Ch 9 and 10 Review Worksheet Solutions

Note Title [redacted]

In 1-~~5~~4, Find all solution in the equation in the interval $[0, 2\pi)$.

1. $\tan^2 3x + \tan 3x = 0$

Step 1:

$\tan 3x(\tan 3x + 1) = 0$

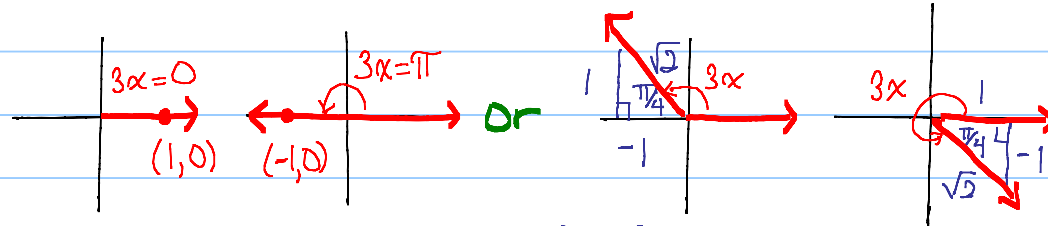
$\tan 3x = 0$ or $\tan 3x + 1 = 0$

$\underline{\qquad -1 \quad -1}$

$\tan 3x = -1$

Step 2:

$\tan 3x = 0$ or $\tan 3x = -1$



$3x = 0 + 2\pi n$ $\quad$ $3x = \pi + 2\pi n$ $\quad$ $3x = \frac{3\pi}{4} + 2\pi n$ $\quad$ $3x = \frac{7\pi}{4} + 2\pi n$

$3x = \frac{3\pi + 8\pi n}{4}$ $\quad$ $3x = \frac{7\pi + 8\pi n}{4}$

$x = \frac{2\pi n}{3}$, $x = \frac{\pi + 2\pi n}{3}$, $x = \frac{3\pi + 8\pi n}{12}$, $x = \frac{7\pi + 8\pi n}{12}$

Step 3: $n = 0, 1, 2$:

$$\boxed{x = 0, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{\pi}{3}, \pi, \frac{5\pi}{3}, \frac{\pi}{4}, \frac{11\pi}{12}, \frac{19\pi}{12}, \frac{7\pi}{12}, \frac{15\pi}{12}, \frac{23\pi}{12}}$$

2. $\sin 2x - \cos x = 0$

Step 1:

Use $\sin 2x = 2\sin x \cos x$

$2\sin x\cos x - \cos x = 0$

$\cos x\,(2\sin x - 1) = 0$

$\cos x = 0$ or $2\sin x - 1 = 0$

$\qquad\qquad\qquad\quad +1 \quad +1$

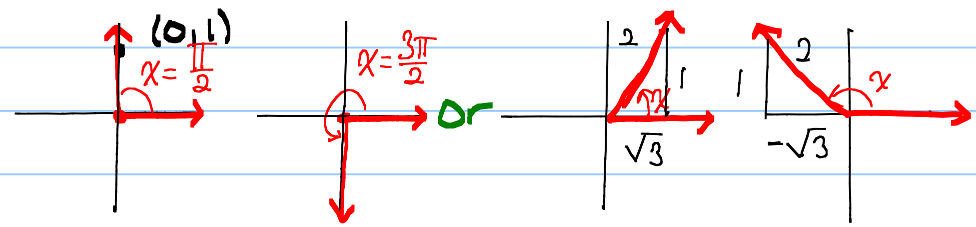
$2\sin x = 1$

$\frac{2\sin x}{2} = \frac{1}{2}$

$\sin x = \frac{1}{2}$

Step 2:

$\cos x = 0$ or $\sin x = \frac{1}{2}$



$x = \frac{\pi}{2} + 2\pi n \qquad x = \frac{3\pi}{2} + 2\pi n \qquad x = \frac{\pi}{6} + 2\pi n \qquad x = \frac{5\pi}{6} + 2\pi n$

Step 3: $n = 0$

$$\boxed{x = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{\pi}{6}, \frac{5\pi}{6}}$$

3. $4\cos^2\frac{x}{2} - 3 = 0$

Step 1:

$$4\cos^2\frac{x}{2} - 3 = 0$$

$$+3 \quad +3$$

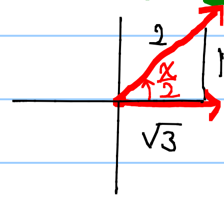
$$\frac{4\cos^2\frac{x}{2}}{4} = \frac{3}{4}$$

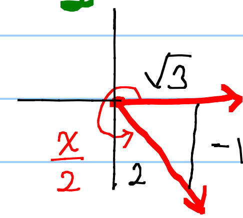
$$\sqrt{\cos^2\frac{x}{2}} = \pm\sqrt{\frac{3}{4}}$$

$$\cos\frac{x}{2} = \frac{\pm\sqrt{3}}{2}$$

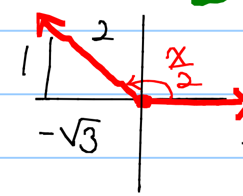
Step 2

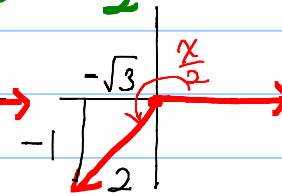
$\cos\frac{x}{2} = \frac{\sqrt{3}}{2}$ or $\cos\frac{x}{2} = \frac{-\sqrt{3}}{2}$





or





$\frac{x}{2} = \frac{\pi}{6} + 2\pi n$ $\quad$ $\frac{x}{2} = \frac{11\pi}{6} + 2\pi n$ $\quad$ $\frac{x}{2} = \frac{5\pi}{6} + 2\pi n$ $\quad$ $\frac{x}{2} = \frac{7\pi}{6} + 2\pi n$

$x = \frac{\pi}{3} + 4\pi n$ $\quad$ $x = \frac{11\pi}{3} + 4\pi n$ $\quad$ $x = \frac{5\pi}{3} + 4\pi n$ $\quad$ $x = \frac{7\pi}{3} + 4\pi n$

larger than 2π $\quad$ larger than 2π

Step 3 $\quad n = 0$

$$\boxed{x = \frac{\pi}{3}, \frac{5\pi}{3}}$$

4. $\csc^2 x - \csc x - 2 = 0$

Step 1:

$(\csc x - 2)(\csc x + 1) = 0$

$\csc x - 2 = 0$ or $\csc x + 1 = 0$

$+2 \quad +2$ $\qquad$ $-1 \quad -1$

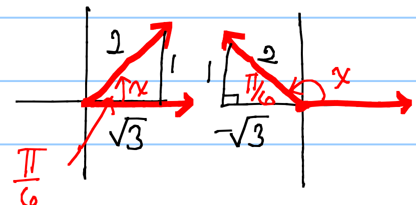
$\csc x = 2$ $\qquad$ $\csc x = -1$

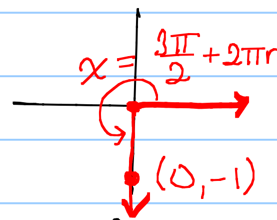
so $\qquad$ so

$\sin x = 1/2$ $\qquad$ $\sin x = -1$

Step 2

$\sin x = \frac{1}{2}$ or $\sin x = -1$





$x = \frac{\pi}{6} + 2\pi n \qquad x = \frac{5\pi}{6} + 2\pi n$ $\qquad$ $x = 3\pi/2 + 2\pi n$

