Name:

Ready, Set, Go!

Ready

Topic: Polynomial division

Use division to determine if the given linear term is a factor of the polynomial. If it is a linear factor, then find the other factors of the polynomial.

1. $2x^3 + 11x^2 + 6x - 16; x + 2$

   Yes
   $(x+2)(2x^2+7x-8)$

2. $8b^3 + 14b^2 - 3b - 9; b + 1$

   Yes
   $(b+1)(4b-3)(2b+3)$

3. $2x^3 + x^2 - 16x - 3; x - 3$

   No

4. $x^3 + 10x^2 + 13x + 36; x + 9$

   Yes
   $(x+9)(x^2+x+4)$

Set

Topic: Simplifying rational expressions & operations on rational expressions

Answer the following questions.

5. Define *rational expression* and give three examples.
   A rational expression is an expression that can be written as the quotient of two polynomials.

6. Circle the expressions below that are rational. Explain why the non-circled terms are irrational.

   [✓ $\frac{x^2+5}{4x-1}$]   [✓ $\frac{3}{x^2-16}$] $\frac{\sin x}{2x+4}$ [✓ $\frac{x^2+6x+5}{x^3+7x-9}$] $\sqrt{x+3}$ [✓ $\frac{x^2+6x+5}{5}$]

   $\frac{\sin x}{2x+4}$ **isn't rational because** $\sin x$ **is not a polynomial.** $\sqrt{x+3}$ **isn't rational because it isn't a polynomial.**

7. Angela simplified the following rational expressions. Circle the one(s) she answered correctly. Then, identify and correct where she went wrong in the other two problems.

a. $\frac{5x}{(x-3)}+\frac{2}{(x-1)}$

$\frac{5x(x-1)}{(x-3)(x-1)}+\frac{2(x-3)}{(x-3)(x-1)}$

$\frac{5x^2-x+2x-6}{(x-3)(x-1)}$

$\frac{5x^2+x-6}{(x-3)(x-1)}$

b. $\frac{x}{(x+3)}-\frac{4(x+3)}{(x-1)}$

$\frac{x}{1}-\frac{4}{(x-1)}$

$\frac{x(x-1)}{(x-1)}-\frac{4}{(x-1)}$

$\frac{x^2-x-4}{(x-1)}$

[✓ c.] $\frac{(x+1)(x-2)}{(x+2)}\cdot\frac{(x+5)}{(x-2)(x+2)}$

$\frac{(x+1)(x-2)(x+5)}{(x+2)(x-2)(x+2)}$

$\frac{(x+1)(x+5)}{(x+2)(x+2)}$

$\frac{x^2+6x+5}{x^2+4x+4}$

A: Distributed incorrectly in the 3rd line. Answer should be $\frac{5x^2-3x-6}{(x-3)(x-1)}$.

B: Canceled the $(x+3)$ terms in the 2nd line when she needed a common denominator. Answer should be: $\frac{-3x^2-25x-36}{(x-1)(x+3)}$.

Simplify each expression. Leave answers in factored form.

8. $\frac{2x+6}{(x+1)}-\frac{4x-3}{(x+1)}$

$\frac{-2x+9}{x+1}$

9. $\frac{2x}{x+2}+\frac{x-1}{x-5}$

$\frac{3x^2-9x-2}{(x+2)(x-5)}$

10. $\frac{3x^2-14x-24}{2x^2-17x+8}\cdot\frac{x^2-2x-48}{6x^2+35x+36}$

$\frac{(x-6)(x+6)}{(2x-1)(2x+9)}$

11. $\frac{8x^2-6x-9}{15x^2+17x-4}\div\frac{4x^2-9}{5x^2-31x+6}$

$\frac{(4x+3)(x-6)}{(3x+4)(2x+3)}$

Go

Topic: Solving radical equations

Find ALL solutions to the following equations. Watch out for extraneous solutions (answers that make the original equation false).

12. $\sqrt{x+5} = 4$

$\mathbf{x = 11}$

13. $\sqrt{2x+15} = x+6$

$\mathbf{x = -3}$
$\mathbf{x = -7}$ **is extraneous**

14. $2\sqrt{x-1} = 2$

$\mathbf{x = 2}$

15. $\sqrt{3x+19} = x-3$

$\mathbf{x = 10}$
$\mathbf{x = -1}$ **is extraneous**

16. a. What causes extraneous solutions when solving **radical equations**?

If the solution makes the radicand negative.

b. What causes extraneous solutions when solving **rational equations**?

If the solution causes the denominator to equal 0.

Topic: Trigonometric ratios in a right triangle

Complete the given trigonometric ratios for ΔABC.

17. $\sin A = \frac{5}{13}$

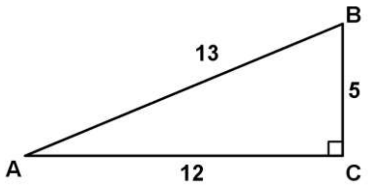
$\cos A = \frac{12}{13}$

$\tan A = \frac{5}{12}$

$\csc A = \frac{13}{5}$

$\sec A = \frac{13}{12}$

$\cot A = \frac{12}{5}$



18. $\sin B = \frac{3}{5}$

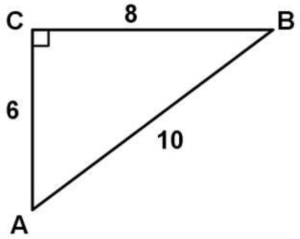
$\cos B = \frac{4}{5}$

$\tan B = \frac{3}{4}$

$\csc B = \frac{5}{3}$

$\sec B = \frac{5}{4}$

$\cot B = \frac{4}{3}$



Name: **Rational Functions** | **2.2H**

Ready, Set, Go!

Ready

Topic: Identifying values that make the expressions undefined

Identify the value(s) of x that make each expression undefined (hint: make the denominator equal to 0 or make the radicand negative).

1. $\frac{2}{x-6}$

$x = 6$

2. $\sqrt{x+4}$

$x < -4$

3. $\frac{x+5}{x+2}$

$x = -2$

4. $\sqrt{2x-1}$

$x < \frac{1}{2}$

5. $\frac{1}{(x+3)(x-9)}$

$x = -3, 9$

6. $\frac{4x}{x^2+7x+10}$

$x = -5, -2$

Set

Topic: Solving rational equations

Find ALL solutions to the following equations. Watch out for extraneous solutions (answers that make the original equation false).

