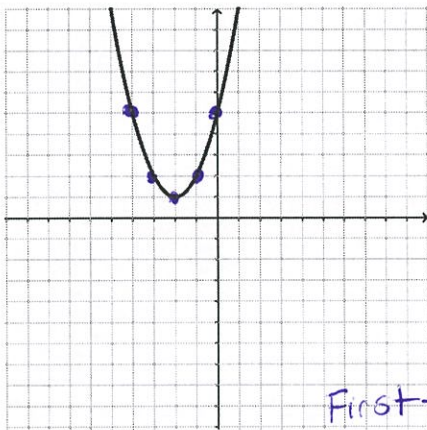
$$x = \frac{1 \pm i\sqrt{359}}{20}$$

3. $49x^4 - 56x^3 + 16x^2 = 0$
 $x^2(49x^2 - 56x + 16) = 0$
 $x^2(7x - 4)^2 = 0$

$$x = 0, \frac{4}{7}$$

4. Find the Standard, Factored, and Vertex form of the given quadratic.

x	y
-	10
-	5
-	2
-	1
-	2
0	5
1	10
2	17
3	26
4	37
5	50



Standard Form:

$$y = x^2 + 4x + 4 + 1$$

$$y = x^2 + 4x + 5$$

Factored Form:

$$y = (x - (-2+i))(x - (-2-i))$$

$$(x + 2 - i)(x + 2 + i)$$

Vertex Form:

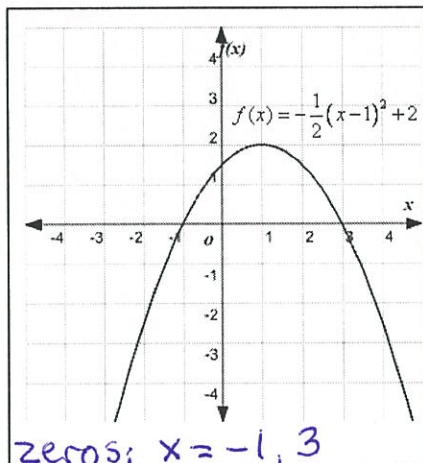
$$y = 1(x + 2)^2 + 1$$

First →

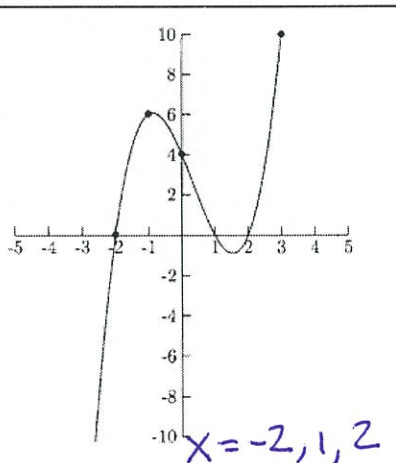
$$0 = (x + 2)^2 + 1 \quad \pm i = x + 2$$

$$-1 = (x + 2)^2 \quad x = -2 \pm i$$

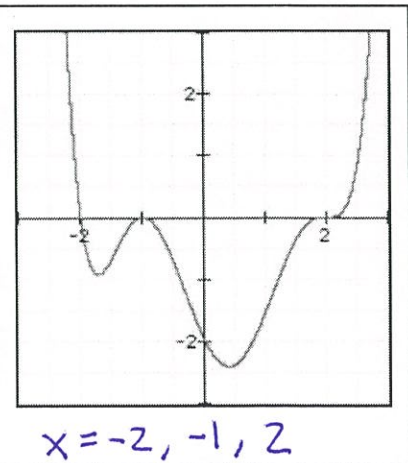
5. Identify the zeros and factors of the following functions?



