Integrated Math 3
Module 6 Honors
Polar and Parametric Functions
Ready, Set, Go! Homework
Solutions

Adapted from

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Name **Modeling with Functions** | **6.1H**

Ready, Set, Go!

Ready

Topic: Equations of lines

Write the equation of the line (in slope-intercept form) that is defined by the given information.

1. A(5, 9) B(7, 17)
   $y = 4x - 11$

2. $P(-3, 8)$ Q(−4, 13)
   $y = -5x - 7$

3. G (3, −10) H(1, −11)
   $y = \frac{1}{2}x - 11\frac{1}{2}$

4. L (−5, 6) M(−8, 8)
   $y = -\frac{2}{3}x + \frac{8}{3}$

5.

x	y
1	−1
6	1
11	3

$y = \frac{2}{5}x - 1\frac{2}{5}$

6.

x	y
1	2
5	5
9	8

$y = \frac{3}{4}x + \frac{5}{4}$

Topic: Key Features of Circles

Reminder: The equation of circle is $(x-h)^2 + (y-k)^2 = r^2$ where the center has coordinates (h, k) and the radius has a length of r. The <u>eccentricity</u> of a conic section describes how much the conic section deviates from being circular (i.e. how "un-circular" the conic section is). The larger the value of eccentricity, the straighter the curve will be.

7. Predict the value of the eccentricity of a circle.
   0

For each circle below, identify the center and radius.

8. $(x+7)^2 + (y-3)^2 = 36$

   Center: $(-7, 3)$
   Radius: **6**

9. $(x-9)^2 + (y-2)^2 = 4$

   Center: $(9, 2)$
   Radius: **2**

10. $(x+4)^2 + (y+1)^2 = 28$

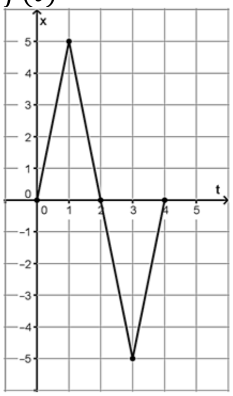
   Center: $(-4, -1)$
   Radius: $2\sqrt{7}$

Set

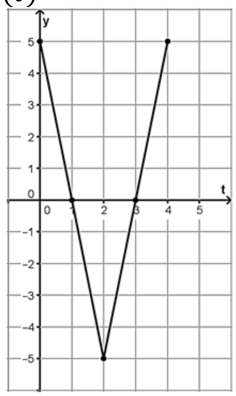
Topic: Parametric equations

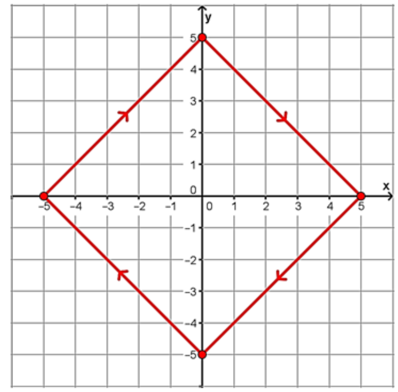
The two given graphs show the motion of an object whose position at time t seconds is given by $x = f(t)$ and $y = g(t)$. Describe the motion of the object. Then graph the two functions as a single graph in the xy-plane. Connect the points to indicate the motion at each second.

11. $f(t)$



$g(t)$

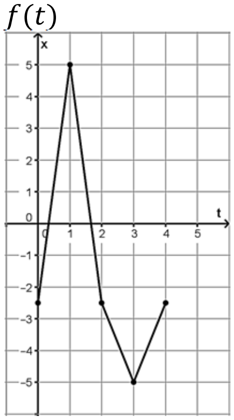




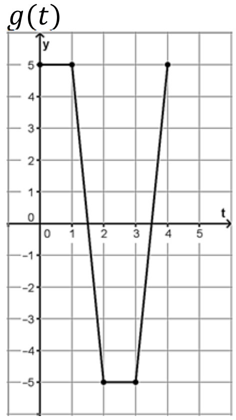
Describe the motion:

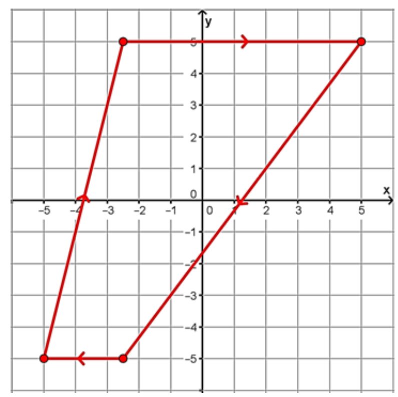
Lines from $(0, 5)$ to $(5, 0)$ to $(0, -5)$ to $(-5, 0)$