7. $\frac{4}{x+2} + \frac{2x}{x-1} = \frac{2x+1}{x+2}$

$x = \frac{1}{3}$

8. $\frac{x^2+9x+18}{x+3} = 2$

$x = -4$

$x = -3$ is extraneous

9. $\frac{3}{x+1} + \frac{4}{x+2} = 5$

$\boldsymbol{x = 0, -\frac{8}{5}}$

10. $\frac{x+3}{x+2} = 1 - \frac{x+1}{x+2}$

no solution

$x = -2$ is extraneous

11. $\frac{x}{x+1} = \frac{5}{2x-2} - \frac{1}{2}$

$\boldsymbol{x = -\frac{2}{3}, 3}$

12. $\frac{4x}{x+1} + \frac{x+5}{x+1} = 3$

no solution

$x = -1$ is extraneous

Topic: Solving rational inequalities

13. Which of the following is/are true?

a. The solution set of $x^2 > 25$ is $(5, \infty)$.

[✓ b.] The inequality $\frac{x-2}{x+3} < 2$ can be solved by multiplying both sides by $(x+3)$, resulting in the equivalent inequality $x - 2 < 2(x+3)$.

c. $(x+3)(x-1) \geq 0$ and $\frac{x+3}{x-1} \geq 0$ have the same solution set.

d. $\frac{x^2+1}{x} > 0$ has no real solution.

Solve each rational inequality. Write your answers in interval notation.

14. $\frac{4}{x-3} < 2$

$(-\infty, \mathbf{3}) \cup (\mathbf{5}, \infty)$

15. $\frac{6}{x} + 2 \geq 0$

$(-\infty, \mathbf{-3}] \cup (\mathbf{0}, \infty)$

16. $\frac{2x}{x-2} \leq 3$

$(-\infty, \mathbf{2}) \cup (\mathbf{2}, \mathbf{6}]$

17. $\frac{2x+7}{x-4} \geq 3$

$(\mathbf{4}, \mathbf{19}]$

Go

Topic: Operations on rational expressions

Simplify each expression completely.

18. $\frac{7}{9x^2} + \frac{x}{3x^2+3x}$

$\frac{3x^2+7x+7}{9x^2(x+1)}$

19. $\frac{7x}{2x-1} \div \frac{x^2-6x}{x^2-11x+30}$

$\frac{7(x-5)}{2x-1}$

20. $\frac{x+2}{2x-2} - \frac{-2x-1}{x^2-4x+3}$

$\frac{x+4}{2(x-3)}$

21. $\frac{x^2+3x-4}{x^2+4x+4} \cdot \frac{2x^2+4x}{x^2-4x+3}$

$\frac{2x(x+4)}{(x+2)(x-3)}$

22. $\frac{x^2-4x-5}{x+5} \div (x^2 + 6x + 5)$

$\frac{x-5}{(x+5)^2}$

23. $\frac{x+3}{x^2-2x-8} - \frac{x-5}{x^2-12x+32}$

$\frac{-2(x+7)}{(x-4)(x+2)(x-8)}$

Name: **Rational Functions** | **2.3H**

Ready, Set, Go!

Ready

Topic: Describing transformations

Describe how the parent function was transformed to obtain each of the following functions.

1. $f(x) = 2(x-3)^2 - 7$

Translate right 3 & down 7, vertically stretch by a factor of 2.

2. $f(x) = \sqrt{x+6} + 2$

Translate left 6 and up 2

3. $f(x) = -\frac{1}{2}|x+4| - 5$

Reflect over the *x*-axis, vertically shrink by a factor of $\frac{1}{2}$, translate left 3 & down 5.

4. $f(x) = -\frac{5}{2}(x-1)^2 + 2$

reflect over the *x*-axis, vertically stretch by a factor of $\frac{5}{2}$, translate right 1 & up 2.

Topic: Proper vs. improper fractions

Classify each fraction as proper or improper. If the fraction is improper, rewrite as a mixed number.

5. $\frac{27}{5}$

improper, $5\frac{2}{5}$

6. $\frac{5}{27}$

proper

7. $\frac{32}{14}$

improper, $2\frac{2}{7}$

8. $\frac{13x^3}{18x^5}$

proper

Set

Topic: Features and families of functions

Find the roots and domain of each function. State the equations of any vertical asymptotes, if they exist.

9. $f(x) = (x+5)(x-2)(x-7)$

Zeros: $\boldsymbol{x = -5, 2, 7}$
Domain: $(-\infty, \infty)$
Asymptotes: **None**

10. $g(x) = 2x^2 - 9x - 35$

Zeros: $\boldsymbol{x = -\frac{5}{2}, 7}$
Domain: $(-\infty, \infty)$
Asymptotes: **None**

11. $k(x) = \frac{x^2+7x+6}{x+2}$

Zeros: $\boldsymbol{x = -1, -6}$
Domain: $(-\infty, \boldsymbol{-2}) \cup (\boldsymbol{-2}, \infty)$
Asymptotes: $\boldsymbol{x = -2}$

12. $h(x) = \frac{-x+2}{x^2+6x+5}$

Zeros: $\boldsymbol{x = 2}$
Domain: $(-\infty, \boldsymbol{-5}) \cup (\boldsymbol{-5, -1}) \cup (\boldsymbol{-1}, \infty)$
Asymptotes: $\boldsymbol{x = -1, x = -5}$

13. $q(x) = \sqrt{x+2} - 1$

Zeros: $\boldsymbol{x = -1}$
Domain: $(\boldsymbol{-2}, \infty)$
Asymptotes: **None**

14. $m(x) = \frac{2x+5}{(x-8)(x+4)}$

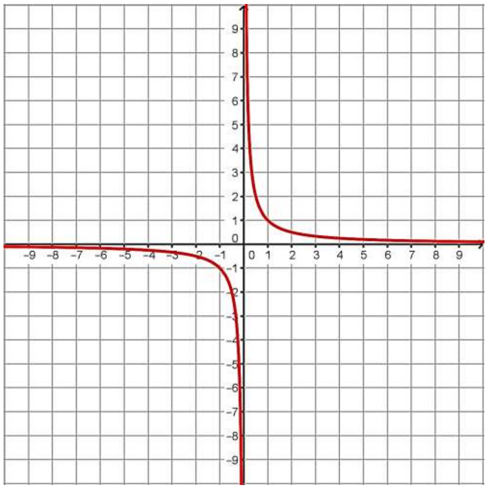
Zeros: $\boldsymbol{x = -\frac{5}{2}}$
Domain: $(-\infty, \boldsymbol{-4}) \cup (\boldsymbol{-4, 8}) \cup (\boldsymbol{8}, \infty)$
Asymptotes: $\boldsymbol{x = -4, x = 8}$

Topic: Features of rational functions

For each function, fill in the table of values and then graph the function. Then list the features of the function (domain/range, continuous/not continuous, intercepts, etc.).

