Name **Polynomial Functions** | **1.1H**

Ready, Set, Go!

Ready

Topic: Inequality statements.

Which is greater? For each problem, make a true statement by placing the appropriate inequality symbol between the two expressions.

If $a > b$*, then:*

1. $3a\ \underline{\;>\;}\ 3b$

2. $b - a\ \underline{\;<\;}\ a - b$

3. $a + x\ \underline{\;>\;}\ b + x$

If $x > 10$*, then:*

4. $x^2\ \underline{\;<\;}\ 2^x$

5. $\sqrt{x}\ \underline{\;<\;}\ x^2$

6. $x^2\ \underline{\;<\;}\ x^3$

Set

Topic: Types of functions

Determine the type of function for each problem. Explain how you know.

7.

x	$f(x)$
1	3
2	6
3	9
4	12
5	15

Linear

8.

x	$f(x)$
1	3
2	6
3	12
4	24
5	48

Exponential

9.

x	$f(x)$
1	3
2	9
3	18
4	30
5	45

Quadratic

10.

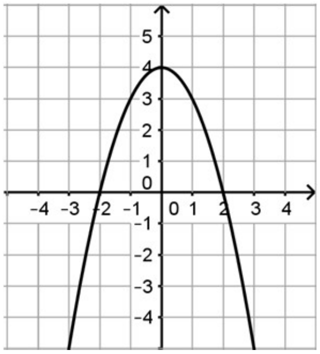
x	$f(x)$
1	3
2	12
3	30
4	60
5	105

Cubic

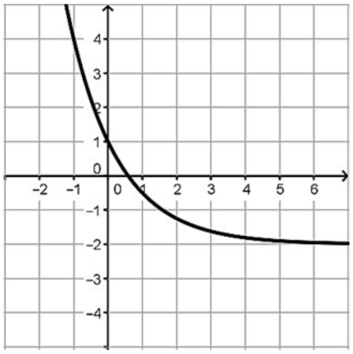
11. $f(x) = -2x^3 + 3x^2 - 5$

Cubic

12. **Quadratic**



13. **Exponential**



14. $g(x) = 2(x - 4) + 7$

Linear

15. $h(x) = 2 \cdot 3^x + 1$

Exponential

16. $g(x) = \log_2(x + 3)$

Logarithmic

Go

Topic: Combining functions.

Use the given functions to solve problems 17 – 21.

$\boldsymbol{f(x) = x - 3}$ $\boldsymbol{g(x) = x + 2}$ $\boldsymbol{h(x) = -x + 1}$

$\boldsymbol{m(x) = x^2 + 3x + 2}$ $\boldsymbol{n(x) = 2x^3 - x^2 + 2x + 1}$ $\boldsymbol{p(x) = 2x + 1}$

17. $f(x) + g(x)$

$\boldsymbol{2x - 1}$

18. $f(x) - h(x)$

$\boldsymbol{2x - 4}$

19. $f(x) \cdot p(x)$

$\boldsymbol{2x^2 - 5x - 3}$

20. $m(x) + g(x)$

$\boldsymbol{x^2 + 4x + 4}$

21. $n(x) - m(x)$

$\boldsymbol{2x^3 - 2x^2 - x - 1}$

Determine if the following statements are ALWAYS or NEVER true. If the statement is NOT true, rewrite it so that it is ALWAYS TRUE.

22. The sum of two linear functions is another linear function.

Always

23. The sum of a linear and a quadratic is a cubic function.

Never; The product of a linear and a quadratic is a cubic function. OR The sum of a linear and a quadratic function is a quadratic function.

24. The sum of a cubic and a quadratic function is a cubic function.

Always

25. The sum of two functions is always a function (passes the vertical line test).

Always

26. The product of two functions is always a function (passes the vertical line test).

Always

Topic: Comparing logarithmic & exponential expressions

Order the following numbers from least to greatest without using a calculator.

27. $\log_7 49$ $\log_7 \sqrt{7}$ $\log_7 \frac{1}{49}$ $\log_7 \sqrt[3]{343}$

$\boldsymbol{\log_7 \frac{1}{49},\ \log_7 \sqrt{7},\ \log_7 \sqrt[3]{343},\ \log_7 49}$

28. $8^{\log_8 19}$ $\ln e^8$ $\ln e^{-8}$ $e^{-\ln 8}$

$\boldsymbol{\ln e^{-8},\ e^{-\ln 8},\ \ln e^8,\ 8^{\log_8 19}}$

29. $\log_3(\log_7 7^{27})$ $\log(\ln e^{100})$ $\ln(\log 10)$ $\log_5(\log_2 32)$

$\boldsymbol{\ln(\log 10),\ \log_5(\log_2 32),\ \log(\ln e^{100}),\ \log_3(\log_7 7^{27})}$

30. $10^{\log 33}$ $10^{\log 53}$ $\log(\ln e^{10})$ $\log 10^7$

$\boldsymbol{\log(\ln e^{10}),\ \log 10^7,\ 10^{\log 33},\ 10^{\log 53}}$

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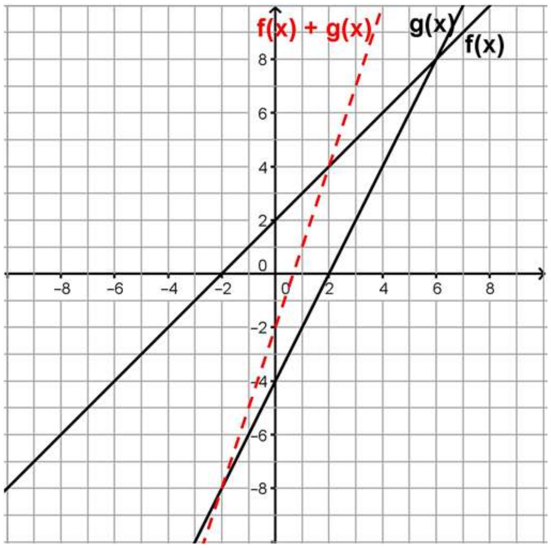
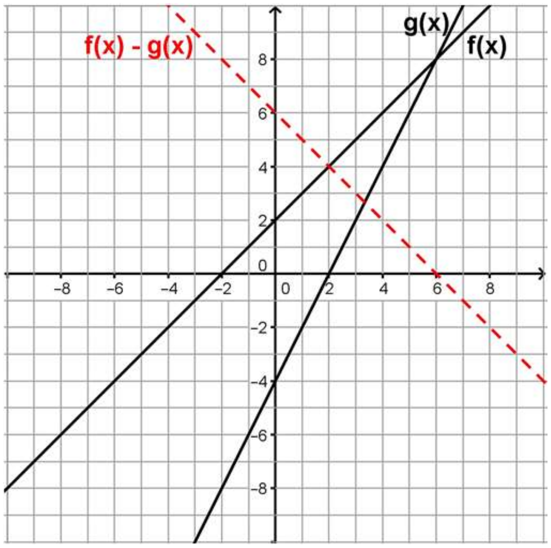
Topic: Combining polynomial functions graphically.

Use the graphs of $f(x)$ and $g(x)$ to sketch the graph of the following. Use the values on the graphs to produce your sketch. Do not write and combine equations.

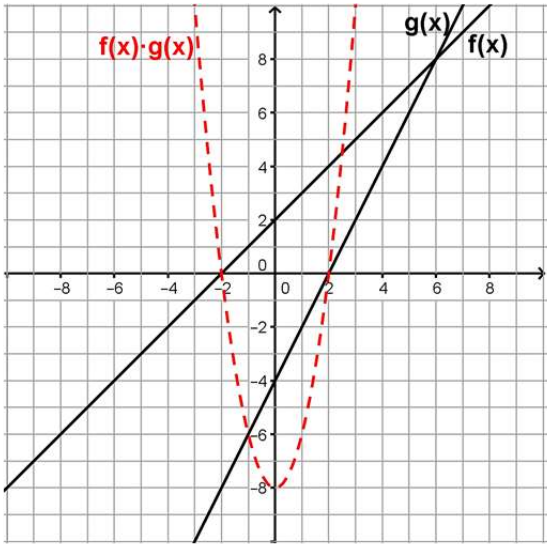
1. $f(x) + g(x)$



2. $f(x) - g(x)$



3. $f(x) \cdot g(x)$



4. Complete each sentence below.
   a. The sum of two linear functions is…
   linear
   b. The difference of two linear functions is…
   linear
   c. The product of two linear functions is…
   quadratic

Set

Topic: Ordering real number expressions

Order the following numbers from least to greatest without using a calculator.

5. 100^3 $\sqrt{100}$ $\log_2 100$ 100

$\mathbf{\log_2 100\ ,\ \sqrt{100}, 100, 100^3}$

6. 2^{-1} $\sqrt{100}$ $\log_2\left(\frac{1}{8}\right)$ 0

$\mathbf{\log_2 \frac{1}{8}, 0, 2^{-1}, \sqrt{100}}$

7. 2^0 $\sqrt{16}$ $\log_2 8$ 2

$\mathbf{2^0, 2, \log_2 8\ , \sqrt{16}}$

8. 2^{100} $\log_2 100$ $\sqrt{100}$ 100^2

$\mathbf{\log_2 100, \sqrt{100}, 100^2, 2^{100}}$

Which is greater? For each problem, make a true statement by placing the appropriate inequality symbol between the two expressions. (Hint: think about what you know about the expression and the end behavior as well as rates of change of a function instead of plugging in values).

