Integrated Math 1 Honors
Module 11H
Exponents
Ready, Set, Go! Homework
Solutions

Adapted from

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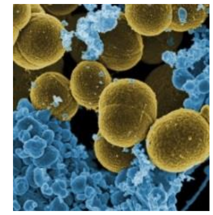
Name: **Solving Quadratic and Other Equations** | **11.1H**



Ready, Set, Go!

Ready

Topic: Comparing additive and multiplicative patterns.

The sequences below exemplify either an additive (arithmetic) or a multiplicative (geometric) pattern. Identify the type of sequence, fill in the missing values on the table and write an equation.

1.

Term	1st	2nd	3rd	4th	5th	6th	7th	8th
Value	-3	9	-27	81	**-243**	**729**	**-2187**	**6561**

Type of Sequence:
Geometric

Equation:
$\boldsymbol{f(n) = (-3)^n}$

2.

Term	1st	2nd	3rd	4th	5th	6th	7th	8th
Value	160	80	40	20	**10**	**5**	$\frac{5}{2}$	$\frac{5}{4}$

Type of Sequence:
Geometric

Equation:
$\boldsymbol{f(n) = 320\left(\frac{1}{2}\right)^n}$ **or** $\boldsymbol{f(n) = 160\left(\frac{1}{2}\right)^{x-1}}$

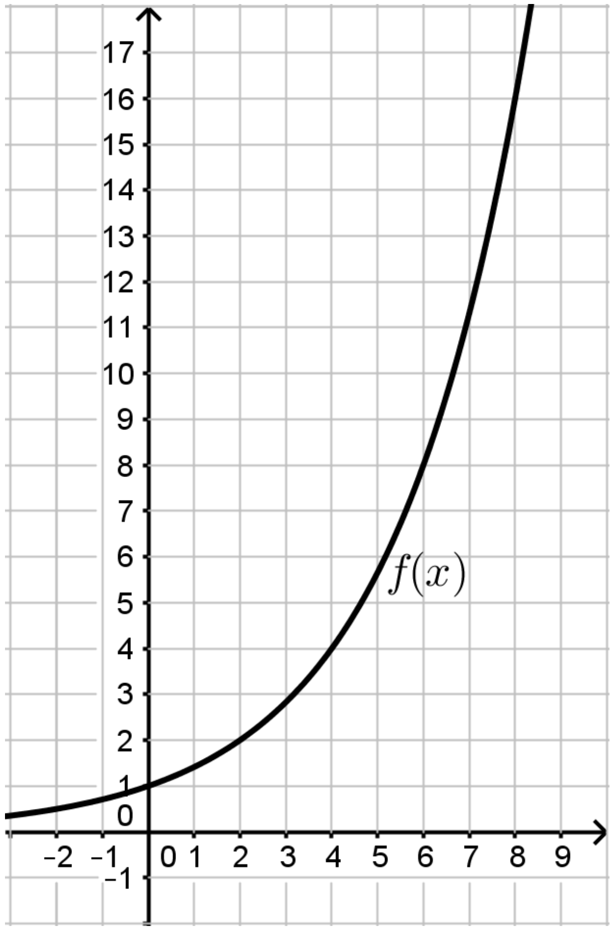
3.

Term	1st	2nd	3rd	4th	5th	6th	7th	8th
Value	-9	-2	5	12	**19**	**26**	**33**	**40**

Type of Sequence:
Arithmetic

Equation:
$\boldsymbol{f(n) = 7n - 16}$ **or** $\boldsymbol{f(n) = 7(n-1) - 9}$

Use the graph of the function to find the desired values of the function. Also create an explicit equation for the function.



4. Find the value of $f(2)$
   2

5. Find where $f(x) = 4$
   $\boldsymbol{x = 4}$

6. Find the value of $f(6)$
   8

7. Find where $f(x) = 16$
   $\boldsymbol{x = 8}$

8. What do you notice about the way that inputs and outputs for this function relate? (Create an in-out table if you need to.)

   exponential

9. What is the explicit equation for this function?
   $\boldsymbol{f(x) = 2^{(x/2)}}$

Set

Topic: Evaluate the expressions with rational exponents.