15. $f(x) = \frac{1}{x}$

x	$f(x)$
-2	$\boldsymbol{-\frac{1}{2}}$
-1	$\boldsymbol{-1}$
$-\frac{1}{2}$	$\boldsymbol{-2}$
0	**Und**
$\frac{1}{2}$	$\boldsymbol{2}$
1	$\boldsymbol{1}$
2	$\boldsymbol{\frac{1}{2}}$



List of Features:
Domain: $\boldsymbol{(-\infty, 0) \cup (0, \infty)}$

Range: $\boldsymbol{(-\infty, 0) \cup (0, \infty)}$

Continuous: **not continuous**

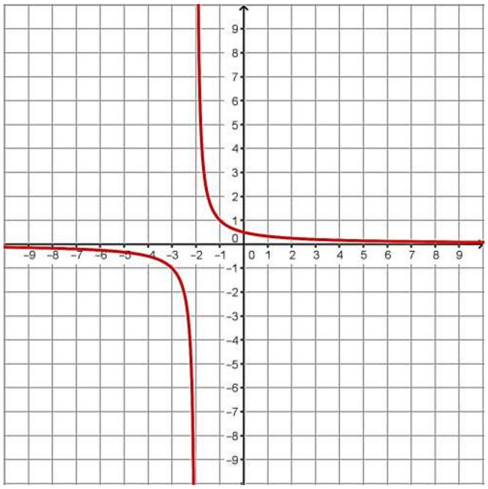
Intercepts: **none**

Asymptotes: $\boldsymbol{x = 0, y = 0}$

End behavior:
As $\boldsymbol{x \to -\infty, f(x) \to 0}$
As $\boldsymbol{x \to \infty, f(x) \to 0}$

16. $f(x) = \frac{1}{x+2}$

x	$f(x)$
-4	$\boldsymbol{-\frac{1}{2}}$
-3	$\boldsymbol{-1}$
$-2\frac{1}{2}$	$\boldsymbol{-2}$
-2	**Und**
$-1\frac{1}{2}$	$\boldsymbol{2}$
-1	$\boldsymbol{1}$
0	$\boldsymbol{\frac{1}{2}}$



List of Features:
Domain: $\boldsymbol{(-\infty, -2) \cup (-2, \infty)}$

Range: $\boldsymbol{(-\infty, 0) \cup (0, \infty)}$

Continuous: **not continuous**

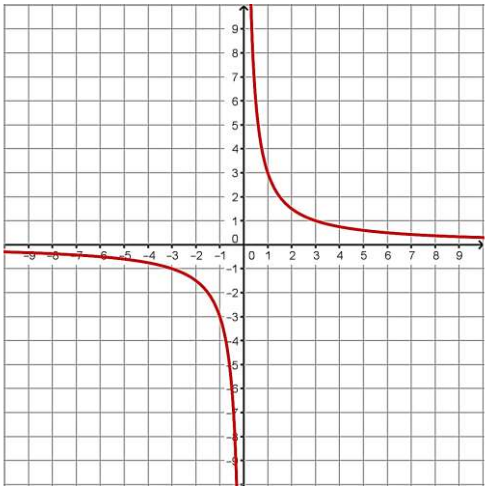
Intercepts: **no** $\boldsymbol{x}$**-intercepts**
$\boldsymbol{y}$**-intercept:** $\boldsymbol{\left(0, \frac{1}{2}\right)}$

Asymptotes: $\boldsymbol{x = -2, y = 0}$

End behavior:
As $\boldsymbol{x \to -\infty, f(x) \to 0}$
As $\boldsymbol{x \to \infty, f(x) \to 0}$

17. $f(x) = \frac{3}{x}$

x	$f(x)$
-2	$\boldsymbol{-\frac{3}{2}}$
-1	**-3**
$-\frac{1}{2}$	**-6**
0	**Und**
$\frac{1}{2}$	**6**
1	**3**
2	$\boldsymbol{\frac{3}{2}}$



List of Features:
Domain: **$(-\infty, 0) \cup (0, \infty)$**

Range: **$(-\infty, 0) \cup (0, \infty)$**

Continuous: **not continuous**

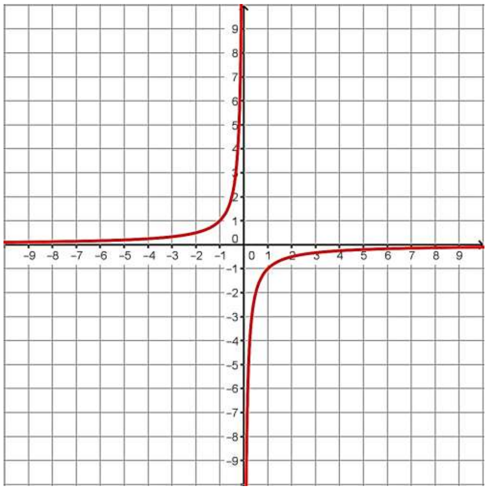
Intercepts: **none**

Asymptotes: **$x = 0, y = 0$**

End behavior:
As $x \to -\infty, f(x) \to 0$
As $x \to \infty, f(x) \to 0$

18. $f(x) = -\frac{1}{x}$

x	$f(x)$
-2	$\boldsymbol{\frac{1}{2}}$
-1	**1**
$-\frac{1}{2}$	**2**
0	**Und**
$\frac{1}{2}$	**-2**
1	**-1**
	$\boldsymbol{-\frac{1}{2}}$



List of Features:
Domain: **$(-\infty, 0) \cup (0, \infty)$**

Range: **$(-\infty, 0) \cup (0, \infty)$**

Continuous: **not continuous**

Intercepts: **none**

Asymptotes: **$x = 0, y = 0$**

End behavior:
As $x \to -\infty, f(x) \to 0$
As $x \to \infty, f(x) \to 0$

19. What happens to $f(x)$ as the x-values get closer to zero in the graph in question 18?
The values of $f(x)$ approach $\pm\infty$

Go

Topic: Operations with rational expressions

Simplify each expression completely.