If $x < -100$, *then:*

9. $x^2 \underline{\ \ >\ \ } 2^x$

10. $x^5 \underline{\ \ <\ \ } x^2$

11. $x^2 \underline{\ \ >\ \ } x^3$

If $x > 100$, *then:*

12. $x^2 \underline{\ \ <\ \ } 2^x$

13. $x^5 \underline{\ \ >\ \ } x^2$

14. $x^2 \underline{\ \ <\ \ } x^3$

Go

Topic: Combining functions

Perform each operation. Write your answers in standard form.

15. $f(x) = x^5 + 3x^2 + 4x^4, g(x) = 3x^5 - x^3 + 3x^2$

$f(x) + g(x) = \mathbf{4x^5 + 4x^4 - x^3 + 6x^2}$

16. $f(x) = 3x^2 + 4x, g(x) = x^2 - 5$

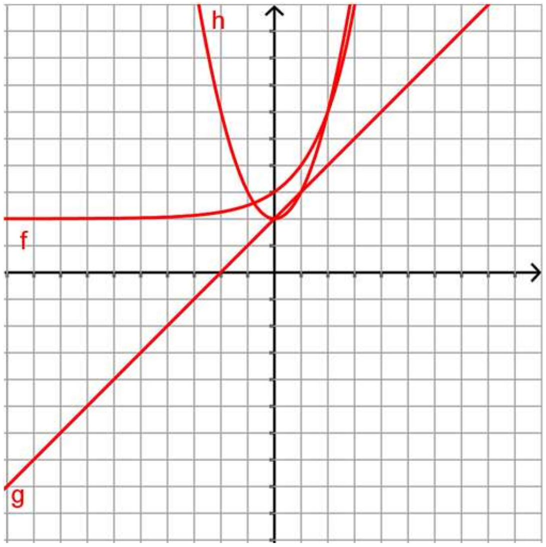
$f(x) \cdot g(x) = \mathbf{3x^4 + 4x^3 - 15x^2 - 20x}$

17. $f(x) = x^4 - 6x^2 + 5x^3, g(x) = 2x^2 - 7x^4 + 6$

$g(x) - f(x) = \mathbf{-8x^4 - 5x^3 + 8x^2 + 6}$

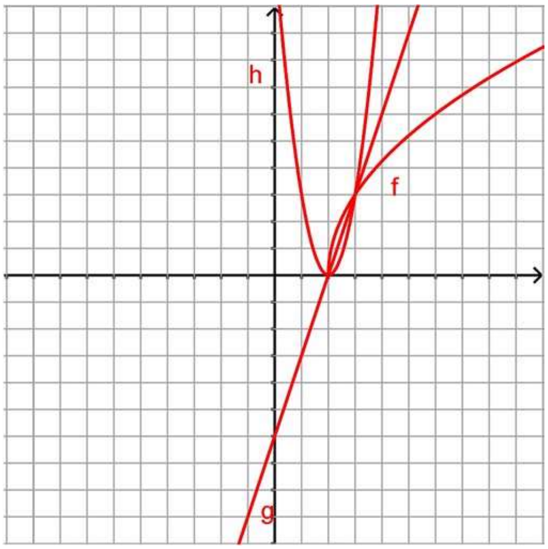
Graph each set of functions on the same axes. Label each function and state how the functions are related to the graphs of their parent functions.

18. $f(x) = 2^x + 2$
$g(x) = x + 2$
$h(x) = x^2 + 2$



Features in common:
All 3 functions have been translated up 2 units from the parent functions.

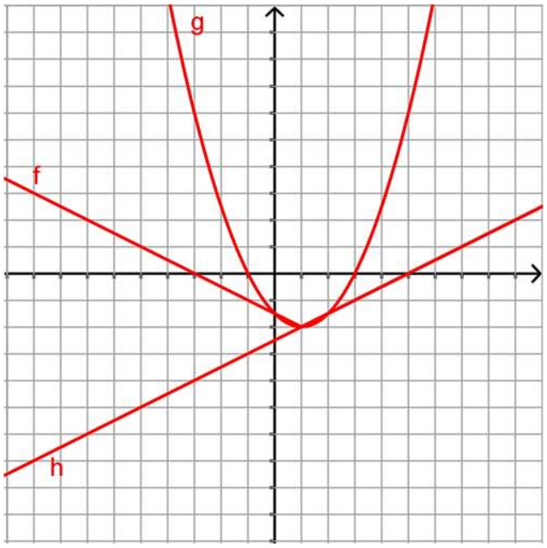
19. $f(x) = 3\sqrt{x-2}$
$g(x) = 3(x-2)$
$h(x) = 3(x-2)^2$



Features in common:

All 3 functions have been translated right 2 units from the parent function and have a stretch factor of 3.

20. $f(x) = \frac{1}{2}|x-1| - 2$
$g(x) = \frac{1}{2}(x-1) - 2$
$h(x) = \frac{1}{2}(x-1)^2 - 2$



Features in common:

All 3 functions have been translated right 1 unit and down 2 units from the parent function and have a stretch factor of $\frac{1}{2}$.

Ready, Set, Go!

Ready

Topic: Forms of linear and quadratic functions

The different forms of linear and quadratic functions are listed below. Determine what features of the function/graph can quickly be determined based upon the structure of each form of linear and quadratic functions.

Linear

1. Standard form: $ax + by = c$
   x- and y-intercepts

2. Slope-intercept form: $y = mx + b$
   Slope and y-intercept

3. Point-slope form: $y = m(x - x_1) + y_1$
   Slope and point on graph

Quadratic

4. Standard form: $y = ax^2 + bx + c$
   y-intercept, direction of opening

5. Factored form: $y = a(x - r_1)(x - r_2)$
   x-intercepts, direction of opening

6. Vertex form: $y = a(x - h)^2 + k$
   Vertex and max/min value, direction of opening

For each, write what you know about the function (including end behavior) and then graph.

7. Equation: $f(x) = (x - 2)(x + 3)$
   What I know about this function:

   Intercepts: $(\mathbf{-3, 0}), (\mathbf{0, -6}), (\mathbf{2, 0})$

   Domain: $(-\infty, \infty)$

   Range: $(\mathbf{-6.25}, \infty)$

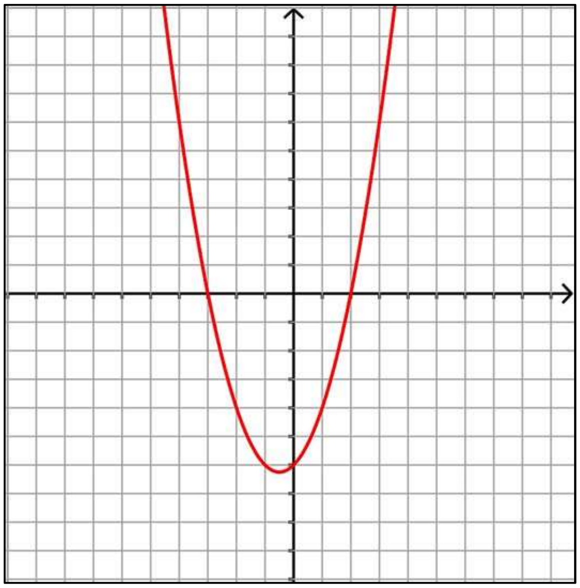
   Maximum(s): **none**

   Minimum(s): $(\mathbf{-0.5, -6.25})$

   Intervals of increase/decrease:
   I: $(\mathbf{-0.5}, \infty)$, **D:** $(-\infty, \mathbf{-0.5})$
   End behavior:
   As $x \to -\infty$, $f(x) \to$ ____∞

   As $x \to \infty$, $f(x) \to$ ____∞.

Graph:



8. Equation: $g(x) = (x + 1)(x - 1)(x - 2)$

What I know about this function:

Intercepts: $\mathbf{(-1, 0), (0, 2), (1, 0), (2, 0)}$

Domain: $(-\infty, \infty)$

Range: $(-\infty, \infty)$

Maximum(s): **about** $\mathbf{(-0.1, 2.1)}$

Minimum(s): **about** $\mathbf{(1.5, -0.5)}$

Intervals of increase/decrease:

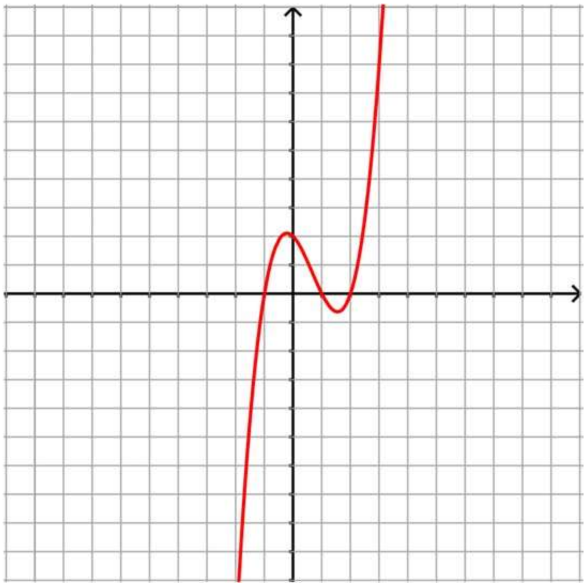
I: $\mathbf{(-\infty, -0.1) \cup (1.5, \infty)}$**, D:** $\mathbf{(-0.1, 1.5)}$