Fill in the missing values of the table based on the growth that is described.

10. The growth in the table is **triple** at each whole year.

Years	0	$\frac{1}{2}$	1	$\frac{3}{2}$	2	$\frac{5}{2}$	3	$\frac{7}{2}$	4
Bacteria	2	$2\sqrt{3}$	6	$6\sqrt{3}$	18	$18\sqrt{3}$	54	$54\sqrt{3}$	162

11. The growth in the table is **triple** at each whole year.

Years	0	$\frac{1}{3}$	$\frac{2}{3}$	1	$\frac{4}{3}$	$\frac{5}{3}$	2	$\frac{7}{3}$	$\frac{8}{3}$
Bacteria	2	$2\sqrt[3]{3}$	$2\sqrt[3]{9}$	6	$6\sqrt[3]{3}$	$6\sqrt[3]{9}$	18	$18\sqrt[3]{3}$	$18\sqrt[3]{9}$

12. The values in the table grow by a **factor of four** at each whole year.

Years	0	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{3}{4}$	1	$\frac{5}{4}$	$\frac{3}{2}$	$\frac{7}{4}$	2
Bacteria	2	$2\sqrt[4]{4}$	$2\sqrt[4]{16}$	$2\sqrt[4]{64}$	8	$8\sqrt[4]{4}$	$8\sqrt[4]{16}$	$8\sqrt[4]{64}$	32
	OR	$2\sqrt{2}$	4	$4\sqrt{2}$	8	$8\sqrt{2}$	16	$16\sqrt{2}$	32

Go

Topic: Simplifying exponents

Simplify the following expressions using exponent rules and relationships. Write your answers with positive exponents only.

13. $(3x^2)^3$

$27x^6$

14. $\frac{x^4y^9}{xy^3}$

x^3y^6

15. x^{-5}

$\frac{1}{x^5}$

16. $(2x^0y^7)(3x^6y^4)^2$

$18x^{12}y^{15}$

17. $\frac{4x^3y^8}{24x^9y^{12}}$

$\frac{1}{6x^6y^4}$

18. $\frac{8x^{-3}y^5}{32x^{-7}y^{-2}}$

$\frac{x^4y^7}{4}$

Topic: Writing functions in vertex and factored forms

Write each quadratic function in vertex and factored forms.

19. $f(x) = x^2 - 10x + 16$

Vertex Form: $\boldsymbol{f(x) = (x-5)^2 - 9}$

Factored Form: $\boldsymbol{f(x) = (x-8)(x-2)}$

20. $f(x) = x^2 + 6x - 27$

Vertex Form: $\boldsymbol{f(x) = (x+3)^2 - 36}$

Factored Form: $\boldsymbol{f(x) = (x+9)(x-3)}$

21. $f(x) = x^2 + 16x + 60$

Vertex Form: $\boldsymbol{f(x) = (x+8)^2 - 4}$

Factored Form: $\boldsymbol{f(x) = (x+10)(x+6)}$

22. $f(x) = (x-4)^2 - 81$

Vertex Form: $\boldsymbol{f(x) = (x-4)^2 - 81}$

Factored Form: $\boldsymbol{f(x) = (x-13)(x+5)}$

Ready, Set, Go!