20. $\frac{7x+7}{x^2+18x+80} \cdot \frac{x+8}{7x+7}$

$\mathbf{\frac{1}{x+10}}$

21. $\frac{x^2+8x+16}{x^2-6x+9} \div \frac{2x+8}{3x-9}$

$\mathbf{\frac{3(x+4)}{2(x-3)}}$

22. $\frac{3}{x+1} + \frac{4}{2x-6} - \frac{x^2-5}{x^2-2x-3}$

$\mathbf{\frac{-x^2+5x-2}{(x-3)(x+1)}}$

23. $\frac{\frac{2}{4x+12}}{\frac{4}{2x+6}+\frac{1}{x+3}}$

$\mathbf{\frac{1}{6}}$

Topic: Solving rational equations

Solve each equation.

25. $\frac{5}{x^3+5x^2} = \frac{4}{x+5} + \frac{1}{x^2}$

$\mathbf{x = -\frac{1}{4}}$

$\mathbf{x = 0}$ **is extraneous**

26. $\frac{x+5}{x^2+x} = \frac{1}{x^2+x} - \frac{x-6}{x+1}$

$\mathbf{x = 4, 1}$

27. $\frac{3}{x^2+5x+6} - \frac{x-6}{x^2+5x+6} = \frac{1}{x+3}$

$\mathbf{x = \frac{7}{2}}$

28. $\frac{x}{x+4} = 3 - \frac{4}{x+4}$

No solution

$\mathbf{x = -4}$ **is extraneous**

Name: **Rational Functions** | **2.4H**

Ready, Set, Go!

Ready

Topic: Distinguishing between proper and improper rational functions.

Determine if each of the following is a proper or an improper rational function. (Hint: look at the degree of the polynomials.)

1. $f(x) = \frac{x^3+3x^2+7}{7x^2-2x+1}$

   Improper

2. $f(x) = x^3 - 5x^2 - 4$

   Improper

3. $f(x) = \frac{3x^2-2x+7}{x^5-5}$

   Proper

4. $f(x) = \frac{x^3+4x^2+2x}{10x+7}$

   Improper

5. $f(x) = \frac{5x^2-4x+4}{7x^5-2x+3}$

   Proper

6. Which of the above functions have the following end behavior:
   as $x \to \infty, f(x) \to 0$ and as $x \to -\infty, f(x) \to 0$

   Questions 3 & 5

7. Complete the statement: ALL proper rational functions have end behavior that…
   approaches zero

Topic: Describing end behavior of polynomial functions

Based on the equations alone, describe what happens as $x \to \infty$ and $x \to -\infty$.

8. $h(x) = 2x - 3x^2 + 7$

   as $x \to \infty, h(x) \to -\infty$

   as $x \to -\infty, h(x) \to -\infty$

9. $g(x) = 14x^5 - 100x^4 + 1$

   as $x \to \infty, g(x) \to \infty$

   as $x \to -\infty, g(x) \to -\infty$

10. $f(x) = 3x - 4 + x^6$

    as $x \to \infty, f(x) \to \infty$

    as $x \to -\infty, f(x) \to \infty$

Set

Topic: Features of rational functions.

Find the x-intercept(s), y-intercept, and any vertical asymptotes of the following functions.

11. $f(x) = \frac{(x-1)(x+4)}{(x-5)(x+1)(x+2)}$

x-intercept(s): $\mathbf{1, -4}$

y-intercept: $\mathbf{\frac{2}{5}}$

vertical asymptotes:
$\boldsymbol{x = 5, -1, -2}$

12. $g(x) = \frac{(x^2-1)}{(x-3)(x+2)^2}$

x-intercept(s): $\mathbf{\pm 1}$

y-intercept: $\mathbf{\frac{1}{12}}$

vertical asymptotes:
$\boldsymbol{x = 3, -2}$

Topic: Improper vs. proper rational expressions

Determine if each rational expression is proper or improper. If improper, divide the polynomials to rewrite the rational expressions such that $\frac{a(x)}{b(x)} = q(x) + \frac{r(x)}{b(x)}$ where $q(x)$ represents the quotient and $r(x)$ represents the remainder.

13. $\frac{2x^3-7x^2+6}{x-3}$

Improper
$\boldsymbol{2x^2 - x - 3 + \frac{-3}{x-3}}$

14. $\frac{(x+1)}{(x-2)(x+2)}$

Proper

15. $\frac{x^3-3x^2+5x-1}{x^2-4x+4}$

Improper
$\boldsymbol{x + 1 + \frac{5x-5}{x^2-4x+4}}$

16. $\frac{x^3-5x+2}{x-10}$

Improper
$\boldsymbol{x^2 + 10x + 95 + \frac{952}{x-10}}$

Go

Topic: Simplifying rational expressions

Simplify each of the rational expressions by canceling common factors. Leave answers in factored form, where possible.

17. $\frac{6x^2-41x-56}{2x^2-17x+8}$

$\frac{6x+7}{2x-1}$

18. $\frac{9x^2-16}{6x^2+17x+12}$

$\frac{3x-4}{2x+3}$

19. $\frac{x^2-1}{4x^2+9x+5}$

$\frac{x-1}{4x+5}$

20. $\frac{4x^2-9}{3x^2-13x+14} \cdot \frac{x-2}{16x^2-26x+3}$

$\frac{2x+3}{(3x-7)(8x-1)}$

21. $\frac{9x^2-1}{2x-12} \div \frac{3x^2-23x-8}{5x^2-27x-18}$

$\frac{(3x-1)(5x+3)}{2(x-8)}$

Topic: Solving rational equations and inequalities

Solve each rational equation. Be sure to check for extraneous solutions.

22. $\frac{1}{x-2} = \frac{3}{x+2} - \frac{6x}{x^2-4}$

No solution
Extraneous: $x = -2$

23. $\frac{1}{x-6} + \frac{x}{x-2} = \frac{4}{x^2-8x+12}$

$x = -1$
Extraneous: $x = 6$

Solve each rational inequality. Write your answers in interval notation.

24. $\frac{x+6}{x^2+6x+8} \geq 0$

$[-6, -4) \cup (-2, \infty)$

25. $\frac{x^2-9}{x+5} < 0$

$(-\infty, -5) \cup [-3, 3]$

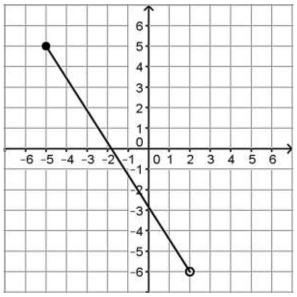
Ready, Set, Go!

Ready

Topic: Domain and range

Based on the graph given in each problem below, identify the domain and range of each function.

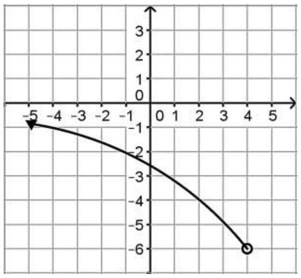
1.