Step 3 $\quad n = 0$

$$\boxed{x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{3\pi}{2}}$$

5. $\sin\frac{x}{2} + \cos x = 0$

Using $\frac{1}{2}$ ∠ formula:

$$\pm\sqrt{\frac{1-\cos x}{2}} + \cos x = 0$$

$$-\cos x \qquad -\cos x$$

$$\left(\pm\sqrt{\frac{1-\cos x}{2}}\right)^2 = \left(-\cos x\right)^2$$

$$\cancel{2}\cdot\frac{1-\cos x}{\cancel{2}} = 2\cos^2 x$$

$$1-\cos x = 2\cos^2 x$$

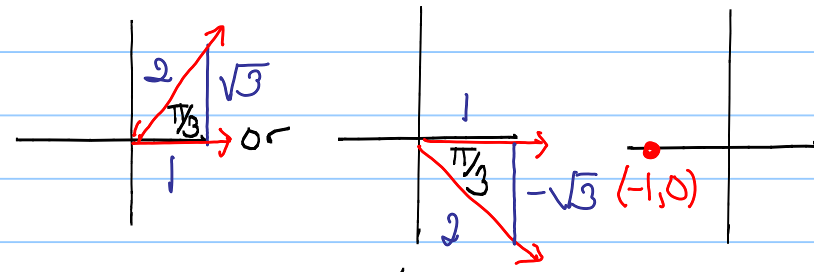
$$-1+\cos x \qquad +\cos x - 1$$

$$2\cos^2 x + \cos x - 1 = 0$$

$$(2\cos x - 1)(\cos x + 1) = 0$$

$2\cos x - 1 = 0$ and $\cos x + 1 = 0$

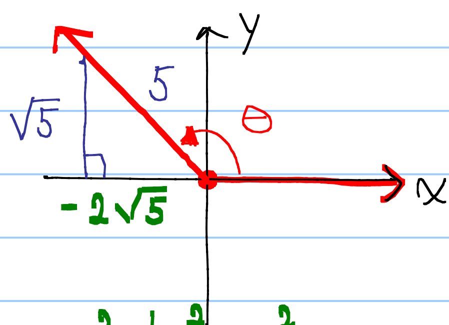
$\cos x = 1/2$ $\qquad$ $\cos x = -1$



$$\boxed{x = \pi/3,\ 5\pi/3,\ \pi}$$

6. Given $\sin\theta = \frac{\sqrt{5}}{5}$ and θ is in the inteval $[\frac{\pi}{2}, \pi]$. Find the exact values of $\sin 2\theta$, $\cos 2\theta$, and $\tan 2\theta$.

Step 1:



$a^2 + b^2 = c^2$

$(\sqrt{5})^2 + b^2 = (5)^2$

$5 + b^2 = 25$

$b^2 = 20$

$b = \pm 2\sqrt{5}$

Step 2: Use double ∠ formulas

$\sin 2\theta = 2\sin\theta\cos\theta$

$= 2\left(\frac{\sqrt{5}}{5}\right)\left(\frac{-2\sqrt{5}}{5}\right)$

$= \frac{-4(5)}{25}$

$\boxed{\sin 2\theta = \frac{-4}{5}}$

$\cos 2\theta = \cos^2\theta - \sin^2\theta$

$= \left(\frac{-2\sqrt{5}}{5}\right)^2 - \left(\frac{\sqrt{5}}{5}\right)^2$

$= \frac{4(5)}{25} - \frac{5}{25}$

$= \frac{4}{5} - \frac{1}{5}$

$\boxed{\cos 2\theta = \frac{3}{5}}$

$\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta} = \frac{2\left(\frac{\sqrt{5}}{-2\sqrt{5}}\right)}{1-\left(\frac{\sqrt{5}}{-2\sqrt{5}}\right)^2} = \frac{2(-\frac{1}{2})}{1-(-\frac{1}{2})^2}$

$= \frac{-1}{1-\frac{1}{4}} = \frac{-1}{\frac{3}{4}} = \frac{-4}{3}$

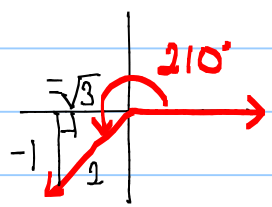
$\boxed{\tan 2\theta = \frac{-4}{3}}$

7. Use the half-angle formulas to find the exact value of $\sin 105^0$, $\cos 105^0$, and $\tan 105^0$.

Step 1:

$$2 \cdot \frac{x}{2} = (105^\circ)(2)$$

$$x = 210^\circ$$



105° is in Quad II.

Step 2:

$$\sin 105^\circ = \sin \frac{210^\circ}{2} = \sqrt{\frac{1-\cos 210^\circ}{2}} = \sqrt{\frac{1-\left(-\frac{\sqrt{3}}{2}\right)}{2}} = \sqrt{\frac{\frac{2+\sqrt{3}}{2}}{2}}$$

$$= \sqrt{\frac{2+\sqrt{3}}{2} \div 2} = \sqrt{\frac{2+\sqrt{3}}{2} \cdot \frac{1}{2}} = \sqrt{\frac{2+\sqrt{3}}{4}} = \frac{\sqrt{2+\sqrt{3}}}{2}.$$

$$\cos 105^\circ = \cos \frac{210^\circ}{2} = -\sqrt{\frac{1+\cos 210^\circ}{2}} = -\sqrt{\frac{1+\left(-\frac{\sqrt{3}}{2}\right)}{2}} = -\sqrt{\frac{2-\sqrt{3}}{4}}$$

$$= \frac{-\sqrt{2-\sqrt{3}}}{2}$$

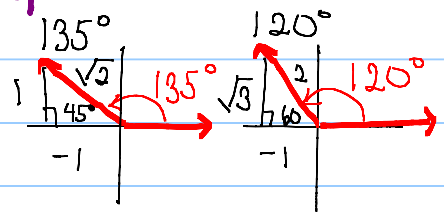
$$\tan 105^\circ = \tan \frac{210^\circ}{2} = \frac{\sin 210^\circ}{1+\cos 210^\circ} = \frac{\frac{-1}{2}}{1-\frac{\sqrt{3}}{2}} = \frac{\frac{-1}{2}}{\frac{2-\sqrt{3}}{2}} = \frac{-1}{2} \div \frac{2-\sqrt{3}}{2}$$

$$= \frac{-1}{(2-\sqrt{3})} \cdot \frac{(2+\sqrt{3})}{(2+\sqrt{3})} = \frac{-2-\sqrt{3}}{4-3} = -2-\sqrt{3}$$

$$\sin 105^\circ = \frac{\sqrt{2+\sqrt{3}}}{2} \quad \cos 105^\circ = \frac{-\sqrt{2-\sqrt{3}}}{2}, \quad \tan 105^\circ = -2-\sqrt{3}$$

8. Use the sum formulas to find the exact value of $\sin 255^0$, $\cos 255^0$, and $\tan 255^0$.

Step1: $255° = 135° + 120°$



step2: Use sum formulas

$\sin 255° = \sin(135° + 120°)$

$= \sin 135° \cos 120° + \cos 135° \sin 120°$

$= \left(\frac{\sqrt{2}}{2}\right)\left(-\frac{1}{2}\right) + \left(-\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right)$

$= \dfrac{-\sqrt{2} - \sqrt{6}}{4}$

step2: continued

$\tan 255° = \tan(135° + 120°)$

$= \dfrac{\tan 135° + \tan 120°}{1 - \tan 135° \tan 120°}$

$= \dfrac{-1 - \sqrt{3}}{1 - (-1)(-\sqrt{3})} = \dfrac{(-1-\sqrt{3})(1+\sqrt{3})}{(1-\sqrt{3})(1+\sqrt{3})}$

$= \dfrac{-1 - 2\sqrt{3} - 3}{-2} = \dfrac{-4 - 2\sqrt{3}}{-2} = 2 + \sqrt{3}$

$\cos 255° = \cos(135° + 120°)$

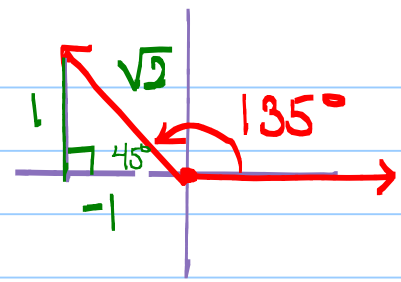
$= \cos 135° \cos 120° - \sin 135° \sin 120°$

$= \left(-\frac{\sqrt{2}}{2}\right)\left(-\frac{1}{2}\right) - \left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right)$