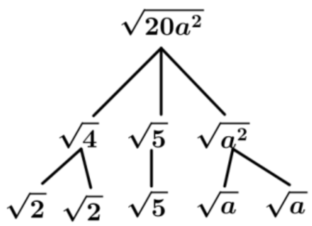
Ready

Topic: Simplifying radicals

A very common radical expression is a square root. One way to think of a square root is the number that will multiply by itself to create a desired value. For example: $\sqrt{2}$ *is the number that will multiply by itself to equal 2. And in like manner* $\sqrt{16}$ *is the number that will multiply by itself to equal 16; in this case the value is 4 because* $4 \times 4 = 16$*. When the square root of a square number is taken you get a nice whole number value. Otherwise an **irrational** number is produced.*

This same pattern holds true for other radicals such as cube roots and fourth roots and so forth. For example: $\sqrt[3]{8}$ *is the number that will multiply by itself three times to equal 8. In this case it is equal to the value of 2 because* $2^3 = 2 \cdot 2 \cdot 2 = 8$*.*

With this in mind, radicals can be simplified. See the examples below.

Example 1:	*Example 2*:
Simplify $\sqrt{20}$  $\sqrt{20a^2} = \sqrt{4 \cdot 5 \cdot a^2} = \sqrt{2 \cdot 2 \cdot 5 \cdot a \cdot a} = \sqrt{2^2} \cdot \sqrt{5} \cdot \sqrt{a^2}$ $= 2a\sqrt{5}$	Simplify $\sqrt[5]{96}$ $\sqrt[5]{96} = \sqrt[5]{2^5} \cdot \sqrt[5]{3} = 2\sqrt[5]{3}$

Simplify each of the radicals.

1. $\sqrt{40}$

   $\mathbf{2\sqrt{10}}$

2. $\sqrt{32}$

   $\mathbf{4\sqrt{2}}$

3. $\sqrt[3]{16}$

   $\mathbf{2\sqrt[3]{2}}$

4. $\sqrt{72a^4}$

   $\boldsymbol{6a^2\sqrt{2}}$

5. $\sqrt[3]{54x^4}$

   $\boldsymbol{3x\sqrt[3]{2x}}$

6. $\sqrt[4]{81y^{10}}$

   $\boldsymbol{3y^2\sqrt[4]{y^2}}$

7. $\sqrt[5]{160b}$

   $\boldsymbol{2\sqrt[5]{5b}}$

Set

Topic: Finding arithmetic and geometric means and making meaning of rational exponents.

You may have found arithmetic and geometric means in your prior work. Finding ***arithmetic and geometric means*** *requires finding values of a sequence between given values from non-consecutive terms. In each of the sequences below determine the means and show how you found them.*

Find the *arithmetic* means for the following. Show your work.

8.

x	1	2	3
y	5	**8**	11

9.

x	1	2	3	4	5
y	18	**11**	**4**	**−3**	−10

10.

x	1	2	3	4	5	6	7
y	12	**9**	**6**	**3**	**0**	**−3**	−6

Find the *geometric* means for the following. Show your work.

11.

x	1	2	3
y	3	**6**	12

12.

x	1	2	3	4
y	7	**35**	**175**	875

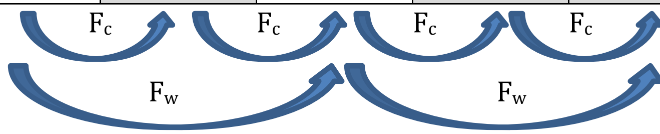
13.

x	1	2	3	4	5	6
y	4	**12**	**36**	**108**	**324**	972

Fill in the tables of values and find the factor (multiplier) used to move between whole number values, F_w, as well as the factor (multiplier), F_c, used to move between each column of the table.

14.

x	0	$\frac{1}{2}$	1	$\frac{3}{2}$	2
y	4	**8**	16	**32**	**64**

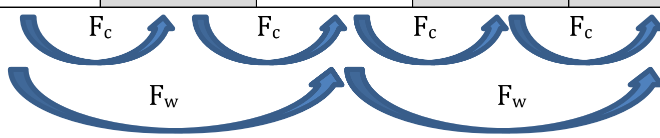


F_w = **4**

F_c = **2**

15.

x	0	$\frac{1}{2}$	1	$\frac{3}{2}$	2
y	4	$\mathbf{4\sqrt{2}}$	8	$\mathbf{8\sqrt{2}}$	**16**