Domain: $[-\mathbf{5},\mathbf{2})$

Range**:** $(-\mathbf{6},\mathbf{5}]$

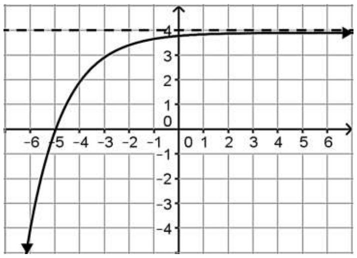
2.



Domain: $(-\infty,\mathbf{4}]$

Range: $(-\mathbf{6},\infty)$

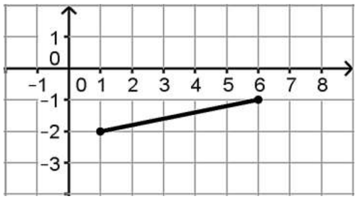
3.



Domain: $(-\infty,\infty)$

Range: $(-\infty,\mathbf{4})$

4.



Domain: $[\mathbf{1},\mathbf{6}]$

Range: $[-\mathbf{2},-\mathbf{1}]$

Set

Topic: Determine end behavior for rational functions

For each of the given functions, describe the end behavior as the *x*-values approach $+\infty$ and also $-\infty$.

5. $f(x) = \frac{(x-1)(x+2)}{x}$

As $x \to \infty, f(x) \to \infty$

As $x \to -\infty, f(x) \to -\infty$

6. $g(x) = \frac{2x+3}{x+1}$

As $x \to \infty, f(x) \to \mathbf{2}$

As $x \to -\infty, f(x) \to \mathbf{2}$

7. $t(x) = \frac{x^3+2x^2+x}{x+2}$

As $x \to \infty, f(x) \to \infty$

As $x \to -\infty, f(x) \to \infty$

8. $g(x) = \frac{2x+1}{x^2}$

As $x \to \infty, f(x) \to \mathbf{0}$

As $x \to -\infty, f(x) \to \mathbf{0}$

Topic: Finding vertical and horizontal asymptotes of rational functions.

Find the vertical, any horizontal asymptotes, and *x*-intercepts for the functions below.

9. $f(x) = \frac{1}{x^2-9}$

Vertical: $\boldsymbol{x = \pm 3}$

Horizontal: $\boldsymbol{y = 0}$

x-intercepts: **NA**

10. $f(x) = \frac{3}{x^3+2x^2-3x}$

Vertical: $\boldsymbol{x = 0, -3, 1}$

Horizontal: $\boldsymbol{y = 0}$

x-intercepts: **NA**

11. $f(x) = \frac{-1}{(x-3)(x+5)}$

Vertical: $\boldsymbol{x = 3, x = -5}$

Horizontal: $\boldsymbol{y = 0}$

x-intercepts: **NA**

12. $f(x) = \frac{3x-1}{x+2}$

Vertical: $\boldsymbol{x = -2}$

Horizontal: $\boldsymbol{y = 3}$

x-intercepts: $\boldsymbol{x = \frac{1}{3}}$

13. $f(x) = \frac{3x^2+4}{2x^2}$

Vertical: $\boldsymbol{x = 0}$

Horizontal: $\boldsymbol{y = \frac{3}{2}}$

x-intercepts: **NA**

14. $f(x) = \frac{(2x-1)(2x+1)}{x+4}$

Vertical: $\boldsymbol{x = -4}$

Horizontal: **none**

x-intercepts: $\boldsymbol{x = \frac{1}{2}, -\frac{1}{2}}$

Topic: Even and odd functions

15. Determine which of the following functions are even, odd or neither. Label them accordingly.

a. $f(x) = x^2 - 3$
even

b. $f(x) = \frac{1}{x}$
odd

c. $f(x) = x^2 + 4x + 4$
neither

d. $f(x) = |x|$
even

e. $f(x) = -x^2 + 7$
even

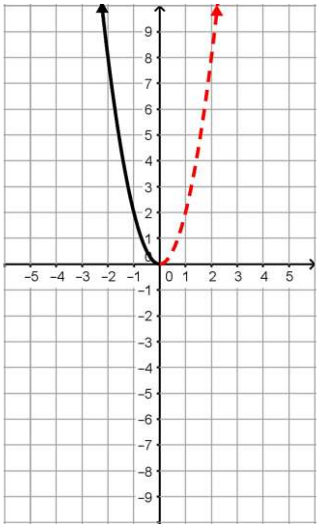
f. $f(x) = x^3 + x + 2$
neither

16. Use technology to graph each of the functions from number 15. What graphical characteristics go with an even function and what characteristics go with an odd function?

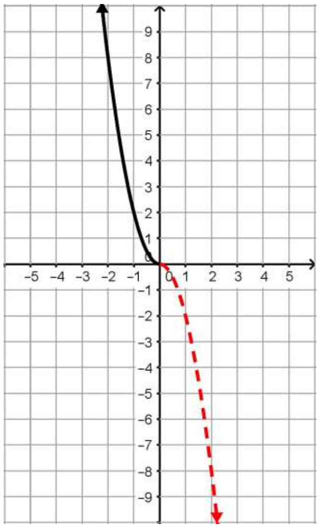
Even functions are symmetric about the *y*-axis. Odd functions have 180° rotational symmetry. Neither functions do not have the symmetries listed.

Given that the partial graphs below are even and odd functions, draw in the rest of the graph.

17. Even function



18. Odd Function



Go

Topic: Rational equations and inequalities.

Solve each rational equation and inequality.

19. $\frac{x-6}{2x^2+2x-4}+\frac{x}{2x-2}=\frac{1}{2}$

$\boldsymbol{x=2}$

20. $\frac{(x-1)(x+2)}{2x}>0$

$(-2,\ 0)\cup(1,\infty)$

21. $0=\frac{x^3+2x^2+x}{x+1}$

$\boldsymbol{x=0}$

Extraneous: $\boldsymbol{x=-1}$

22. $0=\frac{0.001(x^4+4x^2+4)}{x}$

$\boldsymbol{x=\pm i\sqrt{2}}$

23. $\frac{2x+1}{x^2}<0$

$\left(-\infty,-\frac{1}{2}\right)$

Topic: Simplifying rational expressions

Simplify each expression. Where possible, leave your answers in factored form.