$= \dfrac{\sqrt{2} - \sqrt{6}}{4}$

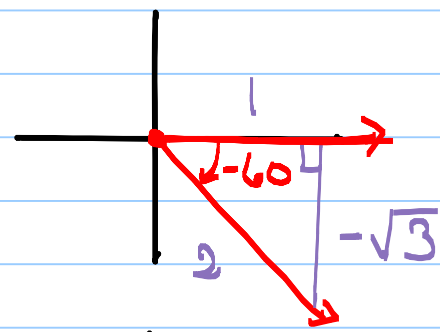
$\sin 255° = \dfrac{-\sqrt{2} - \sqrt{6}}{4}$, $\cos 255° = \dfrac{\sqrt{2} - \sqrt{6}}{4}$, $\tan 255° = 2 + \sqrt{3}$

9. $\tan 135^\circ$



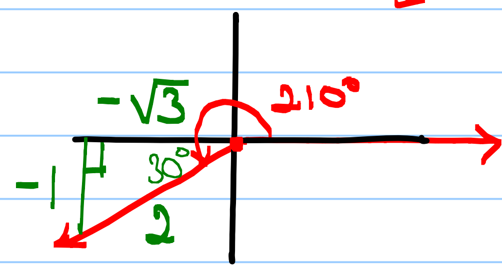
$\tan 135^\circ = \frac{1}{-1} = -1$

10. $\sin(-60^\circ)$



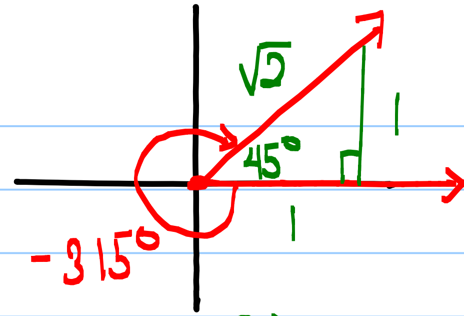
$\sin(-60) = \frac{-\sqrt{3}}{2}$

11. $\cos 210^\circ$



$\cos 210^\circ = \frac{-\sqrt{3}}{2}$

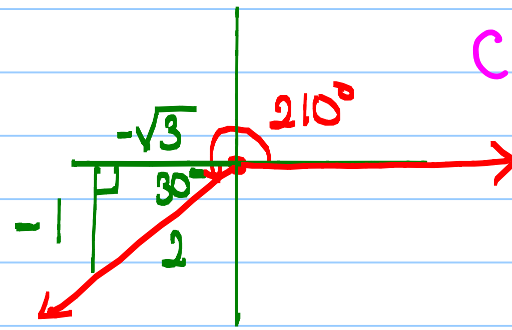
12. $\sec(-315^\circ)$



$\sec(-315^\circ) = \frac{\sqrt{2}}{1} = \sqrt{2}$

13. $\cot\left(\frac{7\pi}{6}\right) = \cot(210^\circ)$

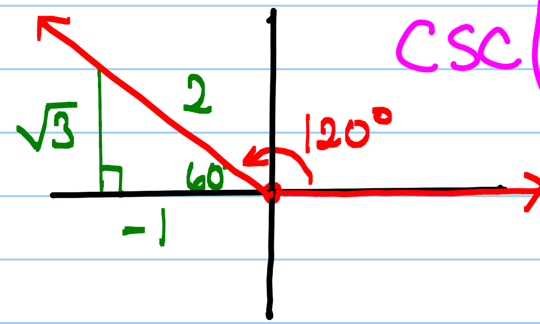
$\frac{7\cancel{\pi}}{\cancel{6}} \cdot \frac{\cancel{180}^{30}}{\cancel{\pi}} = 210^\circ$



$\cot\left(\frac{7\pi}{6}\right) = \sqrt{3}$

14. $\csc\left(\frac{2\pi}{3}\right) = \csc 120^\circ$

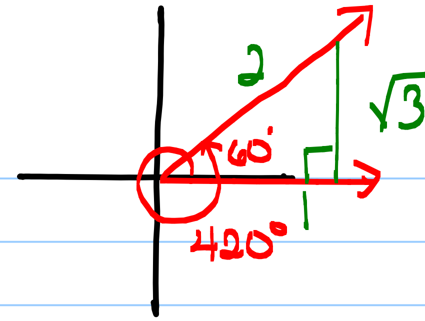
$\frac{2\cancel{\pi}}{\cancel{3}} \cdot \frac{\cancel{180}^{60}}{\cancel{\pi}} = 120^\circ$



$\csc\left(\frac{2\pi}{3}\right) = \frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$

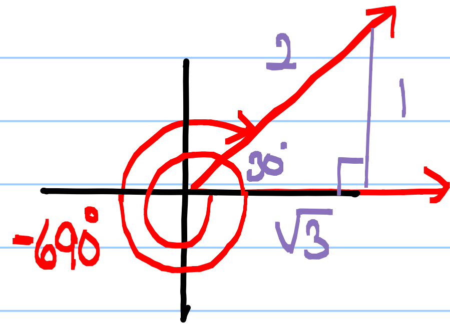
15. $\tan \frac{7\pi}{3} = \tan 420^\circ$

$\frac{7\cancel{\pi}}{\cancel{3}} \cdot \frac{\cancel{180}^{60^\circ}}{\cancel{\pi}} = 420^\circ$