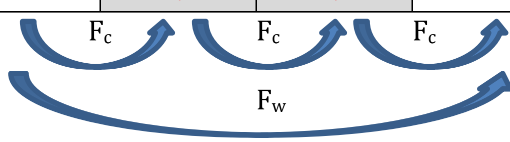


F_w = **2**

F_c = $\mathbf{\sqrt{2}}$

16.

x	0	$\frac{1}{3}$	$\frac{2}{3}$	1
y	5	$\mathbf{5\sqrt[3]{3}}$	$\mathbf{5\sqrt[3]{9}}$	15



F_w = **3**

F_c = $\mathbf{\sqrt[3]{3}}$

Go

Topic: Evaluating functions

Find the desired values for each function below.

17. $f(x) = 2x - 7$

a. Find $f(-3)$
$\mathbf{-13}$

b. Find x if $f(x) = 21$
$\boldsymbol{x = 14}$

c. Find $f\left(\frac{1}{2}\right)$
$\mathbf{-6}$

18. $g(x) = 3^x(2)$

a. Find $g(-4)$
$\mathbf{\frac{2}{81}}$

b. Find x if $g(x) = 162$
$\boldsymbol{x = 4}$

c. Find $g\left(\frac{1}{2}\right)$
$\mathbf{2\sqrt{3} \approx 3.4641}$

19. $I(t) = 210(1.08^t)$

a. Find $I(12)$
$\mathbf{\approx 528.82}$

b. Find t if $I(t) = 420$
$\boldsymbol{t \approx 9}$

c. Find $I\left(\frac{1}{2}\right)$
$\mathbf{\approx 218.24}$

20. $h(x) = x^2 + x - 6$

a. Find $h(-5)$
$\mathbf{14}$

b. Find x if $h(x) = 0$
$\boldsymbol{x = -3, 2}$

c. Find $h\left(\frac{1}{2}\right)$
$\mathbf{-\frac{21}{4}}$

Topic: Finding x-intercepts of quadratic functions

Find the x-intercepts of each quadratic function using the method stated.

21. Quadratic formula: $f(x) = 9x^2 + 4x - 16$

$\boldsymbol{x = \frac{-2 \pm 2\sqrt{37}}{9}}$

22. Completing the square: $f(x) = x^2 - 12x + 26$

$\boldsymbol{x = 6 \pm \sqrt{10}}$

23. Factoring: $f(x) = 3x^2 - 11x + 10$

$\boldsymbol{x = \frac{5}{3}, 2}$

24. Completing the square: $f(x) = 9x^2 - 18x + 8$

$\boldsymbol{x = \frac{4}{3}, \frac{2}{3}}$

Topic: Simplifying expressions with exponents

Simplify each expression as much as possible. Leave answers with positive exponents only.

25. $(3x^2y^8)(-4x^2y^6)^3$

$\boldsymbol{-192x^8y^{26}}$

26. $\frac{18x^{-7}y^{-2}}{24x^{-5}y^6}$

$\boldsymbol{\frac{3}{4x^2y^8}}$

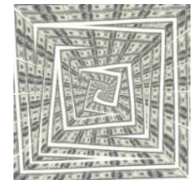
27. $(5x^2y^9)^0(-4x^5y^{-6})^2$

$\boldsymbol{\frac{16x^{10}}{y^{12}}}$

28. $\left(\frac{12x^4y^8}{36x^9y^{-2}}\right)\left(\frac{20x^3y^{-4}}{35x^{-10}y}\right)$

$\boldsymbol{\frac{4x^8y^5}{21}}$

Ready, Set, Go!



Ready

Topic: Exponent properties

Provide at least three other equivalent forms of the exponential expression. Use rules of exponents such as $3^5 \cdot 3^6 = 3^{11}$ and $\left(5^2\right)^3 = 5^6$ as well as division properties and others.