End behavior:

As $x \to -\infty$, $g(x) \to$ _____ $-\infty$

As $x \to \infty$, $g(x) \to$ _____∞

Graph:



9. Equation: $y = -x^2 - 4x - 6$

What I know about this function:

Intercepts: $\mathbf{(0, -6)}$

Domain: $(-\infty, \infty)$

Range: $(-\infty, -\mathbf{2})$

Maximum(s): $\mathbf{(-2, -2)}$

Minimum(s): **none**

Intervals of increase/decrease:

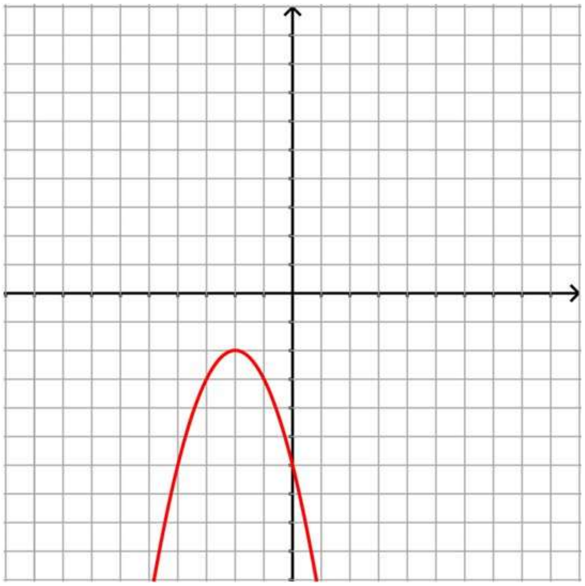
I: $\mathbf{(-\infty, -2)}$**, D:** $\mathbf{(-2, \infty)}$

End behavior:

As $x \to -\infty$, $y \to$ _____ $-\infty$

As $x \to \infty$, $y \to$ _____ $-\infty$

Graph:



10. Equation: $h(x) = \sqrt{x+2}$

What I know about this function:

Intercepts: $\mathbf{(-2, 0), (0, \sqrt{2})}$

Domain: $\mathbf{(-2, \infty)}$

Range: $\mathbf{(0, \infty)}$

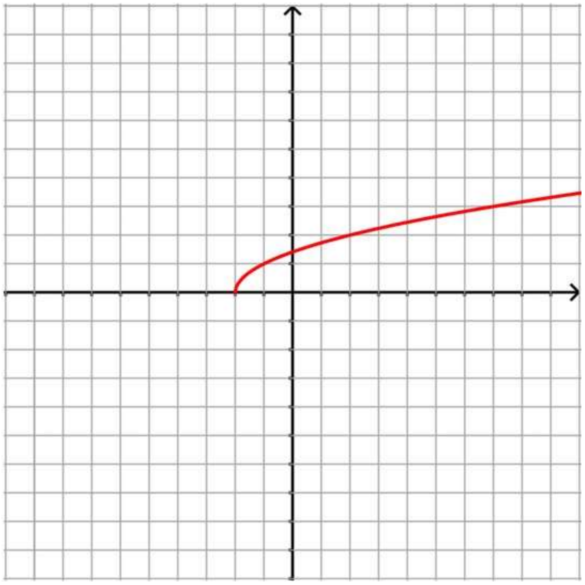
Maximum(s): **none**

Minimum(s): $\mathbf{(-2, 0)}$

Intervals of increase/decrease:
I: $\mathbf{(-2, \infty)}$
End behavior:
As $x \to$ _____ $\mathbf{-2}$, $h(x) \to$ _____ $\mathbf{0}$

As $x \to \infty$, $h(x) \to$ _____ ∞

Graph:



Set

Topic: End behavior of various types of functions

Determine the function type and state the end behavior in the form as $x \to$ __, $f(x) \to$ __.

11. $f(x) = x^2 + 12x - 1$
Quadratic
$\mathbf{x \to -\infty, f(x) \to \infty}$
$\mathbf{x \to \infty, f(x) \to \infty}$

12. $g(x) = 4 \cdot 2^x$
Exponential
$\mathbf{x \to -\infty, g(x) \to 0}$
$\mathbf{x \to \infty, g(x) \to \infty}$

13. $h(x) = -x^3 + 1$
Cubic
$\mathbf{x \to -\infty, h(x) \to \infty}$
$\mathbf{x \to \infty, h(x) \to -\infty}$

14. $p(x) = -x^2 + 3x - 1$
Quadratic
$\mathbf{x \to -\infty, p(x) \to -\infty}$
$\mathbf{x \to \infty, p(x) \to -\infty}$

Use the equations in questions 11-14 to answer the following:

15. Which function above has the greatest value at $x = 1{,}000$? $\mathbf{g(x)}$

16. Which function above is *always* increasing? $\mathbf{g(x)}$

17. Which function above is *always* decreasing? $\mathbf{h(x)}$

18. Which function above has a maximum value? $\mathbf{p(x)}$

19. Which function above has a minimum value? $\mathbf{f(x)}$

Go

Topic: Solving equations

Solve for *x*.

20. $27^{x+3} = 9^{x-1}$

$\boldsymbol{x = -11}$

21. $2x^2 + 4x + 3 = 0$

$\boldsymbol{x = \frac{-2 \pm i\sqrt{2}}{2}}$

22. $\log_2 x + \log_2(x - 7) = 3$

$\boldsymbol{x = 8}$
$\boldsymbol{x = -1}$ **is extraneous**

23. $\log_4(x + 3) = 2$

$\boldsymbol{x = 13}$

24. $(x + 4)(x - 3)(x + 1) = 0$

$\boldsymbol{x = -4, -1, 3}$

25. $4x^4 - 16x^3 + 16x^2 = 0$

$\boldsymbol{x = 0, 2}$

Name **Polynomial Functions | 1.4H**

Ready, Set, Go!

Ready

Topic: Solving equations.

Solve for x.

1. $5x^2 - 16x + 15 = 4x - 5$

   $\mathbf{x = 2}$

2. $3(x+2)^2 + 4 = 13$

   $\mathbf{x = -2 \pm \sqrt{3}}$

3. $8x^2 - 14x - 9 = 0$

   $\mathbf{x = \frac{9}{4}, -\frac{1}{2}}$

4. $e^{2x} - 3e^x + 2 = 0$

   $\mathbf{x = 0, \ln 2}$

5. $(2x-1)(x+4)(5x+2)=0$

   $\mathbf{x = -4, -\frac{2}{5}, \frac{1}{2}}$

6. $81^{3-x} = \left(\frac{1}{3}\right)^{5x-6}$

   $\mathbf{x = -6}$

Set

Topic: Combining polynomial functions.

Given $f(x) = x^2 + 3x + 2$, $g(x) = 5x - 4$, and $h(x) = x^3 + 2x - 5$, find:

7. $f(x) + g(x)$

   $\mathbf{x^2 + 8x - 2}$

8. $f(x) - g(x)$

   $\mathbf{x^2 - 2x + 6}$

9. $f(x) \cdot g(x)$

   $\mathbf{5x^3 + 11x^2 - 2x - 8}$

10. $h(x) + g(x)$

    $\mathbf{x^3 + 7x - 9}$

11. $f(x) - h(x)$

    $\mathbf{-x^3 + x^2 + x + 7}$

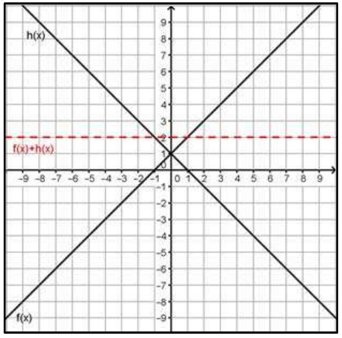
12. $h(x) \cdot g(x)$

    $\mathbf{5x^4 - 4x^3 + 10x^2 - 33x + 20}$

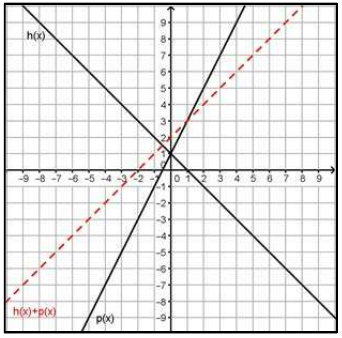
Graphs of the individual functions are given. Graph the solution on the same set of axes.