zeros: $x = -1, 3$
factors: $(x + 1)(x - 3)$



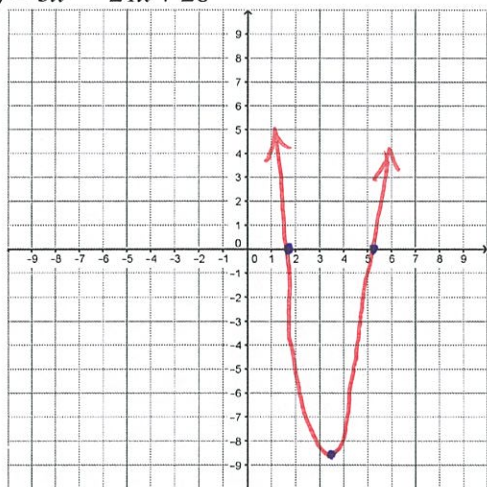
$x = -2, 1, 2$
 $(x + 2)(x - 1)(x - 2)$



$x = -2, -1, 2$
 $(x + 2)(x + 1)(x - 2)$

6. Graph the following function and list the features:

$$f(x) = 3x^2 - 21x + 28$$



$$f(x) = 3\left(x^2 - 7x + \frac{49}{4}\right) + 28 - \frac{3(49)}{4}$$

$$= 3\left(x - \frac{7}{2}\right)^2 - \frac{35}{4}$$

$$\text{vertex: } (3.5, -8.75)$$

$$\text{Zeros: } 0 = 3(x - 3.5)^2 - 8.75$$

$$\pm \sqrt{\frac{8.75}{3}} = x - 3.5$$

$$x = 5.2, 1.8$$

Domain: $(-\infty, \infty)$

Range: $[-8.75, \infty)$

Increasing: $(3.5, \infty)$

Decreasing: $(-\infty, 3.5)$

Min/Max Value: -8.75

Intercepts: $(5.2, 0)$ and $(1.8, 0)$ and $(0, 28)$

Asymptote: None

Roots/X-intercepts/Zeros: $x = 5.2, x = 1.8$

7. Determine if the following points are a solutions in the function $h(x) = x(x-1)(5x-3)$? B, D

a. $(0, -3)$	b. $(1, 0)$	c. $(-2, 36)$	d. $(3, 72)$
--------------	-------------	---------------	--------------

$$-3 \stackrel{?}{=} 0(-1)(-3)$$

$$-3 \neq 0$$

$$0 \stackrel{?}{=} (1)(0)(2)$$

$$0 = 0 \checkmark$$

$$36 \stackrel{?}{=} -2(-3)(-13)$$

$$36 \neq -78$$

$$72 \stackrel{?}{=} 3(2)(12)$$

$$72 = 72 \checkmark$$

8. Evaluate the following trig expressions. (Give an exact value, NO CALCULATORS)

a. $\sin 30^\circ = \frac{1}{2}$

b. $\sin 120^\circ = \frac{\sqrt{3}}{2}$

b. $\sin 120^\circ = \frac{\sqrt{3}}{2}$

c. $\sin 90^\circ = 1$

d. $\cos 30^\circ = \frac{\sqrt{3}}{2}$

d. $\cos 30^\circ = \frac{\sqrt{3}}{2}$

e. $\cos 225^\circ = -\frac{\sqrt{2}}{2}$

e. $\cos 225^\circ = -\frac{\sqrt{2}}{2}$

Remember: $\star a^2 - b^2 = (a-b)(a+b)$ $\star a^2 \pm 2ab + b^2 = (a \pm b)^2$

$\star a^3 \pm b^3 = (a \pm b)(a^2 \mp ab + b^2)$

9. Factor the following expressions completely.

a. $x^3 - 8 = (x-2)(x^2 + 2x + 4)$

b. $27x^6 + 125 = (3x^2 + 5)(9x^4 - 15x^2 + 25)$

c. $16 - x^4 = (4 - x^2)(4 + x^2) = (2 - x)(2 + x)(4 + x^2)$

d. $x^4 y^6 - 121$
 $(x^2 y^3)^2 - 11^2 = (x^2 y^3 - 11)(x^2 y^3 + 11)$
factor again!

e. $x^4 - 2x^2 - 8$
 if $w = x^2$
 then $w^2 - 2w - 8 \rightarrow (w-4)(w+2) \rightarrow (x^2-4)(x^2+2)$

f. $9w^6 - 42w^3y + 49y^2 = (3w^3 - 7y)^2$
 $(3w^3)^2 - 2 \cdot 3 \cdot 7w^3y + (7y)^2$
factor again!
 $= (x-2)(x+2)(x^2+2)$