24. $\frac{8x^2+22x-21}{2x^2-11x-63}$

$\boldsymbol{\frac{4x-3}{x-9}}$

25. $\frac{8x^2+19x+6}{3x+12}\div\frac{4x^2+3x-10}{6x+24}$

$\boldsymbol{\frac{2(8x+3)}{4x-5}}$

26. $\frac{3x-8}{x^2+6x+9}+\frac{5x}{x^2-9}+\frac{2}{x-3}$

$\boldsymbol{\frac{2(5x^2+5x+21)}{(x+3)^2(x-3)}}$

27. $\frac{8x-1}{x^2+x-6}-\frac{4}{x-2}$

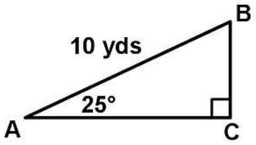
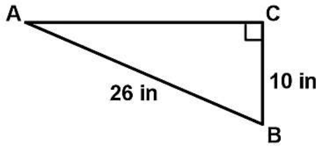
$\boldsymbol{\frac{4x-13}{(x+3)(x-2)}}$

Ready, Set, Go!

Ready

Topic: Solving right triangles

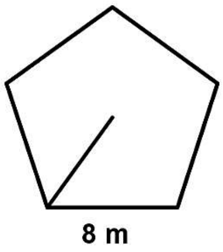
Solve each right triangle by using sine, cosine, and/or tangent to find the missing side lengths and angle measures.

1. $\angle B = 65°$ $\overline{AC} = 9.0631\ yd$ $\overline{BC} = 4.2262\ yd$  	2. $\angle B = 67.38°$ $\angle A = 22.62°$ $\overline{AC} = 24\ in$ 

Topic: Area of regular polygons

3. **Break the polygon into congruent isosceles triangles. Use the isosceles triangles to find the measure of the central angle and the area of the regular polygon.**

Regular Pentagon



Measure of the central angle: **72°**

Area: **110.1106 m^2**

Set

Topic: Features of rational functions

Identify the features of the function, complete the sign line, and then sketch the function.

4. $f(x) = \frac{2x}{x-3}$

Domain: $(-\infty, 3) \cup (3, \infty)$

Intercepts: ***x*-int: 0, *y*-int: 0**

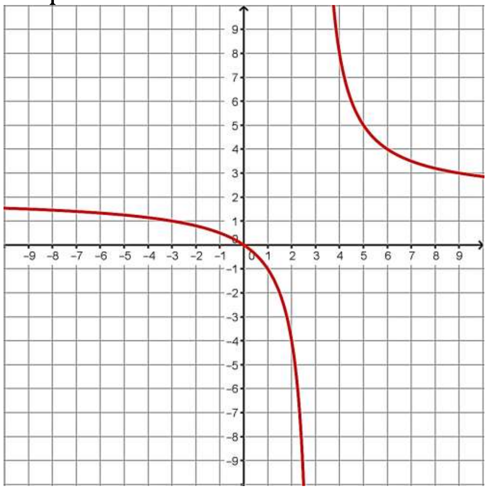
End Behavior: **as $x \to -\infty, f(x) \to 2$**
as $x \to \infty, f(x) \to 2$

Vertical Asymptote(s): $x = 3$

Horizontal Asymptote: $y = 2$

Sign Line:

Graph:



5. $g(x) = \frac{1}{(x-2)(x+2)}$

Domain: $(-\infty, -2) \cup (-2, 2) \cup (2, \infty)$

Intercepts: ***y*-int:** $-\frac{1}{4}$

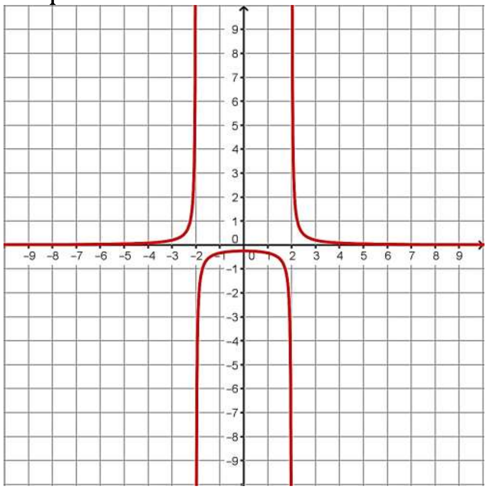
End Behavior: **as $x \to -\infty, f(x) \to 0$**
as $x \to \infty, f(x) \to 0$

Vertical Asymptote(s): $x = 2, -2$

Horizontal Asymptote: $y = 0$

Sign Line:

Graph:



6. $h(x) = \frac{x-2}{x-1}$

Domain: $(-\infty, \mathbf{1}) \cup (\mathbf{1}, \infty)$

Intercepts: **x-int: 2, y-int: 2**

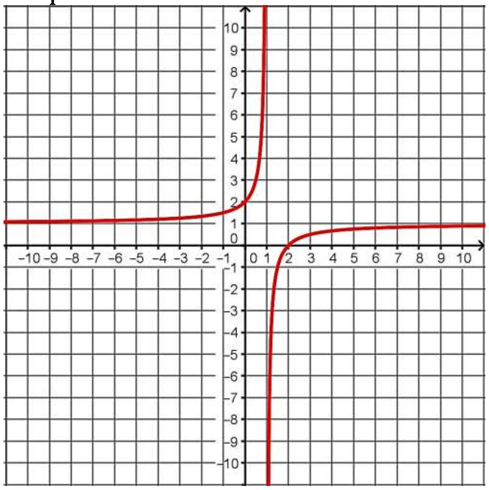
End Behavior: **as $x \to -\infty, f(x) \to 1$**
as $x \to \infty, f(x) \to 1$

Vertical Asymptote(s): **$x = 1$**

Horizontal Asymptote: **$y = 1$**

Sign Line:

Graph:



7. $w(x) = -\frac{1}{x^2}$

Domain: $(-\infty, \mathbf{0}) \cup (\mathbf{0}, \infty)$

Intercepts: **None**

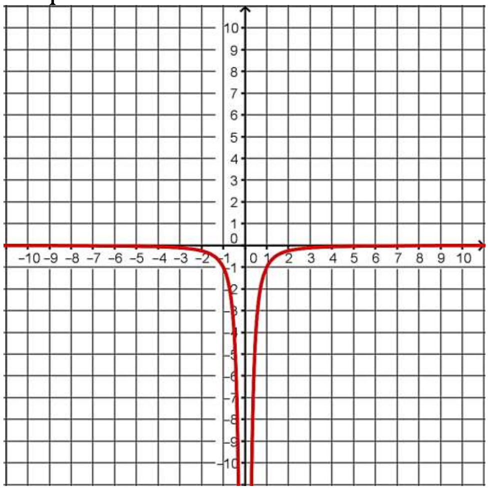
End Behavior: **as $x \to -\infty, f(x) \to 0$**
as $x \to -\infty, f(x) \to 0$

Vertical Asymptote(s): **$x = 0$**

Horizontal Asymptote: **$y = 0$**

Sign Line:

Graph:



Topic: Writing rational functions

Write a rational function that has the indicated features.