$\tan \frac{7\pi}{3} = \sqrt{3}$

16. $\cos(-690^\circ)$



$\cos(-690^\circ) = \frac{\sqrt{3}}{2}$

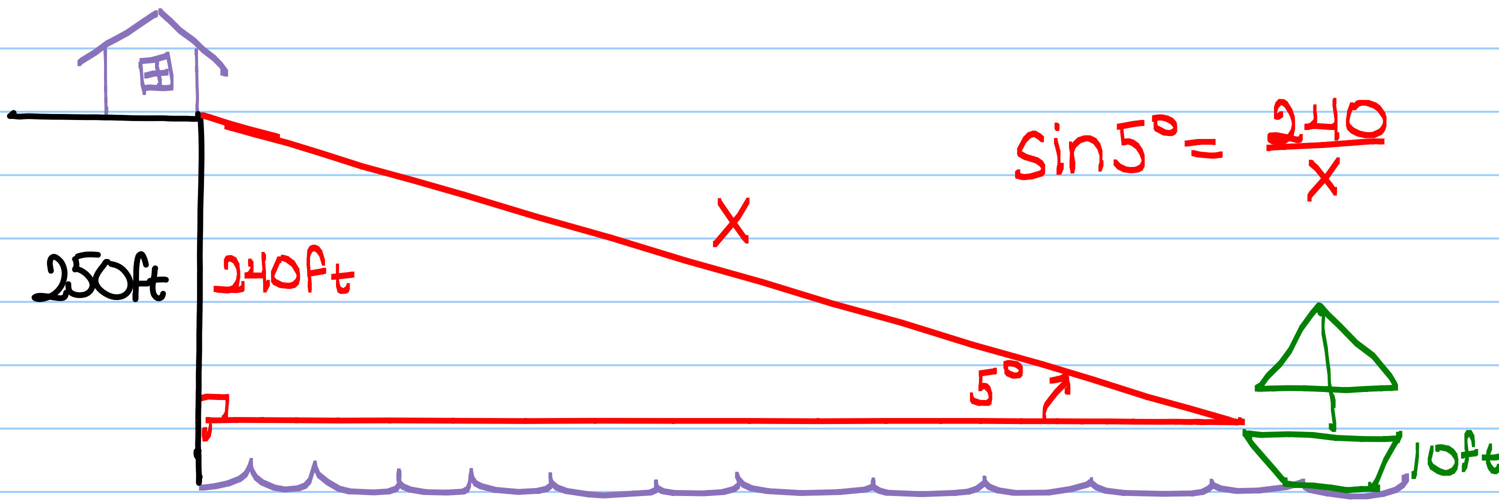
17. $\sin 18^\circ = 0.3090$

18. $\sec 4 = \frac{1}{\cos 4} = -1.530$

19. $\cot(-6.7) = \dfrac{1}{\tan(-6.7)} = -2.2586$

20. $\csc 242^\circ = \dfrac{1}{\sin 242^\circ} = -1.1326$

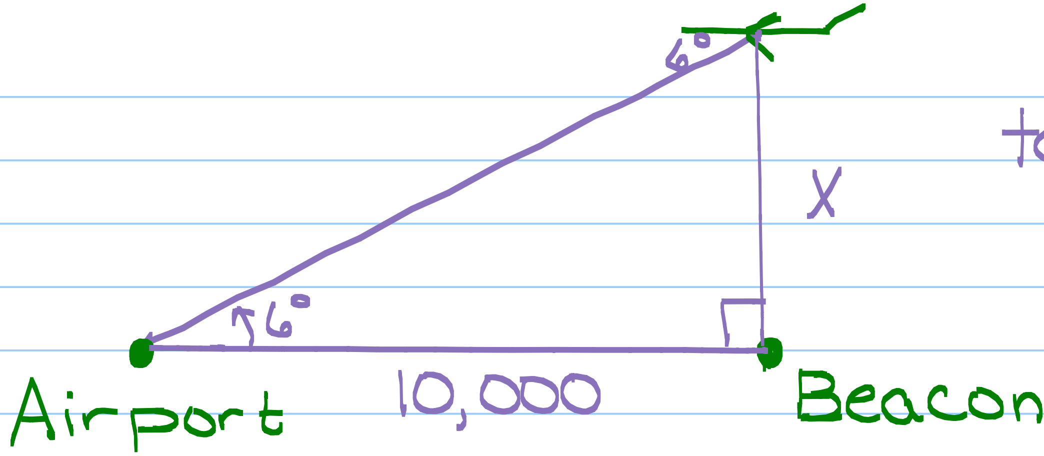
21.



$\sin 5^\circ = \dfrac{240}{x}$

The distance is 2753.7 ft

22.



$$\tan 6^\circ = \frac{x}{10{,}000}$$

The plane is 1051 ft above the beacon.

23. $y = 5\sin\left(\frac{\pi x}{4}\right) + 2$ $a = 5$ $b = \frac{\pi}{4}$ $c = 0$ $d = 2$