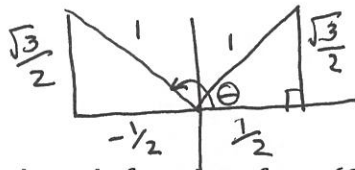
10. Verify the trig identities

a. $\cos x + \sin x \cdot \tan x = \sec x$

$\cos x + \sin x \cdot \frac{\sin x}{\cos x} \rightarrow \frac{\cos^2 x}{\cos x} + \frac{\sin^2 x}{\cos x} \rightarrow \frac{\cos^2 x + \sin^2 x}{\cos x}$
equals 1!
 $\frac{\cos x}{\cos x} + \frac{\sin^2 x}{\cos x} = \frac{1}{\cos x} \rightarrow \boxed{\sec x}$


b. $\frac{1}{\tan x} + \tan x = \frac{1}{\cos x \cdot \sin x}$


$\cot x + \tan x = \frac{\cos^2 x}{\cos x \sin x} + \frac{\sin^2 x}{\cos x \sin x}$
 $\frac{1}{\tan x} = \frac{\cos x}{\sin x}$
 $\frac{\cos x}{\sin x} + \frac{\sin x}{\cos x} = \frac{\cos^2 x + \sin^2 x}{\cos x \cdot \sin x} = \boxed{\frac{1}{\cos x \cdot \sin x}}$



11. Solve the following trig functions for x. (Give an exact value, NO CALCULATORS)

a. $\sin x = \frac{\sqrt{3}}{2}$ $x = 60^\circ$ and 120°

b. $\cos x = -\frac{1}{2}$  $x = 120^\circ$ and 240°

c. $\tan^2 x = 3$
 \downarrow
 $\tan x = \pm \sqrt{3}$ 

all 60° reference angles
 $x = 60^\circ, 120^\circ, 240^\circ, 300^\circ$

12. Convert degrees to radians.

a. $210^\circ \cdot \frac{\pi}{180} = \frac{7\pi}{6}$

b. $60^\circ \cdot \frac{\pi}{180} = \frac{\pi}{3}$

c. $300^\circ \cdot \frac{\pi}{180} = \frac{5\pi}{3}$

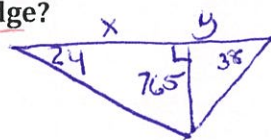
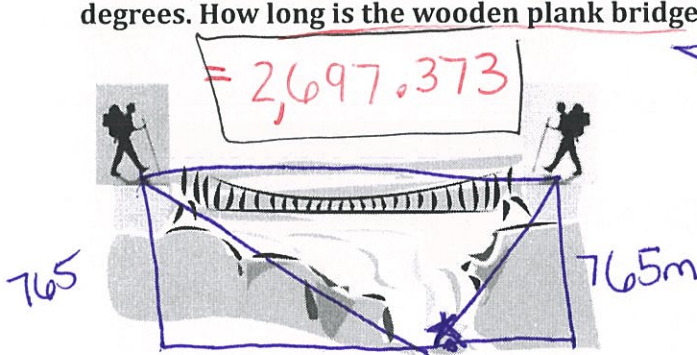
13. Convert radians to degrees.

a. $\frac{2\pi}{3} \cdot \frac{180}{\pi} = 120^\circ$

b. $\frac{5\pi}{6} \cdot \frac{180}{\pi} = 150^\circ$

c. $\frac{7\pi}{4} \cdot \frac{180}{\pi} = 315^\circ$

13. Two hikers are on opposite sides of a wooden plank bridge that spans a canyon. They are each 765 meters above the bottom of the canyon. They both sight the same landmark on the bottom of the canyon floor. The angles of depression from each hiker are 38 degrees and 24 degrees. How long is the wooden plank bridge?