	1st Equivalent Form	2nd Equivalent Form	3rd Equivalent Form
1. 2^{10}	**Answers will vary**		
2. 3^7			
3. 13^{-8}			
4. $7^{\frac{1}{3}}$			
5. 5^1			

Set

Topic: Finding equivalent expressions

Determine whether all three expressions in each problem below are equivalent. Justify why or why they are not equivalent.

6. $5(3^{x-1})$ $\quad 15(3^{x-2})$ $\quad \frac{3}{5}(3^x)$

$\mathbf{5 \cdot 3^{x-1}}$ $\quad \mathbf{5^{-1} \cdot 3^{x+1}}$

Justification:
Only the 1st and 2nd expressions are equivalent. The exponents on the 3rd are different due to the fraction and the additive nature of the exponents for the terms with a base of 3.

7. $64(2^{-x})$ $\quad \frac{64}{2^x}$ $\quad 64\left(\frac{1}{2}\right)^x$

$\mathbf{\frac{64}{2^x}}$ $\quad \mathbf{\frac{64}{2^x}}$

Justification:
All 3 expressions are equivalent since the 1st term can be written with a positive power and the 3rd expression can have the $\left(\frac{1}{2}\right)^x$ be rewritten as $\frac{1}{2^x}$.

8.	$3(x-1)+4$ $\mathbf{3x+1}$	$3x-1$	$3(x-2)+7$ $\mathbf{3x+1}$
	Justification: **Only the 1st and 3rd expressions are equivalent since they both simplifying (by distributing & combining like terms) to $3x+1$.**		
9.	$50(2^{x+2})$ $\mathbf{5^2 \cdot 2^{x+3}}$	$25(2^{2x+1})$ $\mathbf{5^2 \cdot 2^{2x+1}}$	$50(4^x)$ $\mathbf{5^2 \cdot 2^{2x+1}}$
	Justification: **Only the 2nd and 3rd expressions are equivalent since the first term doesn't have the same exponent on the term with a base of 2.**		
10.	$30(1.05^x)$	$30\left(1.05^{\frac{1}{7}}\right)^{7x}$ $\mathbf{30(1.05^x)}$	$30\left(1.05^{\frac{x}{2}}\right)^2$ $\mathbf{30(1.05^x)}$
	Justification: **All expressions are equivalent**		
11.	$20(1.1^x)$	$20(1.1^{-1})^{-1x}$ $\mathbf{20(1.1^x)}$	$20\left(1.1^{\frac{1}{5}}\right)^{5x}$ $\mathbf{20(1.1^x)}$
	Justification: **All expressions are equivalent**		

Go

Topic: Using rules of exponents

Simplify each expression. Write your answers with positive exponents only.

12. $(5x^2y^{-8})^2$

$\mathbf{\frac{25x^4}{y^{16}}}$

13. $\frac{32x^{-3}y^{-4}}{24x^{-5}y^7}$

$\mathbf{\frac{4x^2}{3y^{11}}}$

14. $\left(\frac{4x^6y^2}{6x^9y^{-5}}\right)^2$

$\mathbf{\frac{4y^{14}}{9x^6}}$

15. $(-2x^6y^0)^3(5x^2y^5)$

$\mathbf{-40x^{20}y^5}$

16. $(7x^4y^9)^0(4x^3y^8)^2$

$\mathbf{16x^6y^{16}}$

17. $\left(\frac{12x^3y^7}{28x^5y^{-2}}\right)\left(\frac{3xy^{-6}}{27x^{-3}y^9}\right)$

$\mathbf{\frac{x^2}{21y^6}}$

Topic: Writing quadratic function in vertex form.

Write each quadratic function in vertex form by completing the square.