amp $= 5$

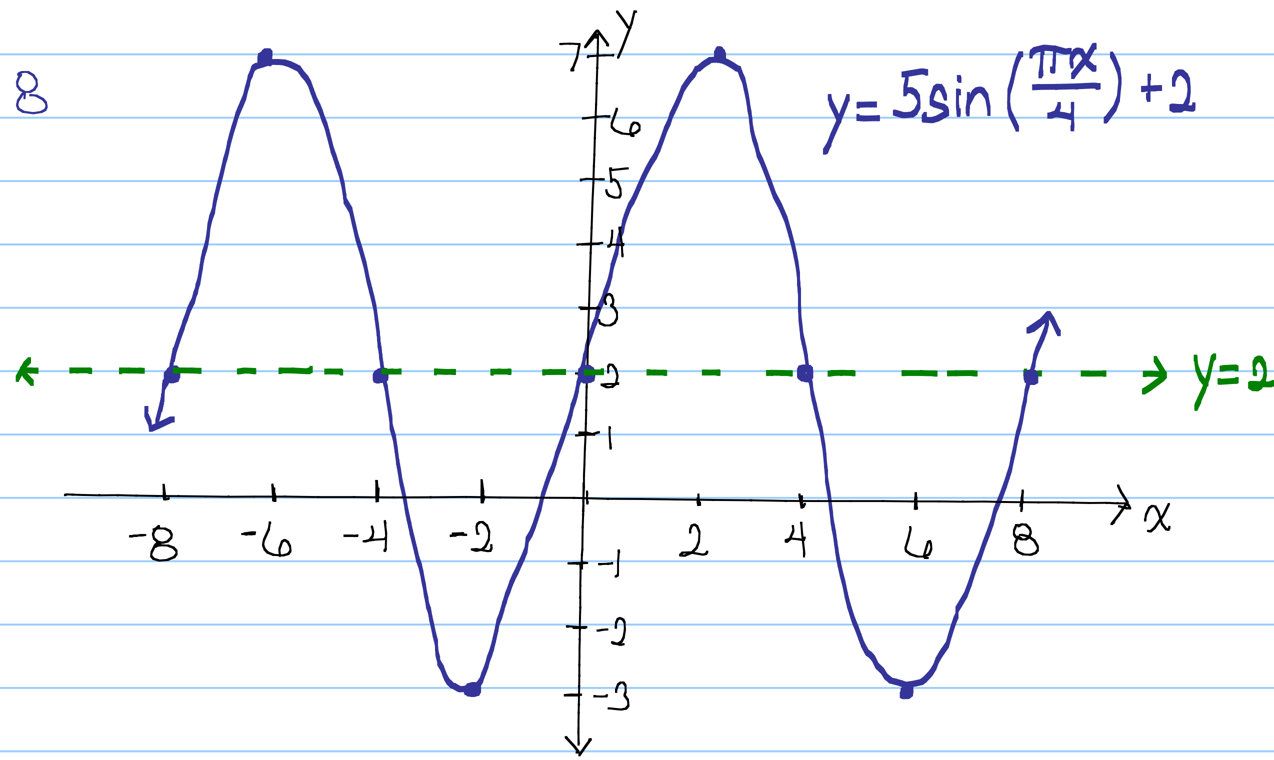
Per $= 2\pi \div \pi/4 = 8$

Interval: 2

Length

P.S.: None

V.S. $y = 2$



24. $y = -2\sin\left(2x - \frac{\pi}{2}\right) - 2$

$a = -2$ $\quad b = 2$ $\quad c = \frac{\pi}{2}$ $\quad d = -2$

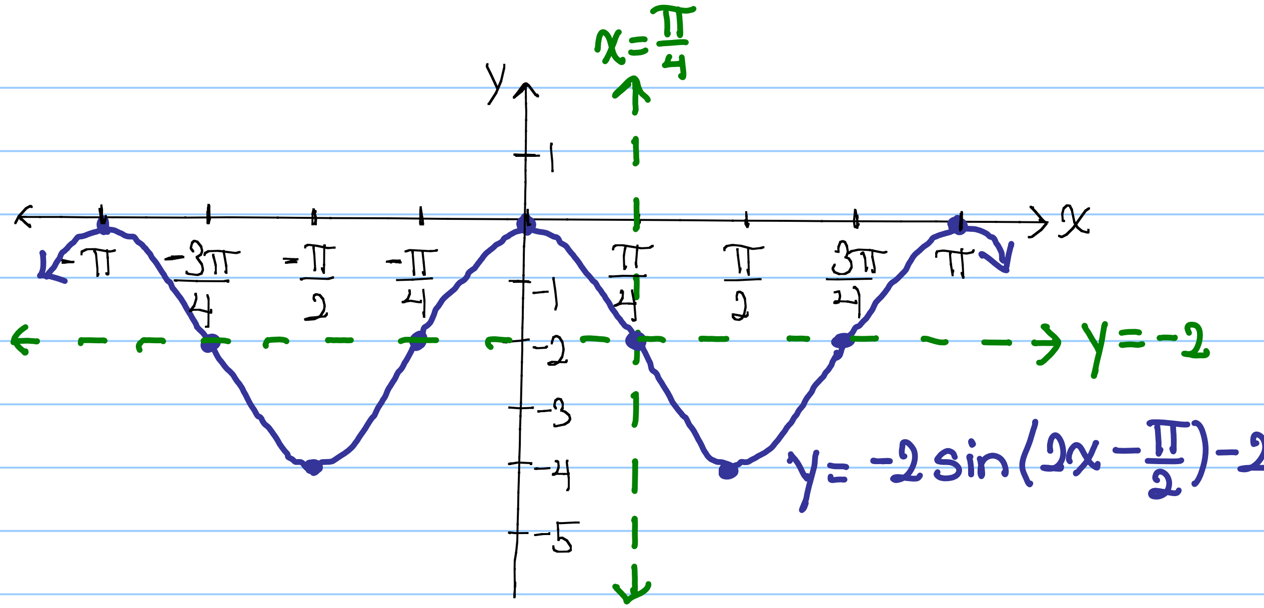
amp $= 2\ (-)$

period $= \pi$

interval length $= \frac{\pi}{4}$

P.S. $x = \pi/4$

V.S. $y = -2$



25. $y = -4\cos x + \pi$

$a = -4$ $b = 1$ $c = 0$ $d = \pi$

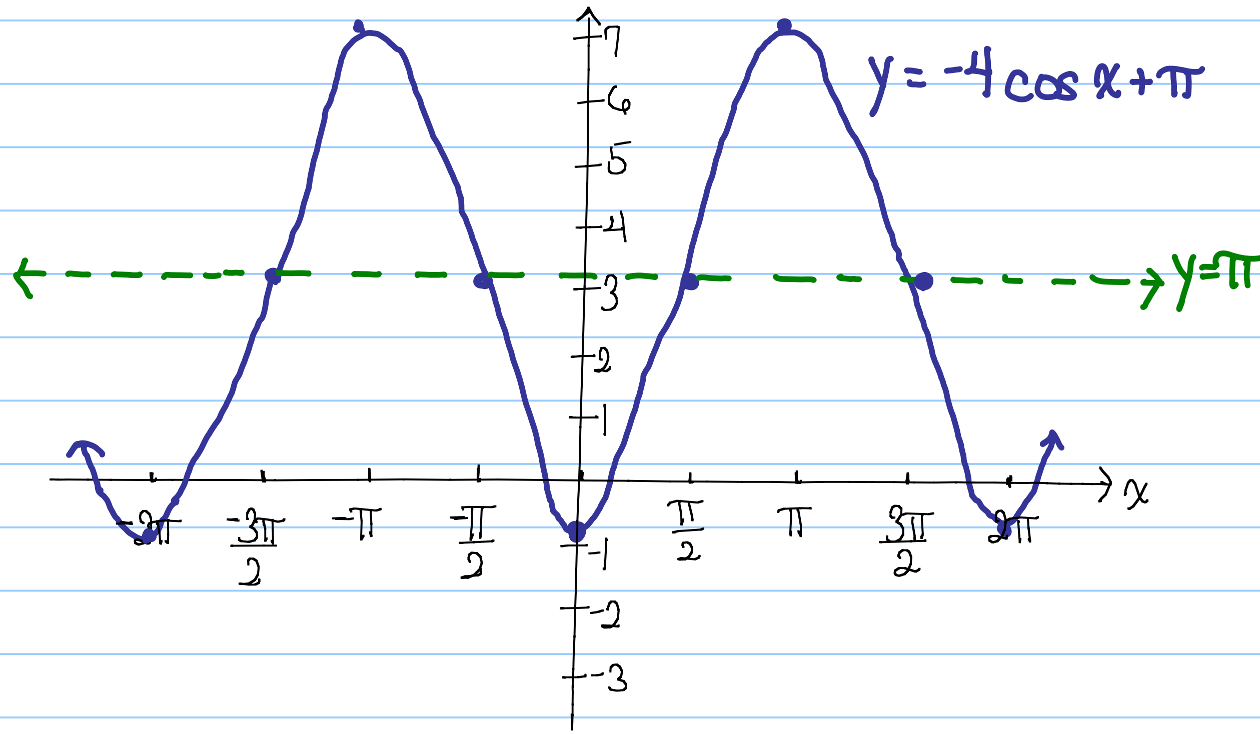
amp $= 4$ $(-)$

period $= 2\pi$

Interval length $= \frac{\pi}{2}$

P.S. None