8. x-intercept at 2, vertical asymptotes at $x = -1$ & $x = 5$.

$\frac{x-2}{(x+1)(x-5)}$

9. horizontal asymptote at $y = 2$, vertical asymptotes at $x = -3$ & $x = 4$.

$\frac{2x+5}{(x+3)(x-4)}$

Go

Topic: Simplifying rational expressions

Rewrite each of the rational expressions in its simplest form. Where possible, leave answers in factored form.

10. $\frac{x^4-16}{3x^2-2x-8}$

$\frac{(x+2)(x^2+4)}{3x+4}$

11. $\frac{x+4}{7x^2+27x-4}$

$\frac{1}{7x-1}$

12. $\frac{x+1}{2x^2-11x-6}+\frac{3x-4}{4x^2-21x-18}$

$\frac{10x^2+2x-1}{(x-6)(2x+1)(4x+3)}$

13. $\frac{81x^2-16}{3x^2-29x+40}\cdot\frac{6x^2-7x-5}{9x^2-5x-4}$

$\frac{(9x-4)(2x+1)}{(x-8)(x-1)}$

14. $\frac{x^2-4x+8}{x^2-4}-\frac{3x-1}{x-2}$

$\frac{-2x^2-9x+10}{(x+2)(x-2)}$

15. $\frac{36x^2-49}{3x-1}\div\frac{6x^2-47x-63}{6x^2+13x-5}$

$\frac{(6x-7)(2x+5)}{x-9}$

Topic: Solving rational equations

Solve each equation.

16. $\frac{3}{x^2-4}=\frac{2}{x+2}+\frac{x}{x-2}$

$x=-2\pm\sqrt{11}$

17. $\frac{2x+8}{x+3}-\frac{2}{x+3}=x$

$x=2$

$x=-3$ is extraneous

18. $\frac{3}{x+1}+\frac{x-2}{3}=\frac{13}{3x+3}$

$x=3,-2$

Ready, Set, Go!

Ready

Topic: Solving rational equations

Solving rational equations, be sure to check your solutions.

1. $\frac{2}{x+2}-\frac{1}{x}=\frac{1}{x}$

No solution

2. $\frac{5}{x-2}+\frac{7}{x+2}=\frac{10x-2}{x^2-4}$

$\boldsymbol{x=1}$

3. $\frac{4}{x}=\frac{9}{x-2}$

$\boldsymbol{x=-\frac{8}{5}}$

4. $\frac{1}{4}+\frac{1}{x}=\frac{1}{6}$

$\boldsymbol{x=-12}$

5. $x+2+\frac{x}{x-2}=\frac{2}{x-2}$

$\boldsymbol{x=-3}$
$\boldsymbol{x=2}$ **is extraneous**

6. $\frac{x}{x-2}+\frac{1}{x-4}=\frac{2}{x^2-6x+8}$

$\boldsymbol{x=-1}$
$\boldsymbol{x=4}$ **is extraneous**

Set

Topic: Key features of rational functions

For each of the given functions list the key features of the function including the domain, the intercepts, end behavior, and asymptotes. Then complete the sign line and sketch a graph of each function.

7. $f(x) = \frac{3(x^2-1)}{x^2-4}$

Domain: $(-\infty, -2) \cup (-2, 2) \cup (2, \infty)$

Intercepts: $x = \pm 1, y = \frac{3}{4}$

End Behavior:

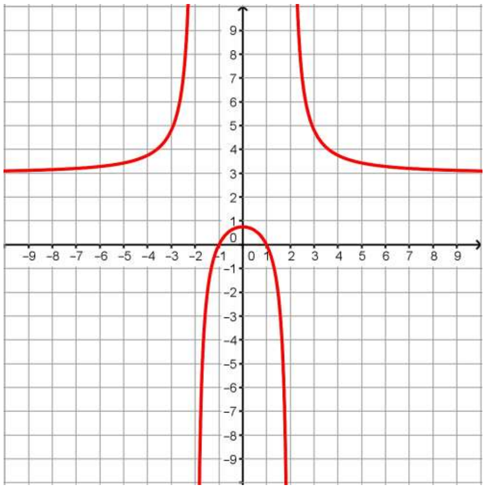
as $x \to -\infty, f(x) \to 3$

as $x \to \infty, f(x) \to 3$

Vertical Asymptote(s): $x = \pm 2$

Horizontal Asymptote: $y = 3$

Sign Line:



8. $r(x) = \frac{x+4}{x^2+5x-6}$

Domain: $(-\infty, -6) \cup (-6, 1) \cup (1, \infty)$

Intercepts: $x = -4, y = -\frac{2}{3}$

End Behavior:

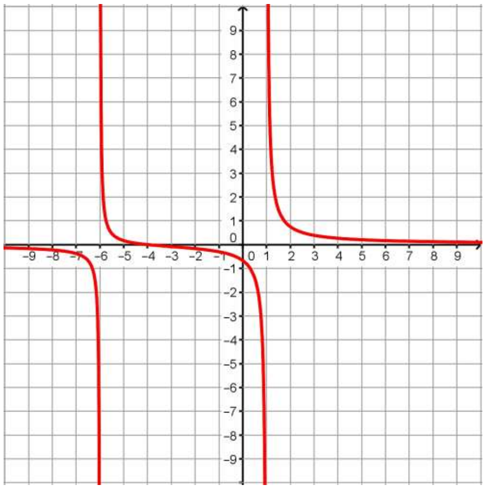
as $x \to -\infty, f(x) \to 0$

as $x \to \infty, f(x) \to 0$

Vertical Asymptote(s): $x = -6, 1$

Horizontal Asymptote: $y = 0$

Sign Line:



9. $q(x) = \frac{x}{x^3} + 3$

Domain: $(-\infty, \mathbf{0}) \cup (\mathbf{0}, \infty)$

Intercepts: **None**

End Behavior:

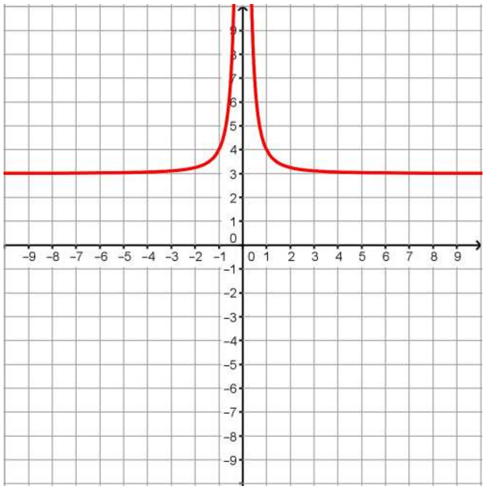
$\mathbf{as}\ \boldsymbol{x \to -\infty, f(x) \to 3}$

$\mathbf{as}\ \boldsymbol{x \to \infty, f(x) \to 3}$

Vertical Asymptote(s): $\boldsymbol{x = 0}$

Horizontal Asymptote: $\boldsymbol{y = 3}$

Sign Line:



10. $m(x) = \frac{x-1}{2x+1}$

Domain: $\left(-\infty, -\frac{1}{2}\right) \cup \left(-\frac{1}{2}, \infty\right)$

Intercepts: $\boldsymbol{x = 1, y = -1}$

End Behavior:

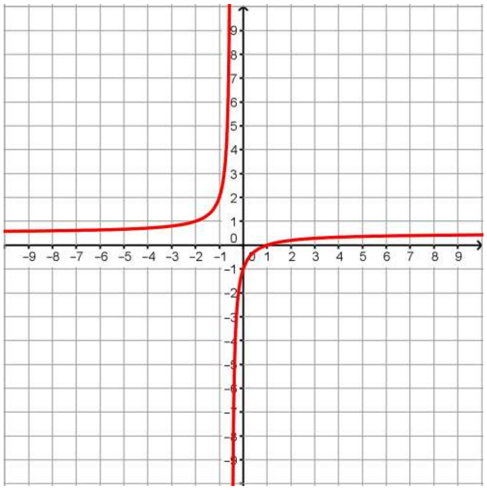
$\mathbf{as}\ \boldsymbol{x \to -\infty, f(x) \to \frac{1}{2}}$

$\mathbf{as}\ \boldsymbol{x \to \infty, f(x) \to \frac{1}{2}}$

Vertical Asymptote(s): $\boldsymbol{x = -\frac{1}{2}}$

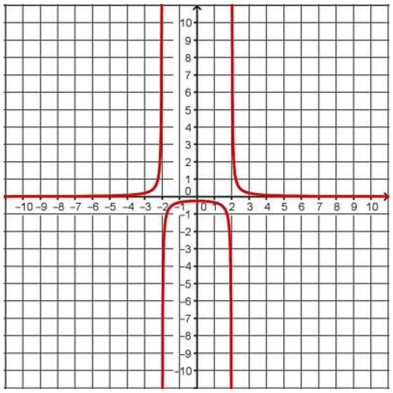
Horizontal Asymptote: $\boldsymbol{y = \frac{1}{2}}$

Sign Line:

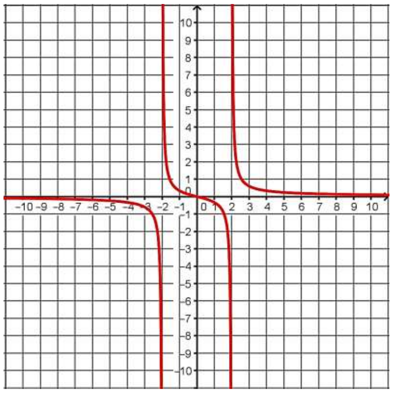


Sketch a graph and compare the given functions. Explain what you think accounts for the similarities and differences between the two functions.

11. a. $f(x) = \frac{1}{(x-2)(x+2)}$



b. $g(x) = \frac{x}{(x-2)(x+2)}$



Similarities:

Answers may vary…same end behavior, same horizontal & vertical asymptotes.

Differences:

Answers may vary. The middle sections of the graphs are different.

Why?

Answers may vary.

Topic: Identifying equations of slant asymptotes

Find the equation of the slant asymptote for each rational function below.

12. $f(x) = \frac{-3x^2+2}{x-1}$

$\boldsymbol{y = -3x - 3}$

13. $f(x) = \frac{x^2+3x+2}{x-2}$

$\boldsymbol{y = x + 5}$

14. $f(x) = \frac{x^2-6x-1}{x+3}$

$\boldsymbol{y = x - 9}$

15. $f(x) = \frac{2x^2+x-5}{x+1}$

$\boldsymbol{y = 2x - 1}$

Go

Topic: Attributes of rational functions

16. How do you know if a function is even, odd, or neither?
    Even functions: $f(-x) = f(x)$ and contain y-axis symmetry
    Odd functions: $f(-x) = -f(x)$ and contains 180° symmetry about the origin.

17. How do you determine the end behavior of a rational function?
    Compare the degrees of the numerator and denominator to determine the horizontal asymptotes.

18. Is the end behavior for a rational function always the same?
    No

19. What is the difference between a proper and an improper rational function?
    Proper: degree in the numerator is less than the degree in the denominator
    Improper: degree in the numerator is greater than or equal to the degree in the denominator

20. What attributes do all proper rational functions have in common?
    Horizontal asymptote is at $y = 0$.

Topic: Solving rational inequalities

Solve each rational inequality. Write your answers in interval notation.

21. $\frac{4}{x+2} > 2$

    $\mathbf{(-2, 0)}$

22. $\frac{3}{x-4} \leq -1$

    $\mathbf{[1, 4)}$

23. $\frac{(x^2-3x-10)}{1-x} \geq 2$

    $\mathbf{(-\infty, -3] \cup (1, 4]}$

24. $\frac{3}{2-x} \leq \frac{1}{x+4}$

    $\mathbf{\left(-4, -\frac{5}{2}\right] \cup (2, \infty)}$

Topic: Operations on rational expressions

Perform the indicated operation. Be sure to simplify your solutions.

25. $\frac{2x}{x+1} - \frac{3x-2}{x}$

$\mathbf{\frac{-x^2-x+2}{x(x+1)}}$

26. $\frac{x-4}{x-7} \cdot \frac{4x}{2x^2-32}$

$\mathbf{\frac{2x}{(x-7)(x+4)}}$

27. $\frac{5x}{x^2+2x} \div \frac{30x^2}{x+2}$

$\mathbf{\frac{1}{6x^2}}$

28. $\frac{x-11}{x^2+6x-40} + \frac{5}{x-4}$

$\mathbf{\frac{6x+39}{(x+10)(x-4)}}$