$\boldsymbol{f(x) = x + 1}$ $\boldsymbol{h(x) = -x + 1}$ $\boldsymbol{p(x) = 2x + 1}$ $\boldsymbol{m(x) = x^2}$

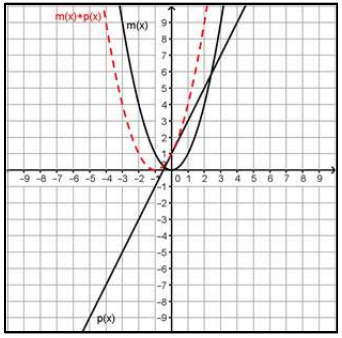
13. $f + h$



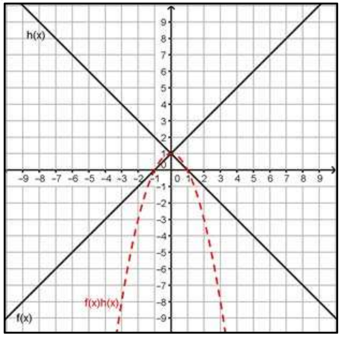
14. $h + p$



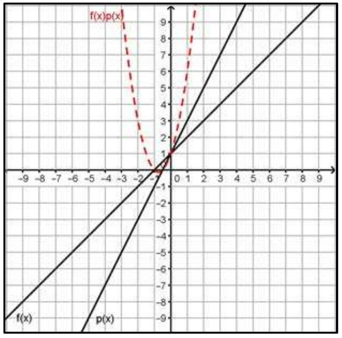
15. $m + p$



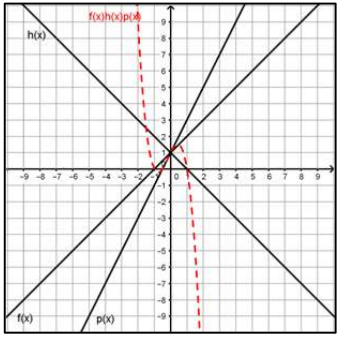
16. $f \cdot h$



17. $f \cdot p$



18. $f \cdot h \cdot p$



Go

Topic: Simplifying expressions containing exponents

19. $\frac{6x^3y^5}{9yz^4}$

$\boldsymbol{\frac{2x^3y^4}{3z^4}}$

20. $\left(\frac{2x^{-2}y^3z^4}{yz^4}\right)^2$

$\boldsymbol{\frac{4y^4}{x^4}}$

21. $\frac{6x+9}{3}$

$\boldsymbol{2x + 3}$

Topic: Solving logarithmic and exponential equations.

Solve each equation.

22. $\log_2(3x-5)=\log_2(x+17)$

$\boldsymbol{x=11}$

23. $64^{x-1}=512$

$\boldsymbol{x=\frac{5}{2}}$

24. $\log_3(6x+9)=5$

$\boldsymbol{x=39}$

25. $\left(\frac{1}{3}\right)^x=81^{2x-3}$

$\boldsymbol{x=\frac{4}{3}}$

Topic: Multiplying polynomials.

Multiply each. Simplify solutions by combining like terms

26. $(a+b)(a+b)$

$\boldsymbol{a^2+2ab+b^2}$

27. $(x-3)(x^2+3x+9)$

$\boldsymbol{x^3-27}$

28. $(x-5)(x^2+5x+25)$

$\boldsymbol{x^3-125}$

29. $(x+1)(x^2-x+1)$

$\boldsymbol{x^3+1}$

30. $(x+7)(x^2-7x+49)$

$\boldsymbol{x^3+343}$

31. $(a-b)(a^2+ab+b^2)$

$\boldsymbol{a^3-b^3}$

Name **Polynomial Functions** **1.5H**

Ready, Set, Go!

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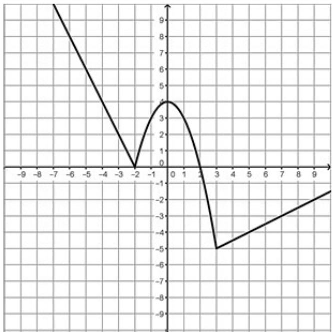
Topic: Describe the features of various functions.

Identify the features of the following functions. (Features include domain, range, intercepts, and end behavior).

1.



Domain: $(-\infty, \infty)$

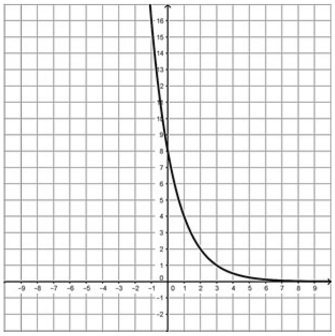
Range: $[-5, \infty)$

x-intercepts: ± 2

y-intercept: 4

End Behavior:
As $x \to -\infty, f(x) \to \infty$
As $x \to \infty, f(x) \to \infty$

2.



Domain: **integers**

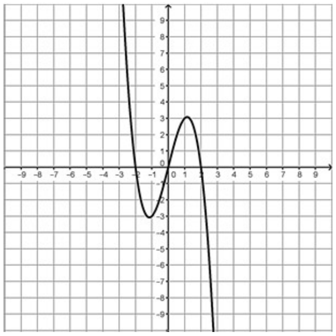
Range: $\{16, 8, 4, 2, 1, \ldots\}$

x-intercepts: **NA**

y-intercept: 8

End Behavior:
As $x \to -\infty, f(x) \to \infty$
As $x \to \infty, f(x) \to 0$

3.



Domain: $(-\infty, \infty)$

Range: $(-\infty, \infty)$

x-intercepts: $\pm 2, 0$

y-intercept: 0

End Behavior:
As $x \to -\infty, f(x) \to \infty$
As $x \to \infty, f(x) \to -\infty$

Topic: Combinations and permutations

Permutations	Combinations
${}_nP_r = \frac{n!}{(n-r)!}$	${}_nC_r = \frac{n!}{r!(n-r)!}$

Find the number of permutations or combinations.

4. ${}_5C_2$
10

5. ${}_4P_2$
12

6. ${}_6C_3$
20

Set

Topic: Features of polynomial functions

Write the key features of each function (intercepts, end behavior, and where the function is increasing/decreasing), then graph.

7. Equation: $f(x) = (x - 1)^2$

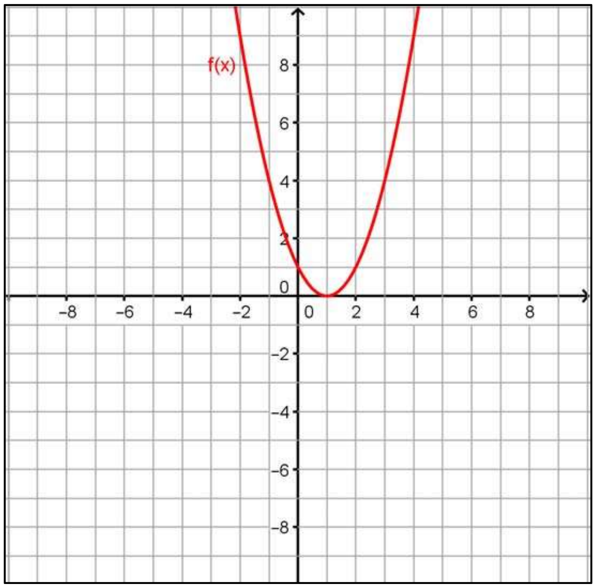
What I know about this function:

Vertex: $(1, 0)$
x-intercept: 1
y-intercept: 1
increasing: $(1, \infty)$
decreasing: $(-\infty, 1)$

End behavior:
As $x \to -\infty$, $f(x) \to$ _____ ∞

As $x \to \infty$, $f(x) \to$ _____ ∞

Graph:



8. Equation: $h(x) = (x^2 - 1)(x^2 - 1)$

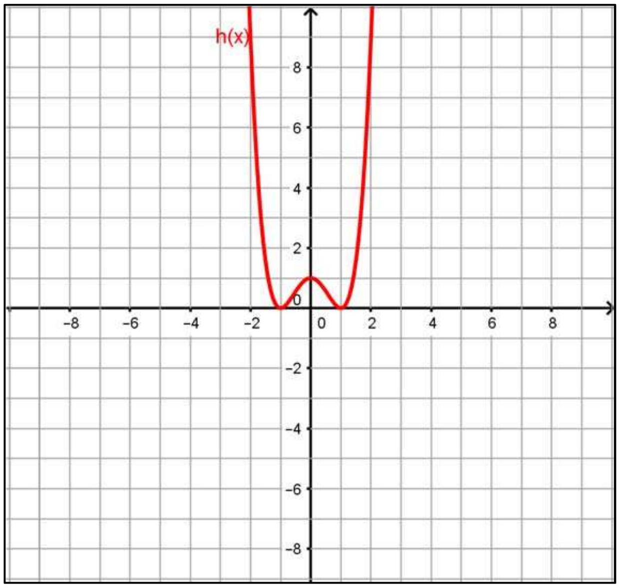
What I know about this function:

x-intercepts: ± 1
y-intercept: 1
increasing: $(-1, 0), (1, \infty)$
decreasing: $(-\infty, -1), (0, 1)$

End behavior:
As $x \to -\infty$, $h(x) \to$ _____ ∞

As $x \to \infty$, $h(x) \to$ _____ ∞

Graph:



9. Equation: $h(x) = (x-3)(x+4)(x+1)$

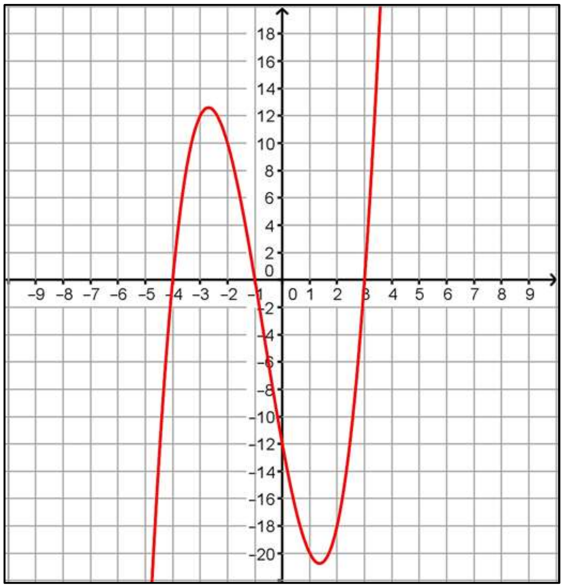
What I know about this function:

x-intercepts: $-4, -1, 3$
y-intercept: -12
increasing: Approximately
$(-\infty, 2.7), (1.4, \infty)$
decreasing: Approximately $(2.7, 1.4)$