18. $f(x) = x^2 - 16x + 68$

    $\boldsymbol{f(x) = (x - 8)^2 + 4}$

19. $f(x) = -x^2 - 4x - 13$

    $\boldsymbol{f(x) = -(x + 2)^2 - 9}$

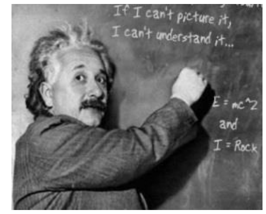
20. $f(x) = 2x^2 + 12x + 30$

    $\boldsymbol{f(x) = 2(x + 3)^2 + 12}$

21. $f(x) = -3x^2 + 24x - 40$

    $\boldsymbol{f(x) = -3(x - 4)^2 + 8}$

Name: **Solving Quadratic and Other Equations | 11.4H**



Ready, Set, Go!

Ready

Topic: Factoring quadratics

Write each of the quadratic expressions in factored form.

1. $2x^2 + 3x + 1$

   $(2x+1)(x+1)$

2. $x^2 - 5x - 6$

   $(x-6)(x+1)$

3. $x^2 + x - 12$

   $(x+4)(x-3)$

4. $2x^2 - 5x - 12$

   $(2x+3)(x-4)$

5. $6x^2 - 7x - 5$

   $(3x-5)(2x+1)$

6. $49x^2 - 4$

   $(7x-2)(7x+2)$

7. $x^4 + 6x^2 + 5$

   $(x^2+5)(x^2+1)$

8. $x^4 - 81$

   $(x+1)(x-1)(x^2+1)$

Topic: Simplifying radicals

Simplify each radical as much as possible.

9. $\sqrt{560}$

   $4\sqrt{35}$

10. $\sqrt{972}$

    $18\sqrt{3}$

11. $\sqrt{1050}$

    $5\sqrt{42}$

12. $\sqrt[3]{648}$

    $6\sqrt[3]{3}$

13. $\sqrt[3]{640}$

    $4\sqrt[3]{10}$

14. $\sqrt[3]{3584}$

    $8\sqrt[3]{7}$

Set

Topic: Radical notation and rational exponents

Each of the expressions below can be written using either radical notation, $\sqrt[n]{a^m}$, or rational exponents $a^{m/n}$. Rewrite each of the given expressions in the form that is missing.

	Radical Form	Exponential Form
15.	$\sqrt[3]{5^2}$	$5^{2/3}$
16.	$\sqrt[4]{16^3}$	$16^{3/4}$
17.	$\sqrt[3]{5^7 \cdot 3^5}$	$5^{7/3} \cdot 3^{5/3}$
18.	$\sqrt[3]{9^2 \cdot 9^4}$	$9^{2/3} \cdot 9^{4/3}$
19.	$\sqrt[5]{x^{13}y^{21}}$	$x^{13/5}y^{21/5}$
20.	$\sqrt[3]{27a^5b^2}$	$27^{1/3}a^{5/3}b^{2/3}$
21.	$\sqrt[5]{\dfrac{32x^{13}}{243y^{15}}}$	$\dfrac{32^{1/5}x^{13/5}}{243^{1/5}y^{15/5}}$
22.	$\sqrt{9^3t} \cdot \sqrt[3]{s^6}$	$9^{3/2} \cdot s^{6/3} \cdot t^{1/2}$

Topic: Solving equations with exponents

Solve the equations below, use radicals or rational exponents as needed.