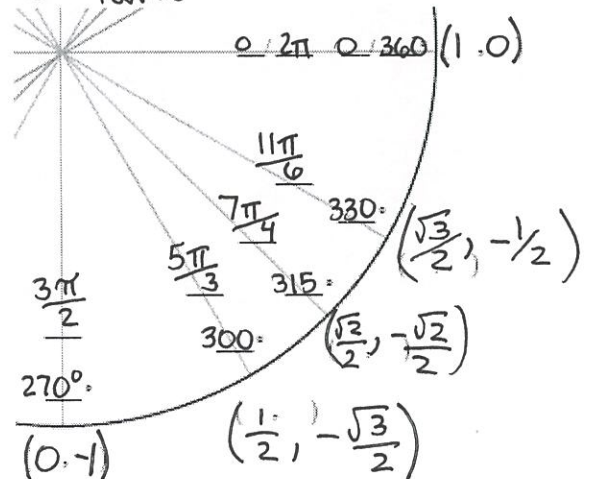


$\tan 24 = \frac{765}{x}$

$x = \frac{765}{\tan 24} = 1718.218$

$\tan 38 = \frac{765}{y}$

$y = \frac{765}{\tan 38} = 979.155$



14. Complete the fourth quadrant of the unit circle.

15. State the domain, range, minimum and maximum values.

	<p>Domain: $(-\infty, \infty)$ Range: $(-\infty, 2)$ Min: $(-\infty)$ Max: undefined at $(1, 2)$</p>
	<p>Domain: $[-4, 3]$ Range: $[-1, 3]$ Min: -1 Max: 3</p>
	<p>Domain: $(-\infty, \infty)$ Range: $(-\infty, 2] \cup (3, \infty)$ Min: $(-\infty)$ Max: (∞)</p>

16. Write the explicit and recursive functions for the following tables.

(a)	(b)	(c)																																				
<table border="1"> <thead> <tr> <th>x</th> <th>f(x)</th> </tr> </thead> <tbody> <tr> <td>-1</td> <td>16</td> </tr> <tr> <td>0</td> <td>2 -14</td> </tr> <tr> <td>1</td> <td>-2 -4 +10</td> </tr> <tr> <td>2</td> <td>4 +6 +10</td> </tr> <tr> <td>3</td> <td>20 +16 +10</td> </tr> </tbody> </table>	x	f(x)	-1	16	0	2 -14	1	-2 -4 +10	2	4 +6 +10	3	20 +16 +10	<table border="1"> <thead> <tr> <th>x</th> <th>f(x)</th> </tr> </thead> <tbody> <tr> <td>-4</td> <td>7</td> </tr> <tr> <td>-1 +3</td> <td>1 -6</td> </tr> <tr> <td>2 +3</td> <td>-5 -6</td> </tr> <tr> <td>3 +1</td> <td>-7 -2</td> </tr> <tr> <td>5 +2</td> <td>-11 -4</td> </tr> </tbody> </table> <p>$f(x) = f(x-1) - 2, f(-1) = 1$</p>	x	f(x)	-4	7	-1 +3	1 -6	2 +3	-5 -6	3 +1	-7 -2	5 +2	-11 -4	<table border="1"> <thead> <tr> <th>x</th> <th>f(x)</th> </tr> </thead> <tbody> <tr> <td>0</td> <td>10</td> </tr> <tr> <td>1</td> <td>17</td> </tr> <tr> <td>2</td> <td>26</td> </tr> <tr> <td>3</td> <td>37</td> </tr> <tr> <td>4</td> <td>50</td> </tr> </tbody> </table>	x	f(x)	0	10	1	17	2	26	3	37	4	50
x	f(x)																																					
-1	16																																					
0	2 -14																																					
1	-2 -4 +10																																					
2	4 +6 +10																																					
3	20 +16 +10																																					
x	f(x)																																					
-4	7																																					
-1 +3	1 -6																																					
2 +3	-5 -6																																					
3 +1	-7 -2																																					
5 +2	-11 -4																																					
x	f(x)																																					
0	10																																					
1	17																																					
2	26																																					
3	37																																					
4	50																																					

Quadratic.

linear $m = -2$
 $f(x) = -2(x+1) + 1$
 $f(x) = -2x - 1$

* Answers to 16a, c are on the next page (i)

#16 (a)

x	f(x)	1st diff	2nd diff
-1	16		
→ 0	2	-14	+10
1	-2	-4	+10
2	4	6	+10
3	20	16	+10

$a = \frac{1}{2}$ (second difference)

$a = 5$

$y = ax^2 + bx + c$ ← y-int

$y = 5x^2 + bx + \underline{\underline{2}}$

plug in (-1, 16)

$16 = 5 - b + 2$

$16 = 7 - b$

$9 = -b$

$b = -9$

explicit: $f(x) = 5x^2 - 9x + 2$

Recursive

$f(x) = f(x-1) + 10x - 14$

$f(0) = 2$

(c)

x	f(x)		
→ 0	10	+5	←
1	17	+7	+2
2	26	+9	+2
3	37	+11	+2
4	50	+13	+2

$y = 1x^2 + bx + \underline{\underline{10}}$

Plug in (1, 17)

$17 = 1 + b + 10$

$17 = 11 + b$

$b = 6$

explicit

$f(x) = x^2 + 6x + 10$

Recursive

$f(x) = f(x-1) + 2x + 5$

$f(0) = 10$