End behavior:
As $x \to -\infty$, $h(x) \to$ ______ $-\infty$

As $x \to \infty$, $h(x) \to$ ______ ∞

Graph:



10. Equation: $f(x) = x^3$

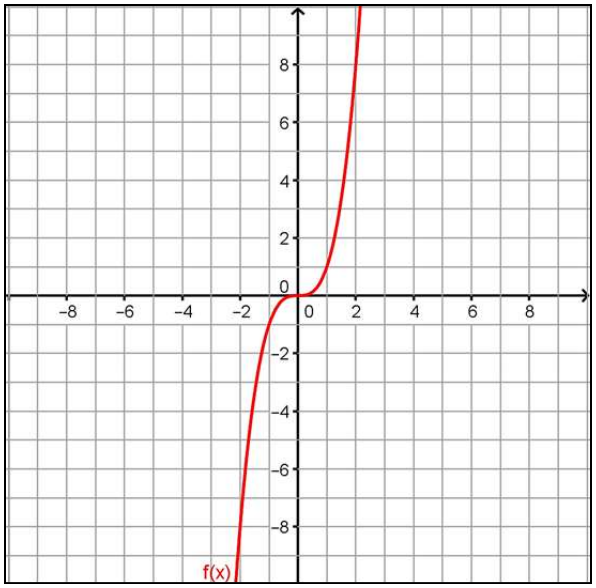
What I know about this function:

x-intercept: 0
y-intercept: 0
increasing: $(-\infty, \infty)$
decreasing: NA

End behavior:
As $x \to -\infty$, $f(x) \to$ ______ $-\infty$

As $x \to \infty$, $f(x) \to$ ______ ∞

Graph:



Go

Topic: Comparing functions in different forms.

Use functions a-h to answer the questions below.

a. $f(x) = 3 - 2x$

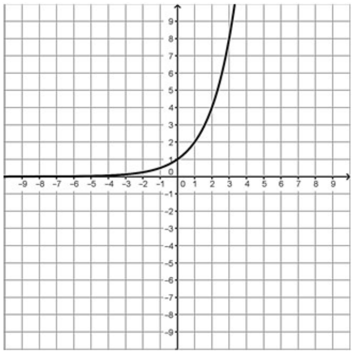
b. $f(x) = \log_2 x$

c. $f(x) = \sqrt{x+1}$

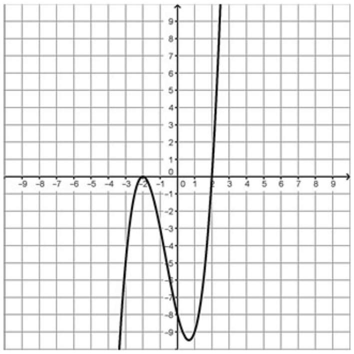
d. $f(x) = 3(x-1)(x+2)(x-4)$

e. $f(x) = -2x^3 + 2x^2 - x + 5$

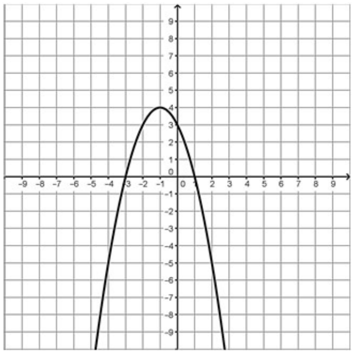
f.



g.



h.



11. Which function(s) do not have a domain of all real numbers?
    b, c

12. Which function(s) do not have a range of all real numbers?
    c, f, h

13. Which function(s) have exactly two *x*-intercepts?
    g, h

14. Compare *a* and *c*: which has the greatest value as $x \to \infty$?
    c

15. Compare *d* and *f*: which has the greatest value as $x \to \infty$?
    f

16. Compare *f* and *g*: which has the greatest value as $x \to \infty$?
    f

17. Compare *e* and *h*: which has the greatest value as $x \to \infty$?
    h

18. Compare *g* and *h*: which has the highest relative maximum value?
    h

19. Compare *b* and *f*: which has the greatest average rate of change over the interval $[15, 20]$?
    f

Name **Polynomial Functions** | **1.6H**

Ready, Set, Go!

Ready

Topic: Arithmetic of polynomials

In the task *To Be Determined . . .* we defined polynomials to be expressions of the following form:

$$a_0x^n + a_1x^{n-1} + a_2x^{n-2} + \cdots + a_{n-3}x^3 + a_{n-2}x^2 + a_{n-1}x + a_n$$

where all of the exponents are positive integers and all of the coefficients $a_0 \ldots a_n$ are constants.

Do the following for each of the problems below:

- **A. Choose the best word to complete each conjecture.**
- **B. After you have made a conjecture, create at least four examples to show why your conjecture is true.**
- **C. If you find a counter-example, change your conjecture to fit your work.**

1. Conjecture #1: The sum of two polynomials is **[always, sometime, never]** a polynomial.
   A. Best word choice:
   Always

   B. At least 4 examples:
   Answers may vary

   C. Counter-example

2. Conjecture #2: The difference of two polynomials is **[always, sometime, never]** a polynomial.
   A. Best word choice:
   Always

   B. At least 4 examples:
   Answers may vary

   C. Counter-example

3. Conjecture #3: The product of two polynomials is **[always, sometime, never]** a polynomial.
   A. Best word choice:
   Always

   B. At least 4 examples:
   Answers may vary

   C. Counter-example

Set

Topic: Binomial expansion

Use Pascal's Triangle to help you expand each binomial.

4. $(x+3)^4$
   $\mathbf{x^4+12x^3+54x^2+108x+81}$

5. $(x+2)^5$
   $\mathbf{x^5+10x^4+40x^3+80x^2+80x+32}$

6. $(x^2+y^3)^3$
   $\mathbf{x^6+3x^4y^3+3x^2y^6+y^9}$

7. $(2x-y)^5$
   $\mathbf{32x^5-80x^4y+80x^3y^2-40x^2y^3+10xy^4-y^5}$

8. Find the 3rd term in the expansion of $(x-3)^5$
   $\mathbf{90x^3}$

9. Find the 2nd term in the expansion of $(2x-3)^7$
   $\mathbf{-1344x^6}$

Go

Topic: Solving polynomial equations

Factor each polynomial. Then use the zero product property to solve for the variable.

10. $a^4 - 7a^2 + 6 = 0$

$\boldsymbol{a = \pm 1, \pm\sqrt{6}}$

11. $a^4 + 7a^2 + 6 = 0$

$\boldsymbol{a = \pm i, \pm i\sqrt{6}}$

12. $16m^4 - 1 = 0$

$\boldsymbol{m = \pm\frac{1}{2}, \pm\frac{1}{2}i}$

13. $4y^2 + 8y - 20 = 0$

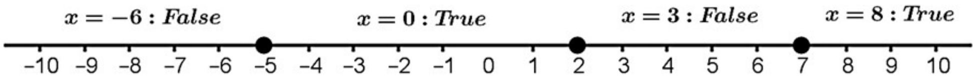
$\boldsymbol{y = -1 \pm \sqrt{6}}$

Topic: Solving inequalities

Solve each inequality by placing the zeros of the related equation on a number line and checking a value in each interval. Express your solutions to the inequality in interval notation.

Example: $(x+5)(x-2)(x-7) \geq 0$

Related equation: $(x+5)(x-2)(x-7) = 0$
Zeros of the related equation: $-5, 2, 7$



Solution to the inequality: $[-5, 2] \cup [7, \infty)$

14. $x^2 + 7x + 6 < 0$

$(-6, -1)$

15. $3x - 5 > 2$

$\left(\frac{7}{3}, \infty\right)$

16. $(x+1)(x-1)(x-5) < 0$

$(-\infty, -1) \cup (1, 5)$

17. $5x^2 - 17x + 14 > 0$

$\left(-\infty, \frac{7}{5}\right) \cup (2, \infty)$

Ready, Set, Go!

Ready

Topic: Factoring special products

Factor.

1. $4x^2 - 25$

   $(2x+5)(2x-5)$

2. $9x^2 - 16y^2$

   $(3x+4y)(3x-4y)$

3. $a^2x^2 - b^2$

   $(ax+b)(ax-b)$

Factoring Rule for the Sum of Cubes: $a^3 + b^3 = (a+b)(a^2 - ab + b^2)$

Factoring Rule for the Difference of Cubes: $a^3 - b^3 = (a-b)(a^2 + ab + b^2)$

4. $64x^3 - 125$

   $(4x-5)(16x^2+20x+25)$

5. $27x^3 + 8$

   $(3x+2)(9x^2-6x+4)$

6. $1000x^3 - y^3$

   $(10x-y)(100x^2+10xy+y^2)$

Set

Topic: Finding zeros of polynomial functions.

Find all zeros of each polynomial, then sketch the graph. Use technology to check your answer.