V.S. $y = \pi$



26. $y = \frac{1}{2}\cos(4\pi x - 2\pi) + 3$

$a = \frac{1}{2}$ $b = 4\pi$ $c = 3$ $d = 3$

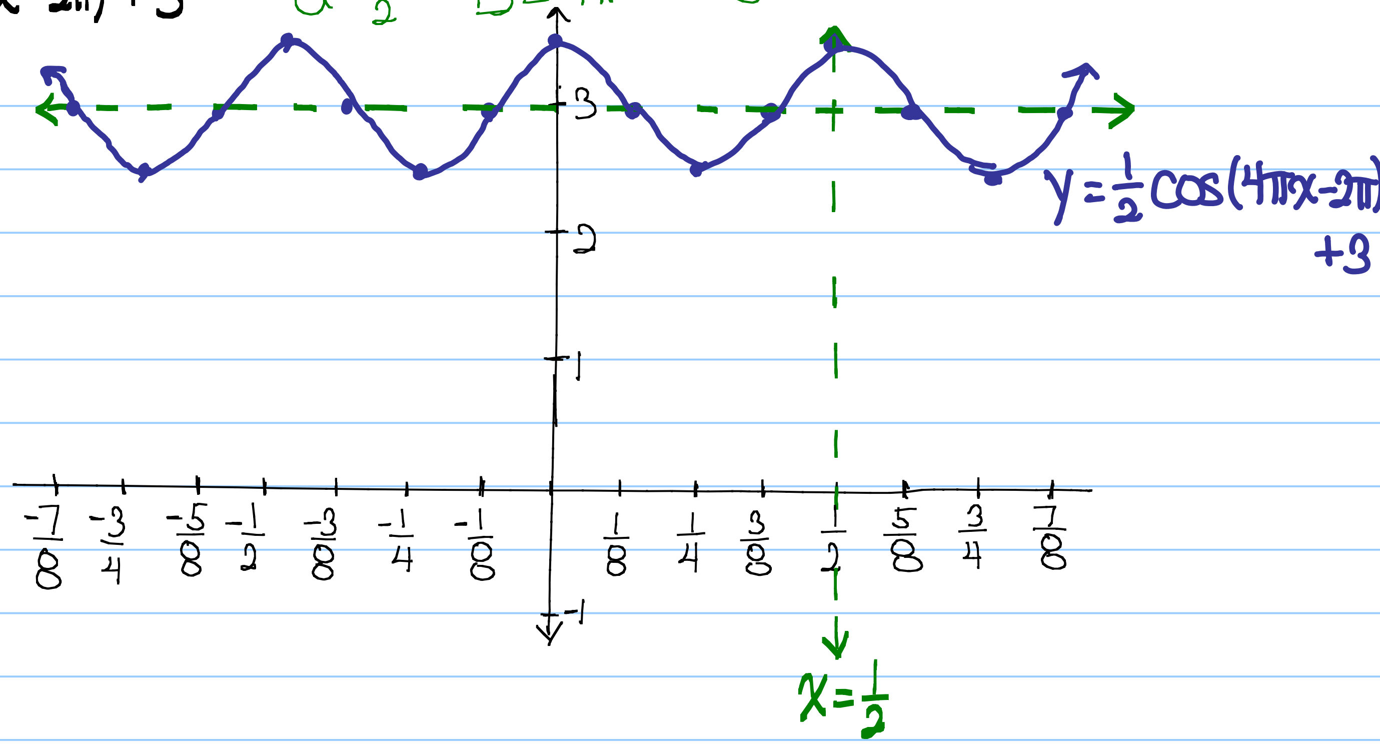
amp $= \frac{1}{2}$

period $= \frac{1}{2}$

interval length $= \frac{1}{8}$

P.S. $= \frac{1}{2}$

V.S. $= 3$



27. $y = 2\tan \pi x$

$a = 2 \quad b = \pi \quad c = 0 \quad d = 0$

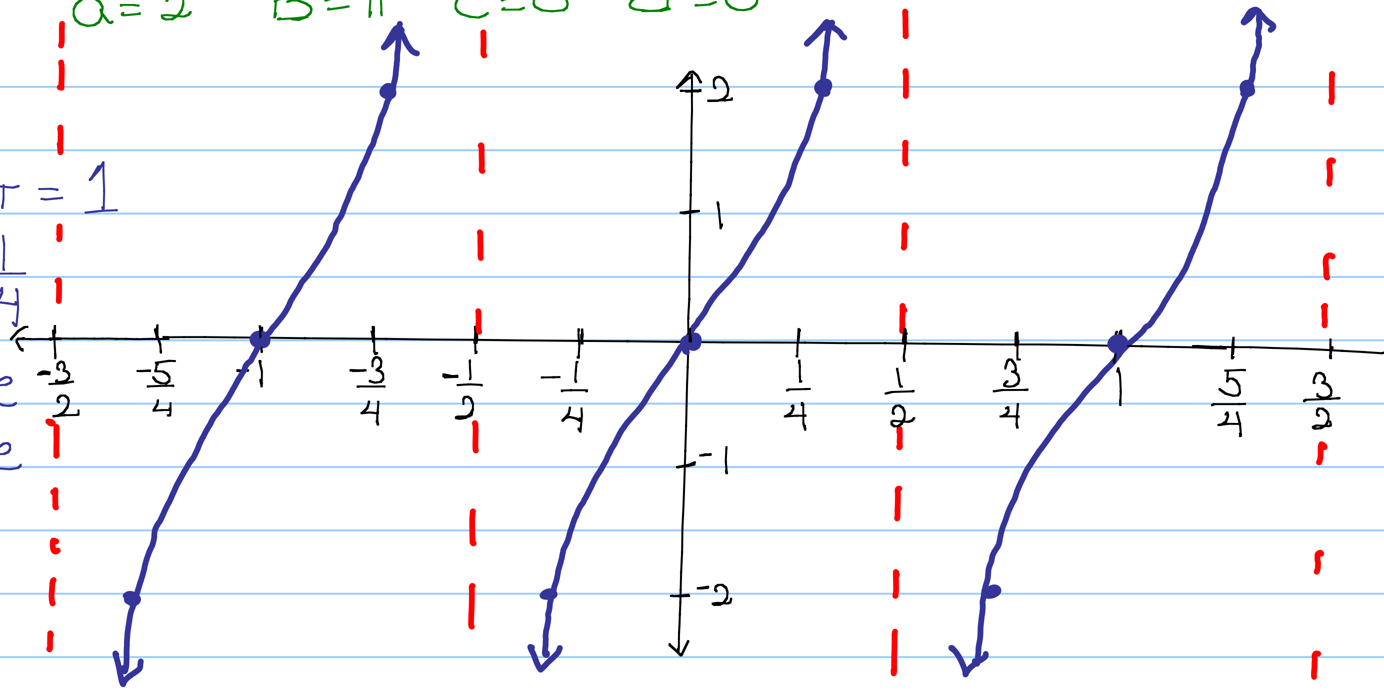
amp $= 2$

period $= \pi/\pi = 1$

interval length $= \frac{1}{4}$

P.S. : None

V.S. : None



28. $y = \tan\left(\frac{\pi x}{2} + \pi\right)$ $a = 1$ $b = \frac{\pi}{2}$ $c = -\pi$ $d = 0$