23. $(x+5)^4 = 81$
 $x = -2, -8$

24. $2(x-7)^5 + 3 = 67$
 $x = 9$

25. $(x-7)^3 = 8$
 $x = 9$

26. $2(x+4)^3 = 162$
 $x = -4 + 3\sqrt[3]{3}$

27. $5(x+2)^3 = 540$

$\boldsymbol{x = -2 + 3\sqrt[3]{4}}$

28. $4(x-8)^5 = 384$

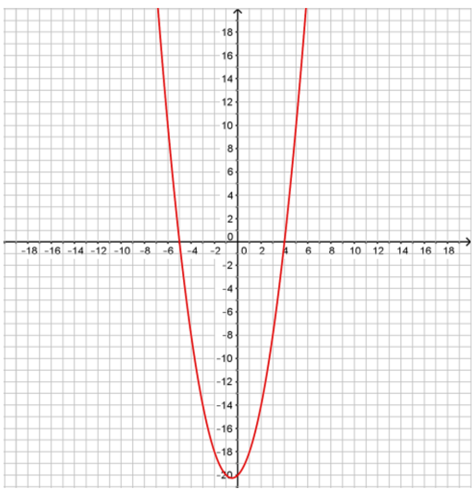
$\boldsymbol{x = 8 + 2\sqrt[5]{3}}$

Go

Topic: x-intercepts and y-intercepts for linear, exponential and quadratic functions

Given the function, find the x-intercept(s) and y-intercept if they exist and then use them to graph a sketch of the function.

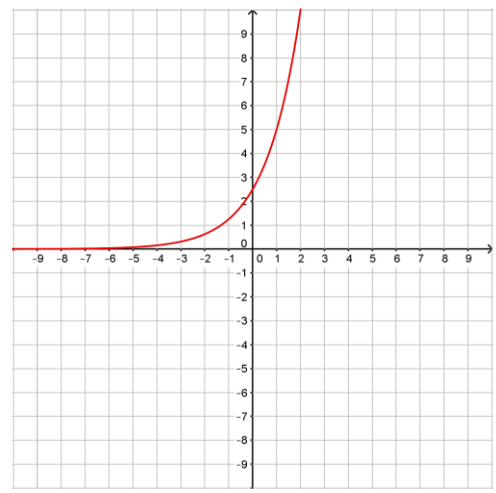
29. $f(x) = (x+5)(x-4)$



a. x-intercept(s): $\boldsymbol{-5, 4}$

b. y-intercept: $\boldsymbol{-20}$

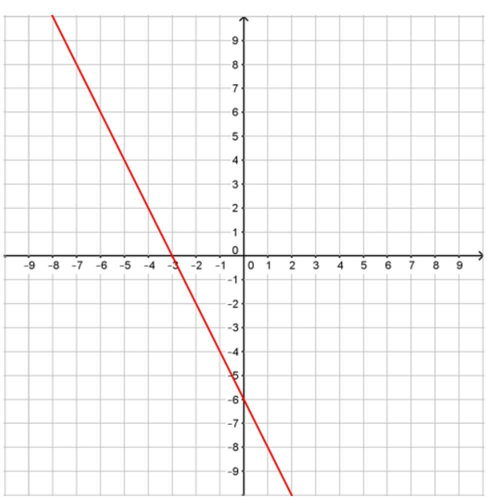
30. $g(x) = 5(2^{x-1})$



a. x-intercept(s): **None**

b. y-intercept: $\boldsymbol{\frac{5}{2}}$

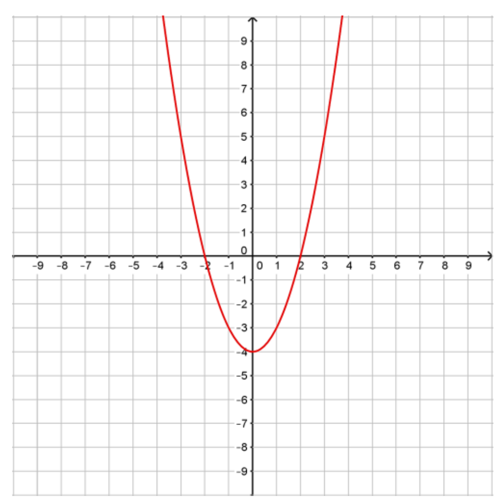
31. $h(x) = -2(x+3)$



a. x-intercept(s): $\boldsymbol{-3}$

b. y-intercept: $\boldsymbol{-6}$

32. $k(x) = x^2 - 4$



a. x-intercept(s): $\boldsymbol{-2, 2}$

b. y-intercept: $\boldsymbol{-4}$