7. $f(x) = x^2 - 25$

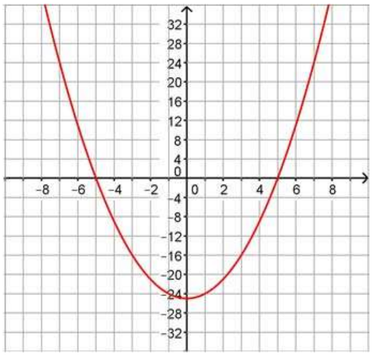
   $x = \pm 5$

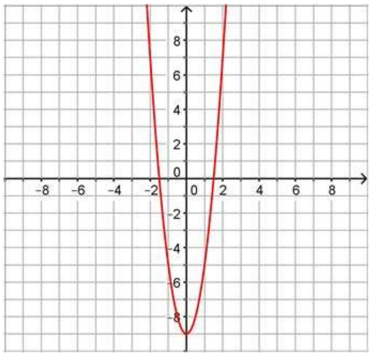
8. $g(x) = 4x^2 - 9$

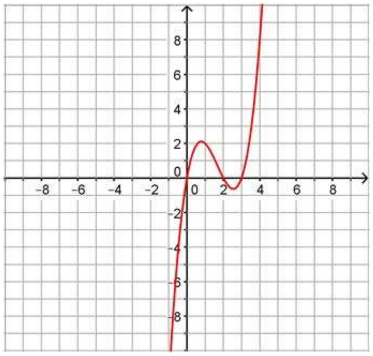
   $x = \pm\frac{3}{2}$

9. $h(x) = x(x^2 - 5x + 6)$

   $x = 0, 2, 3$







10. $f(x) = x^2 + 25$

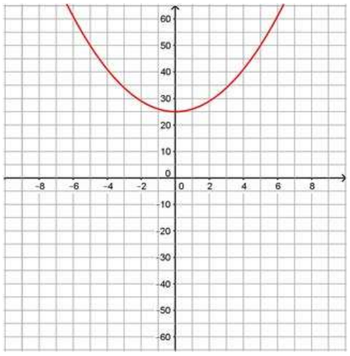
$\boldsymbol{x = \pm 5i}$

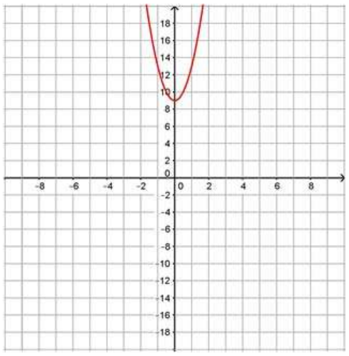
11. $g(x) = 4x^2 + 9$

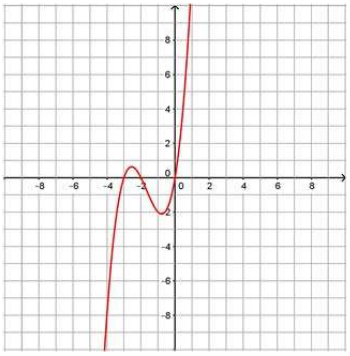
$\boldsymbol{x = \pm \frac{3}{2} i}$

12. $h(x) = x(x^2 + 5x + 6)$

$\boldsymbol{x = 0, -2, -3}$







Topic: Using polynomial division

13. The product of two polynomials is $x^3 + 4x^2 + x - 6$. One of the factors is $x - 1$. Use the box method to find the other factors.

	$\boldsymbol{x^2}$	$\boldsymbol{5x}$	$\boldsymbol{6}$
$\boldsymbol{x}$	$\boldsymbol{x^3}$	$\boldsymbol{5x^2}$	$\boldsymbol{6x}$
$\boldsymbol{-1}$	$\boldsymbol{-x^2}$	$\boldsymbol{-5x}$	$\boldsymbol{-6}$

$\boldsymbol{(x+3)(x+2)}$

14. Use the box method to divide the following polynomials.
$(x^3 - 10x^2 + 29x - 56) \div (x - 7)$

	$\boldsymbol{x^2}$	$\boldsymbol{-3x}$	$\boldsymbol{8}$
$\boldsymbol{x}$	$\boldsymbol{x^3}$	$\boldsymbol{-3x^2}$	$\boldsymbol{8x}$
$\boldsymbol{-7}$	$\boldsymbol{-7x^2}$	$\boldsymbol{21x}$	$\boldsymbol{-56}$

$\boldsymbol{x^2 - 3x + 8}$

Go

Topic: Multiply polynomials

Multiply each expression. Express your solutions in simplest form by combining like terms.

15. $(3x-5)(3x+5)$

$\mathbf{9x^2-25}$

16. $(7x+4)(7x+4)$

$\mathbf{49x^2+56x+16}$

17. $(x-2)(x^2+2x+4)$

$\mathbf{x^3-8}$

18. $(x+1)(x^2-x+1)$

$\mathbf{x^3+1}$

Hint: Binomial expansion

19. Expand: $(2x+5)^3$

$\mathbf{8x^3+60x^2+150x+125}$

20. Find the 6th term in: $(2x-5)^7$

$\mathbf{-262500x^2}$

Name **Polynomial Functions** **1.8H**

Ready, Set, Go!

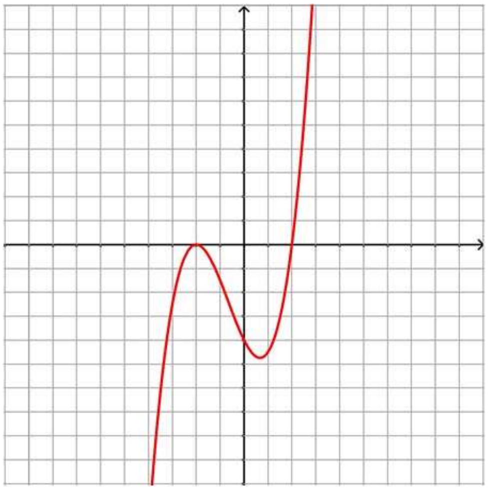
Ready

Topic: Graphing polynomial functions

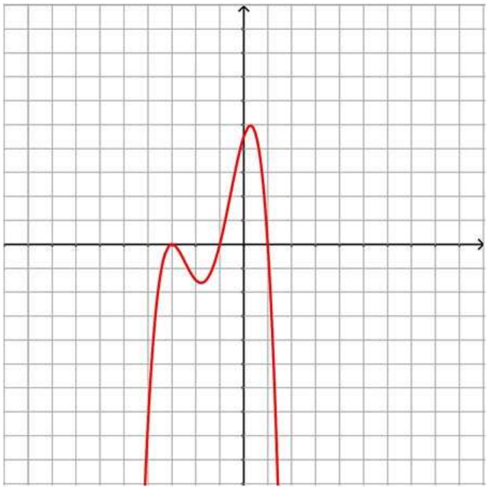
Without using technology, sketch a graph of the polynomial function described (if possible). If not possible, state why not.

Answers will vary – sample answers given

1. A cubic function with one negative zero (multiplicity 2) and one positive.

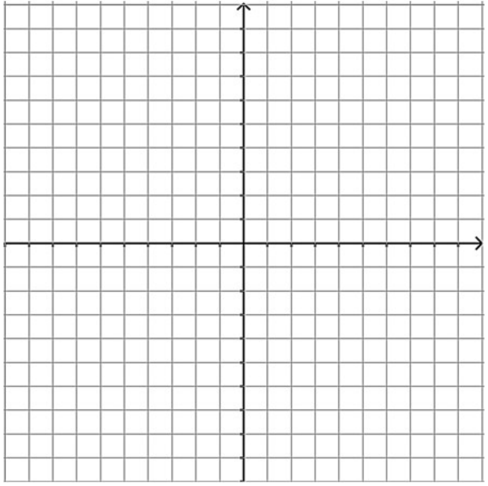


2. A quartic function (4th degree) with a negative leading coefficient, a positive y-intercept, one negative double root, one positive zero, and one additional zero.

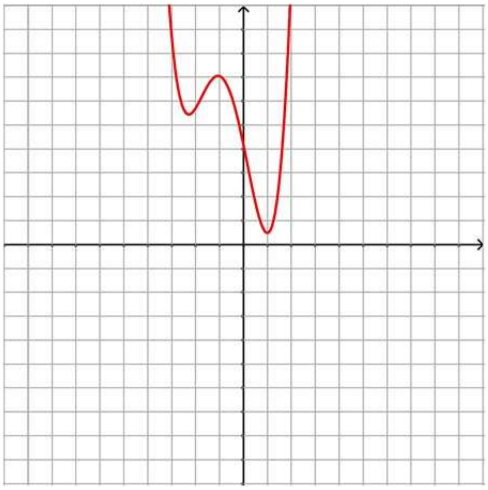


3. A cubic function with zero real roots.

not possible, must have at least one zero



4. A quartic function with zero real roots, a positive leading coefficient, and a positive y-intercept.



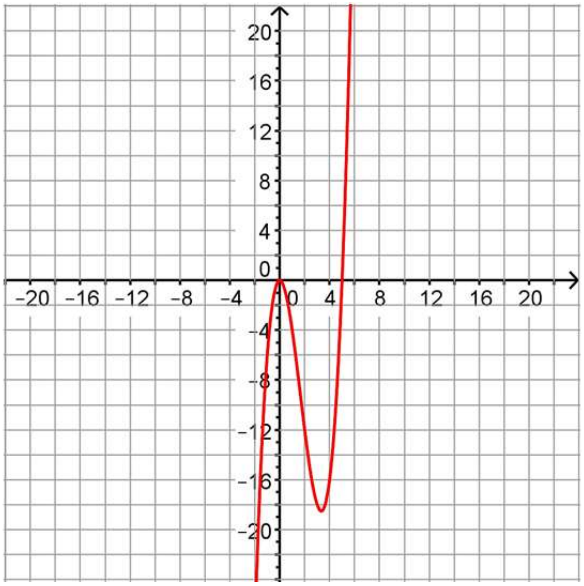
Set

Topic: Finding factors of polynomial functions

Find all linear factors and sketch the graph of the polynomial functions (unless you see another method that allows for quicker graphing. If so, explain method).