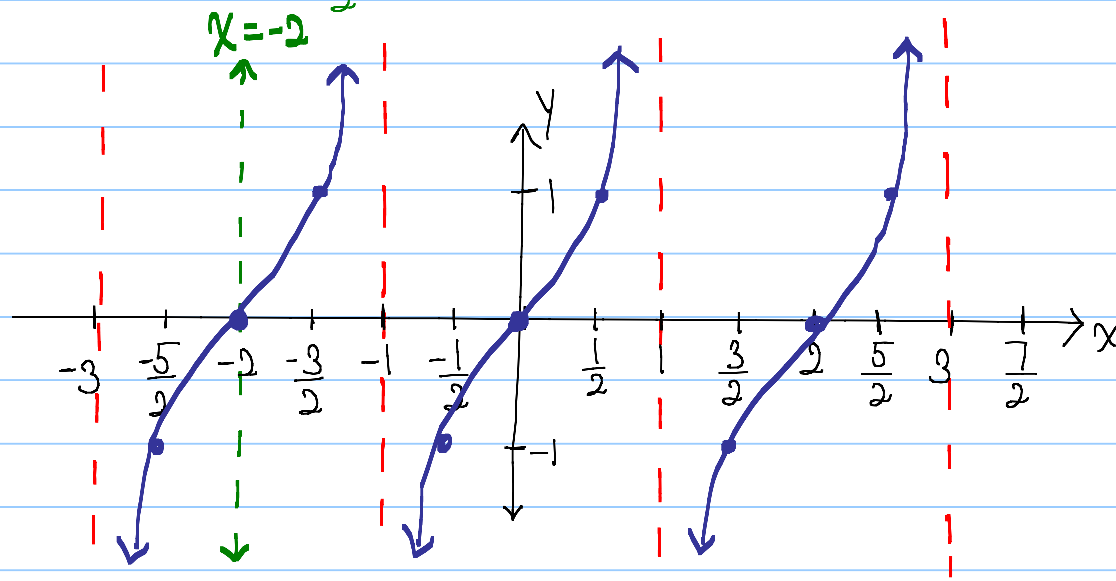
amp = 1

period = 2

interval length = $\frac{1}{2}$

P.S. $x = -2$

V.S. None



cos

29. $y = 3\sec(\frac{\pi x}{4} - \frac{\pi}{4}) + 1$

amp = 3

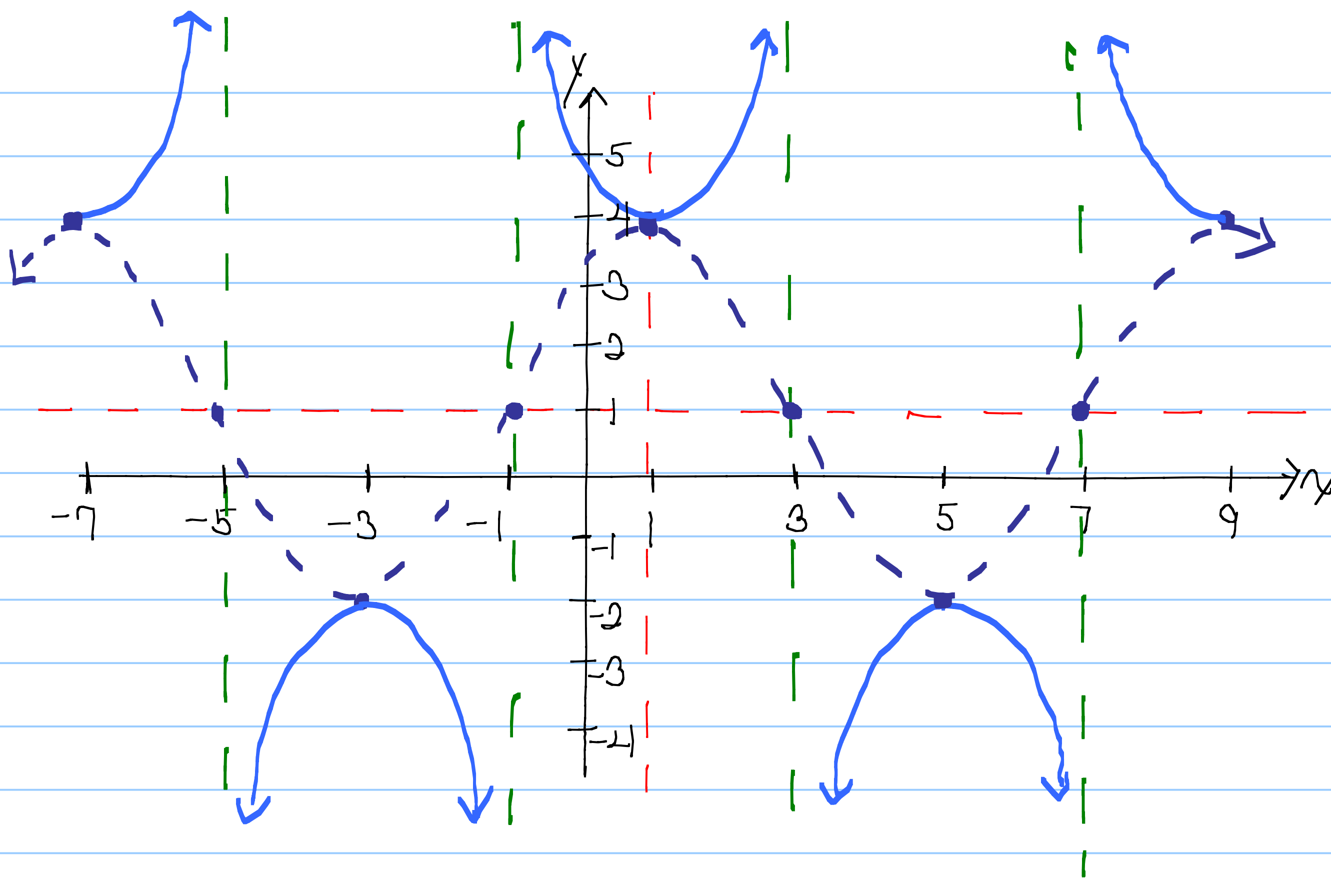
period = 8

interval = 2

length

P.S. $x = 1$

V.S. $y = 1$



sin

30. $y = \frac{1}{2}\csc(2x) - 3$

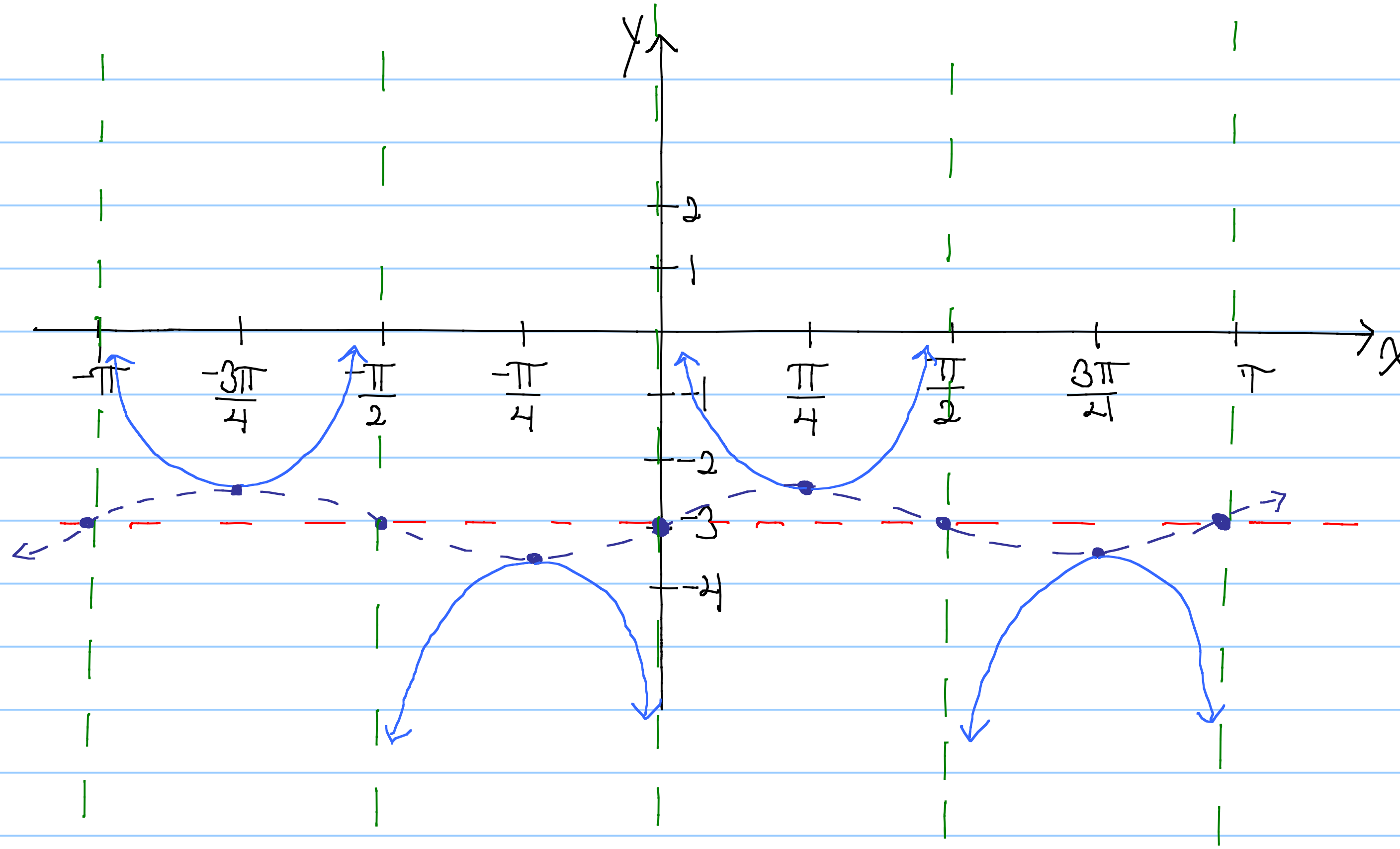
amp $= \frac{1}{2}$

period $= \pi$

interval length $= \frac{\pi}{4}$

P.S. $x = 0$

V.S. $y = -3$



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[redacted]. $y = \cot(\pi x)$

amp = 1

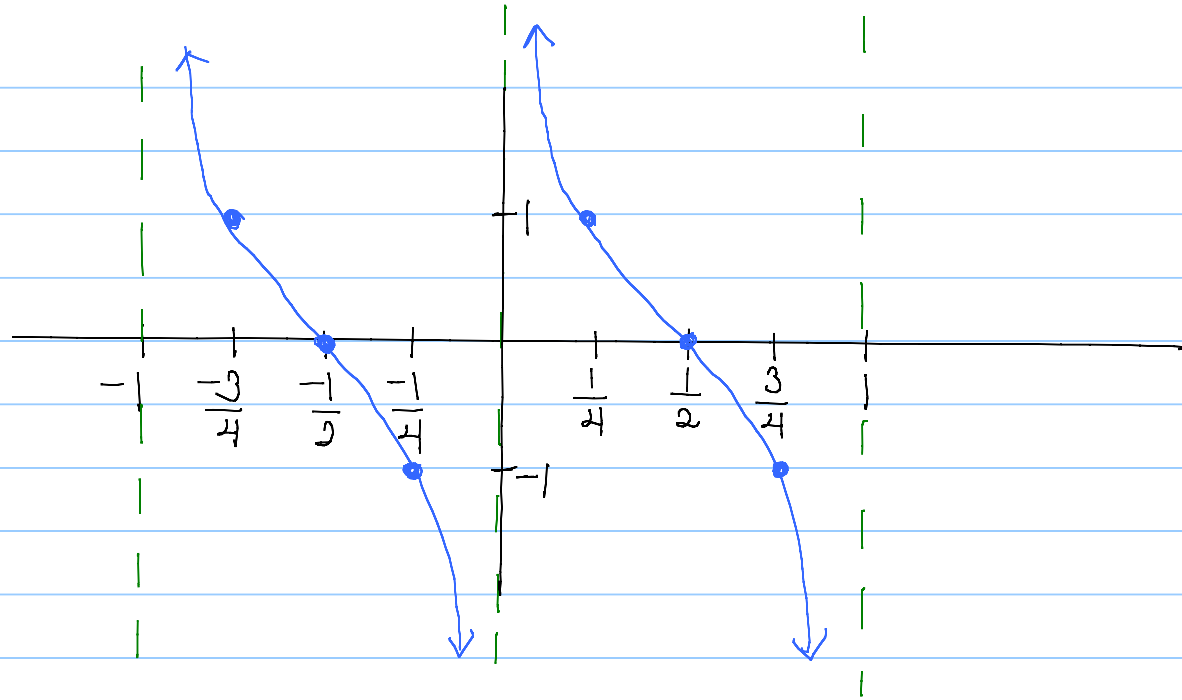
period = 1

interval = $\frac{1}{4}$

length

P.S. $x = 0$

V.S. $y = 0$



32. Use the substitution x = 3tanθ to write the algebraic expression $\sqrt{x^2+9}$ as a trigonometric function of θ where $0<\theta<\frac{\pi}{2}$.