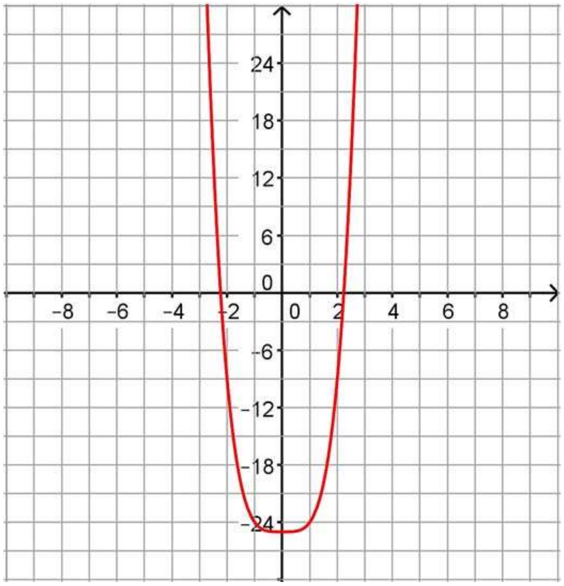
5. $f(x) = x^3 - 5x^2$

   $\boldsymbol{x \cdot x(x - 5)}$



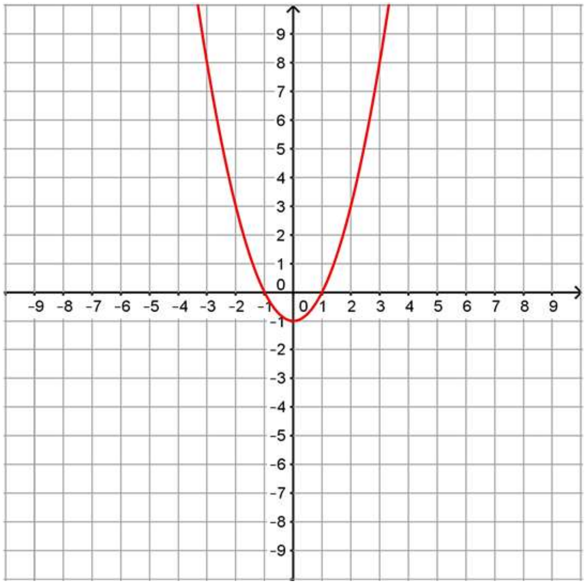
6. $f(x) = x^4 - 25$

   $\boldsymbol{(x + \sqrt{5})(x - \sqrt{5})(x + i\sqrt{5})(x - i\sqrt{5})}$



7. $f(x) = x^2 - 1$

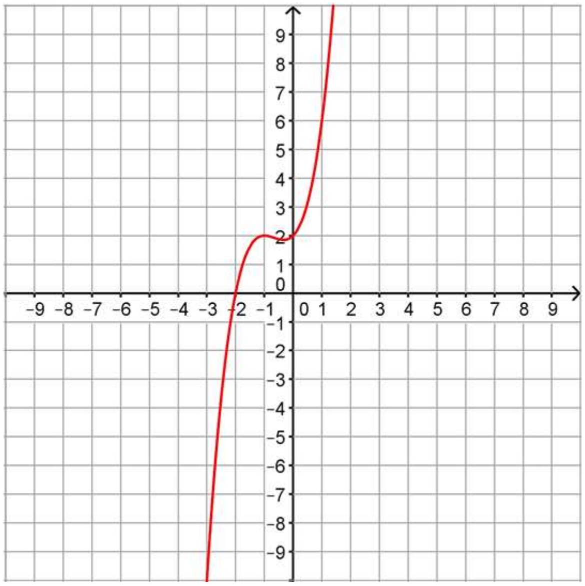
   $\boldsymbol{(x - 1)(x + 1)}$



8. $f(x) = x^3 + 2x^2 + x + 2$

   Hint: one root is i

   $\boldsymbol{(x + 2)(x - i)(x + i)}$



Use the Remainder Theorem to determine if the following are roots to the given equation. If so, find the other roots and graph the equation. Then write the function in factored form.

9. $f(x) = x^3 + 5x^2 + 2x - 8;\ \ f(1)$

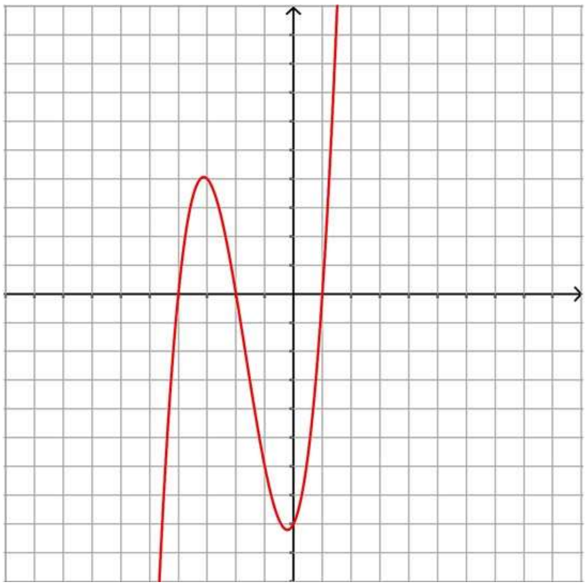
$f(1)$ is a root, other roots $x = -4, -2$

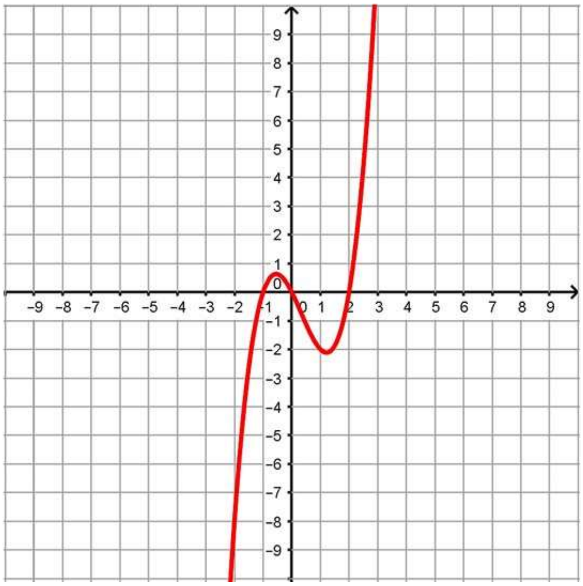
$f(x) = (x - 1)(x + 4)(x + 2)$

10. $f(x) = x^3 - x^2 - 2x;\ \ f(1)$

$f(1)$is a root, other roots $-2, 0$

$f(x) = x(x - 2)(x + 1)$





Topic: Writing polynomial functions given roots

Write the polynomial function in standard form with least degree using the given information. Make sure to include any missing conjugate pairs.

11. Leading coefficient: 2; roots: $2, \sqrt{2}$

$y = 2x^3 - 4x^2 - 4x + 8$

12. Leading coefficient: -1; roots: $1, 1 + \sqrt{3}$

$y = -x^3 + 3x^2 - 2$

13. Leading coefficient: 2; roots: $4i$

$y = 2x^2 + 32$

14. Passes through the points $(2, 0), (-3, 0), (1, 0), (0, 1)$

$y = \frac{1}{6}x^3 - \frac{7}{6}x + 1$

Go

Topic: Expanding binomials

Use Pascal's triangle to help expand the following binomials.

15. $(2x-3)^4$

$\mathbf{16x^4 - 96x^3 + 216x^2 - 216x + 81}$

16. $(3a+2b)^3$

$\mathbf{27a^3 + 54a^2b + 36ab^2 + 8b^3}$

Topic: Finding roots of polynomial functions

Find the roots of the polynomial functions using the given information.

17. $f(x) = x^4 + x^3 - 3x^2 - x + 2$, $x = 1$ is a double root (multiplicity of 2)

$\mathbf{f(x) = (x-1)^2(x+1)(x+2), x = 1, -1, -2}$

18. $g(x) = x^3 - 7x^2 + 3x - 21$, $g(7) = 0$

$\mathbf{g(x) = (x-7)(x^2+3), x = 7, \pm\sqrt{3}i}$

Topic: Factoring polynomials

Factor each polynomial completely.

19. $x^4 - 7x^2 + 12$

$\mathbf{(x+2)(x-2)(x+\sqrt{3})(x-\sqrt{3})}$

20. $x^4 - 15x^2 - 16$

$\mathbf{(x+4)(x-4)(x+i)(x-i)}$

Name **Polynomial Functions** | **1.9H**

Ready, Set, Go!

Ready

Topic: Solving polynomial, logarithmic, and rational equations.

Solve for *x*.

1. $2(x-2)(x+1)^2=0$

   $\boldsymbol{x=-1, 2}$

2. $6x^2+x=12$

   $\boldsymbol{x=\frac{3}{2}, -\frac{4}{3}}$

3. $x^4-2401=0$

   $\boldsymbol{x=\pm 7, \pm 7i}$

4. $x^2+4x-9=0$

   $\boldsymbol{x=-2\pm\sqrt{13}\approx 1.61\ \&\ 5.61}$

5. $\log_2 9=x$

   $\boldsymbol{x=\frac{\log 9}{\log 2}\approx 3.17}$

6. $\frac{3}{x+1}=6$

   $\boldsymbol{x=-\frac{1}{2}}$

Topic: Using the Remainder Theorem

Find $f(3)$ for each polynomial and state whether or not $(x-3)$ is a factor.

7. $f(x)=x^3-9x+3$

   $\boldsymbol{f(3)=3}$
   not a factor

8. $f(x)=x^3-9x^2+27x-28$

   $\boldsymbol{f(3)=-1}$
   not a factor

9. $f(x)=2x^3-5x^2-12x+27$

   $\boldsymbol{f(3)=0}$
   yes it is a factor

Set

Topic: Graphing polynomial functions.

Complete the information below using the graph

10. Function:

$f(x) = (x+2)^2(x-2)$

End Behavior:

As $x \to -\infty$, $f(x) \to$ __$-\infty$

As $x \to \infty$, $f(x) \to$ ____∞

Roots (with multiplicity):

$x = -2$ **(multiplicity of 2),** $x = 2$

Value of Leading Coefficient:

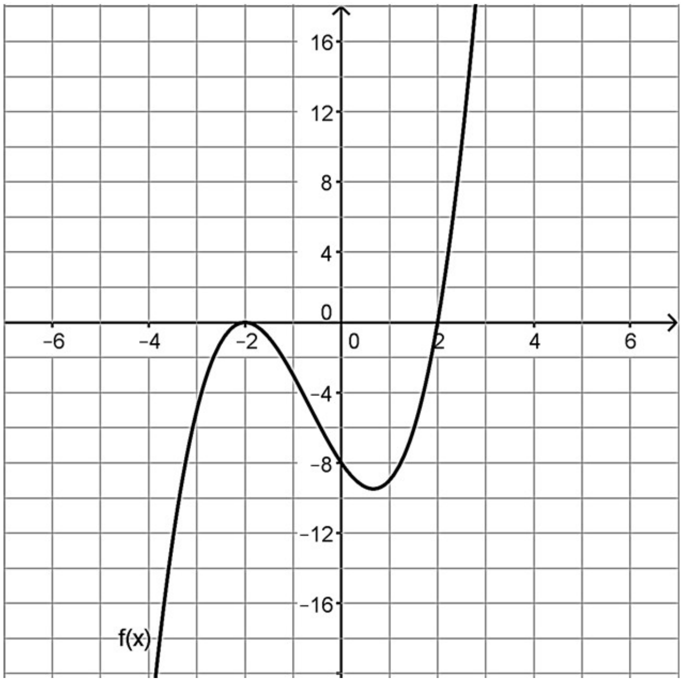
$a = 1$

Domain:

$(-\infty, \infty)$

Range:

$(-\infty, \infty)$



11. Write the polynomial function with least degree, in both factored and standard forms, given the following roots and a point that the function passes through.

Roots: $\pm 1, 3$, Point on the graph: $(0, 9)$

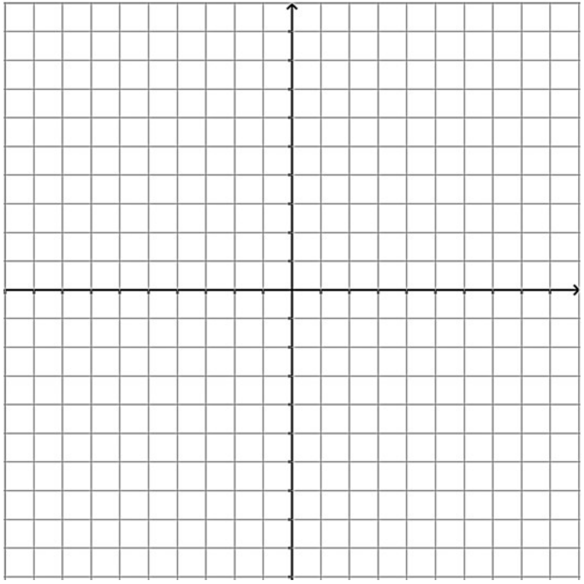
Factored Form: $f(x) = 3(x+1)(x-1)(x-3)$

Standard Form: $f(x) = 3x^3 - 9x^2 - 3x + 9$

Without using technology, sketch the graph of the polynomial function described.

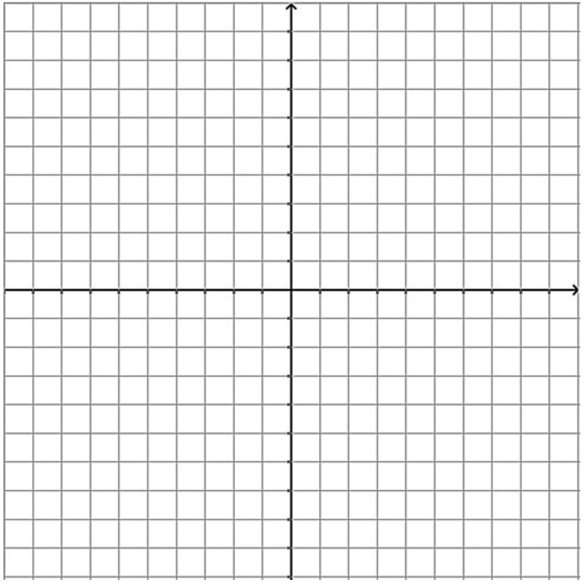
12. A cubic function with a leading coefficient of -1, with one positive zero.

Answers will vary



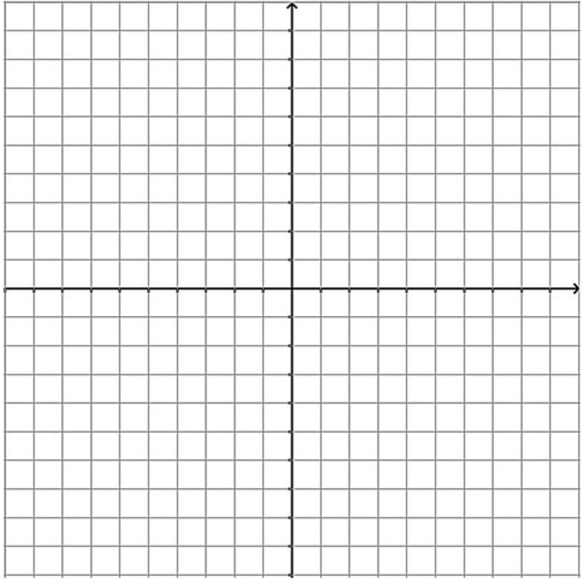
13. A quartic function with a leading coefficient of 1, with two double zeros.

Answers will vary



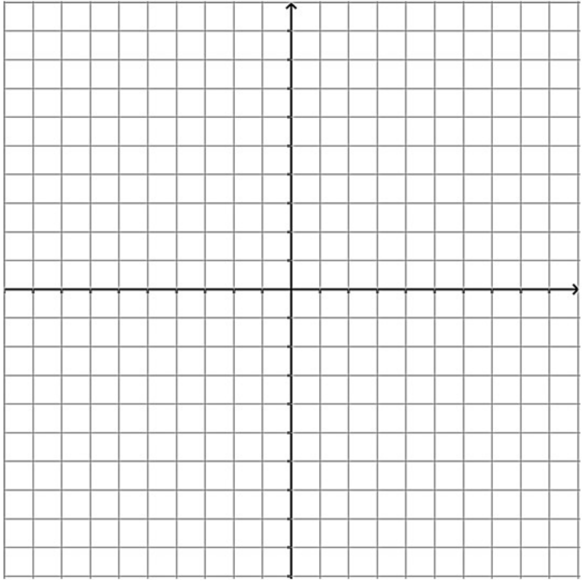
14. A cubic function with a leading coefficient of -3, with one positive triple root.

Answers will vary



15. A quartic function with a leading coefficient of 2, with two negative zeros and two imaginary roots.

Answers will vary



Go

Topic: End behavior

Circle the expression that has the greatest value of $f(x)$ as $x \to \infty$.

16. [✓ 2^x] $x^2 - 2x + 10$ $x + 5$ $\log x$

17. $\left(\frac{1}{2}\right)^x$ $x^2 - 2x + 10$ [✓ $x^5 - 4x^2$] $3\left(\sqrt{x}\right)^7$

18. $3 \cdot 2^x$ $x^3 + x^2 - 4$ [✓ $2(3^x)$] x^{10}

Topic: Determining the type of a function based on a table of data.

19. Determine the type of each function. Then find an explicit equation for each.

x	$a(x)$	$b(x)$	$c(x)$	$d(x)$	$e(x)$
−5	26	35	27	0.015625	−251
−4	17	24	22	0.03125	−129
−3	10	15	17	0.0625	−55
−2	5	8	12	0.125	−17
−1	2	3	7	0.25	−3
0	1	0	2	0.5	−1
1	2	−1	−3	1	1
2	5	0	−8	2	15
3	10	3	−13	4	53
4	17	8	−18	8	127
5	26	15	−23	16	249

quadratic, $a(x) = x^2 + 1$
quadratic, $b(x) = (x - 1)^2 - 1$
linear, $c(x) = -5x + 2$
exponential, $d(x) = \frac{1}{2} \cdot 2^x$
cubic, $e(x) = 2x^3 - 1$