$$\sqrt{(3\tan\theta)^2+9}$$

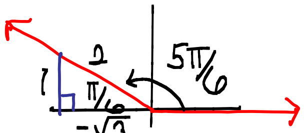
$$\sqrt{9\tan^2\theta+9}$$

$$\sqrt{9(\tan^2\theta+1)}$$

Using identity: $\frac{\sin^2\theta}{\cos^2\theta}+\frac{\cos^2\theta}{\cos^2\theta}=\frac{1}{\cos^2\theta} \rightarrow \tan^2\theta+1=\sec^2\theta$

$$\sqrt{9\sec^2\theta}$$

$$\boxed{3\sec\theta}$$



33. Use the sum or difference formula to evaluate: $\tan\frac{13\pi}{12}$

$$\frac{\pi}{4} + ? = \frac{13\pi}{12} \rightarrow ? = \frac{13\pi}{12} - \frac{\pi}{4} = \frac{10\pi}{12} = \frac{5\pi}{6}$$

$$\tan\left(\frac{\pi}{4} + \frac{5\pi}{6}\right) = \frac{\tan\frac{\pi}{4} + \tan\frac{5\pi}{6}}{1 - \tan\frac{\pi}{4}\tan\frac{5\pi}{6}} = \frac{1 - \frac{\sqrt{3}}{3}}{1-(1)\left(-\frac{\sqrt{3}}{3}\right)} = \frac{\frac{3-\sqrt{3}}{3}}{\frac{3+\sqrt{3}}{3}} = \frac{3-\sqrt{3}}{3+\sqrt{3}} \cdot \frac{(3-\sqrt{3})}{(3-\sqrt{3})}$$

$$= \frac{9 - 2\sqrt{3} + 3}{9-3} = \frac{12 - 2\sqrt{3}}{6} = \boxed{\frac{6-\sqrt{3}}{3}}$$

34. Expand and simplify: $\cos(2x - y)\cos y - \sin(2x - y)\sin y$

$(\cos 2x \cos y + \sin 2x \sin y)\cos y - [(\sin 2x \cos y - \cos 2x \sin y)\sin y]$

$\cos 2x \cos^2 y + \cancel{\sin 2x \sin y \cos y} - \cancel{\sin 2x \sin y \cos y} + \cos 2x \sin^2 y$

$\cos 2x \cos^2 y + \cos 2x \sin^2 y$

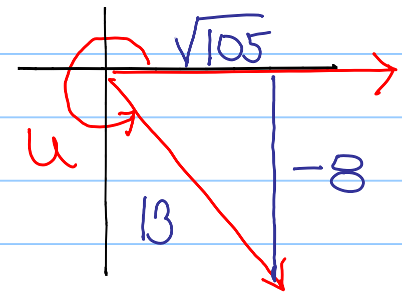
$\cos 2x(\cos^2 y + \sin^2 y)$

$\cos 2x (1)$

$\boxed{\cos 2x}$

35. Given $\sin u = \frac{-8}{13}$, find $\cos\frac{u}{2}$. Assume angle u is in the fourth quadrant. if $u = 4$ then $\frac{u}{2} = \frac{4}{2}$

$$\cos\frac{u}{2} = \pm\sqrt{\frac{1+\cos u}{2}}$$



so $\frac{u}{2}$ = Quad II

$a^2 + 64 = 169$

$a^2 = 105$

so $\cos\frac{u}{2}$ = Neg because cos is Neg. in quad II.

$$\cos\frac{u}{2} = +\sqrt{\frac{1+\frac{\sqrt{105}}{8}}{2}} = \sqrt{\frac{8+\sqrt{105}}{16}} = \boxed{\frac{\sqrt{8+\sqrt{105}}}{4}}$$

36-37. Use the sum-to-product formulas to write the sum or difference as a product.

36. $\cos 120^\circ + \cos 30^\circ = 2\cos\left(\frac{120^\circ + 30^\circ}{2}\right)\cos\left(\frac{120^\circ - 30^\circ}{2}\right)$

$= \boxed{2\cos 75^\circ \cos 45^\circ}$

37. $\sin(x+\frac{\pi}{2}) + \sin(x-\frac{\pi}{2}) = \boxed{2\sin\left(x+\frac{\pi}{2}\right)\cos\left(x-\frac{\pi}{2}\right)}$

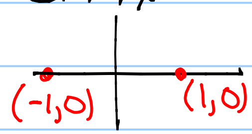
38. Find all solution in the equation in the interval $[0, 2\pi)$: $\cos 2x - \cos 6x = 0$ Using sum-to-product

$-2\sin\left(\frac{2x+6x}{2}\right)\sin\left(\frac{2x-6x}{2}\right) = 0$

$-2\sin 4x \sin(-2x) = 0$

$-2\sin 4x = 0 \qquad \sin(-2x) = 0$

$\sin 4x = 0$



$4x = 0 + 2\pi n \rightarrow x = \pi n/2$

$4x = \pi + 2\pi n \rightarrow x = \pi/4 + \pi n/2$

$\boxed{x = 0, \pi/2, \pi, 3\pi/2, \pi/4, 3\pi/4, 5\pi/4, 7\pi/4}$

39. Verify the identity: $\cos^4 x - \sin^4 x = \cos 2x$

$$(\cos^2 x - \sin^2 x)(\cos^2 x + \sin^2 x) = \cos 2x$$

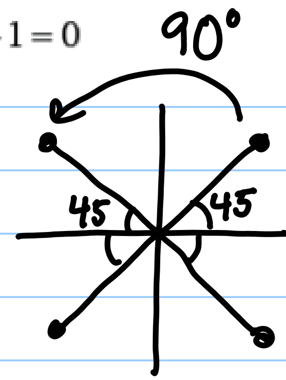
Double ∠ identity

Pythagorean identity

$$(\cos 2x)(1) = \cos 2x$$

$$\cos 2x = \cos 2x$$

40.) Find all solutions in the interval $[0,2\pi)$: $2\cos^2(2\theta)-1=0$



$$\cos^2(2\theta)=1/2$$

$$\cos(2\theta)=\frac{\pm 1}{\sqrt{2}}=\pm\frac{\sqrt{2}}{2}$$

$$2\theta=\frac{\pi}{4}+\frac{\pi n}{2}$$

$$\theta=\frac{\pi}{8}+\frac{\pi n}{4}=\frac{\pi+2\pi n}{8}$$

$$\boxed{X=\pi/8,\ 3\pi/8,\ 5\pi/8,\ 7\pi/8,\ 9\pi/8,\ 11\pi/8,\ 13\pi/8,\ 15\pi/8}$$

$n=0,\ 1,\ 2,\ 3,\ 4,\ 5,\ 6,\ 7$

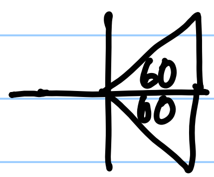
41.) Find all solutions in the interval $[0,2\pi)$: $2\sin^2\frac{x}{4} - 3\cos\frac{x}{4} = 0 \rightarrow 2\sin^2\theta - 3\cos\theta = 0$

$$2(1-\cos^2\theta) - 3\cos\theta = 0$$

$$2 - 2\cos^2\theta - 3\cos\theta = 0$$

$$2\cos^2\theta + 3\cos\theta - 2 = 0$$

$$(2\cos\theta - 1)(\cos\theta + 2)$$



$$\cos\theta = \tfrac{1}{2}$$

$$\frac{x}{4} = \frac{\pi}{3} + 2\pi n$$

$$\frac{x}{4} = \frac{5\pi}{3} + 2\pi n$$

$$\cos\theta = -2$$

$$\varnothing$$

$$x = \frac{4\pi}{3} + 8\pi n$$

$$x = \frac{20\pi}{3} + 8\pi n$$

$$\boxed{x = \frac{4\pi}{3}}$$

42.) Verify the identity: $\tan^2 x \cos^2 x + \cot^2 x \sin^2 x = 1$

$$\frac{\sin^2 x}{\cos^2 x} \cdot \cos^2 x + \frac{\cos^2 x}{\sin^2 x} \cdot \sin^2 x = \sin^2 x + \cos^2 x = 1$$

Q.E.D.