12. $f(t)$



$g(t)$

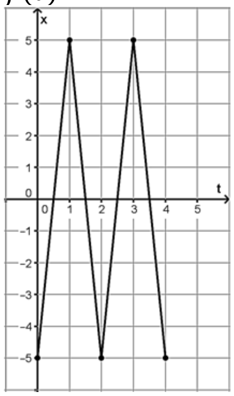




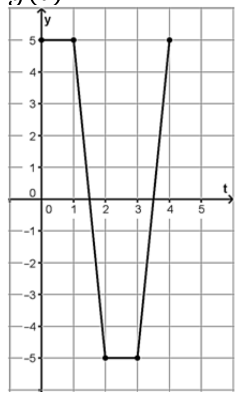
Describe the motion:

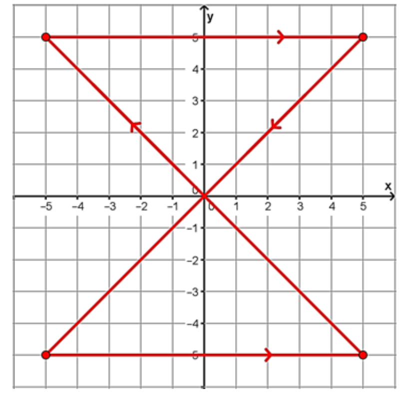
Lines from $(-2.5, 5)$ to $(5, 5)$ to $(-2.5, -5)$ to $(-5, -5)$ to $(-2.5, 5)$

13. $f(t)$



$g(t)$



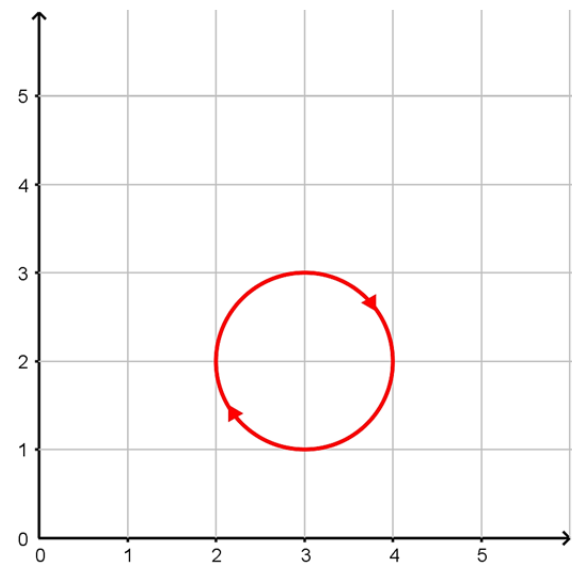


Describe the motion:

Lines from $(-5, 5)$ to $(5, 5)$ to $(-5, -5)$ to $(5, -5)$ to $(-5, 5)$

14. Fill in the table of values for the pair of parametric equations $x = 3 + \sin t$ and $y = 2 + \cos t$. Sketch the graph of the two equations in the xy-plane. Indicate the direction of the curve. Use the graph to write an equation for y as a function of x.

Time (t)	$x = 3 + \sin t$	$y = 2 + \cos t$	(x, y)
0	**3**	**3**	$(3, 3)$
1	**3.84**	**2.54**	$(3.84, 2.54)$
2	**3.91**	**1.58**	$(3.91, 1.58)$
3	**3.14**	**1.01**	$(3.14, 1.01)$
4	**2.24**	**1.35**	$(2.24, 1.35)$
5	**2.04**	**2.28**	$(2.04, 2.28)$
6	**2.72**	**2.96**	$(2.72, 2.96)$
7	**3.66**	**2.75**	$(3.66\ 2.75)$



Equation:

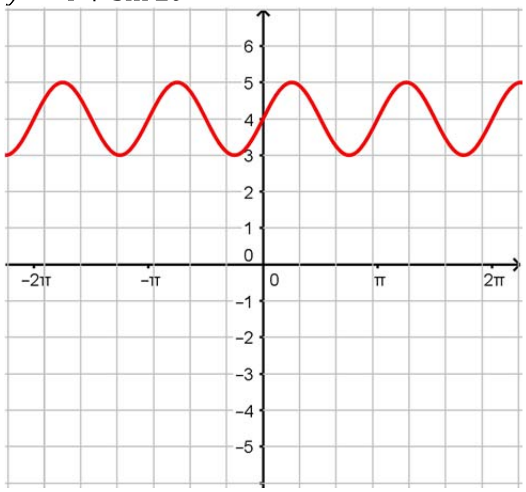
$\boldsymbol{(x-3)^2 + (y-2)^2 = 1}$

Go

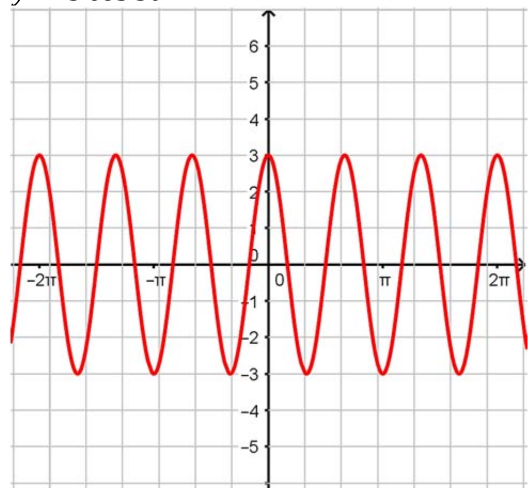
Topic: Graphs of the trigonometric functions

Graph the functions.

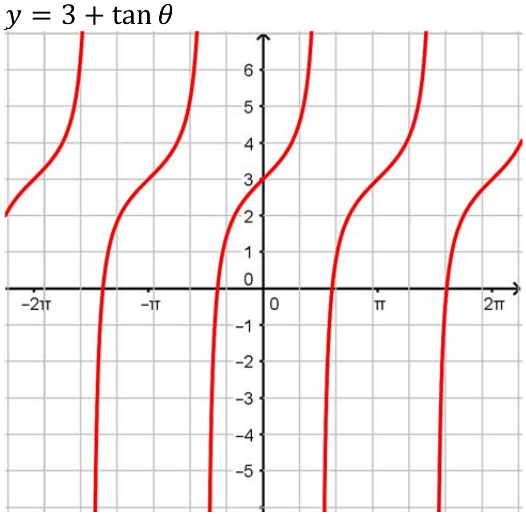
15. $y = 4 + \sin 2\theta$



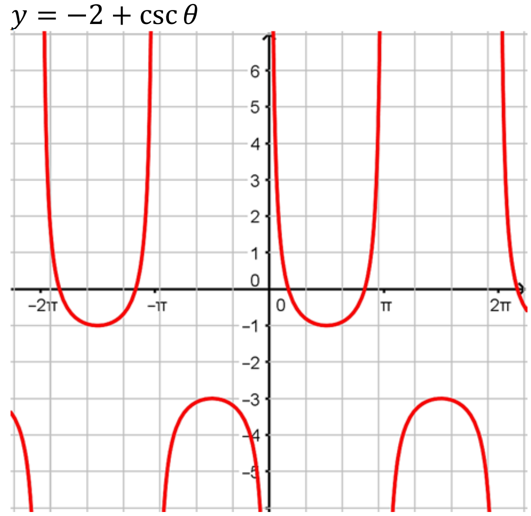
16. $y = 3\cos 3\theta$



17. $y = 3 + \tan\theta$



18. $y = -2 + \csc\theta$



Topic: Composition of inverse trigonometric functions

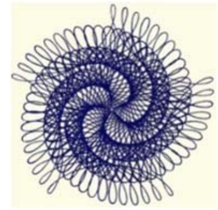
Evaluate each expression.

19. $\sin^{-1}\left(\cos\frac{\pi}{3}\right)$

$\boldsymbol{\frac{\pi}{6}}$

20. $\cos^{-1}\left(\tan\frac{\pi}{4}\right)$

0



Ready, Set, Go!

Ready

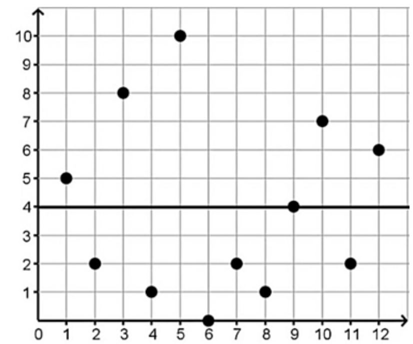
Topic: Measures of central tendency

1. Find the mean, median, and mode of the following test scores:
   98, 74, 70, 68, 85, 82, 85, 94, 90, 91, 99, 85, 88, 79, 96, 98, 85, 82, 80, 86

   Mean = 85.75, Median = 85, Mode = 85

2. The graph to the right shows 12 points whose y-values add up to 48. The $y = 4$ line is graphed. Use the position of the points on the graph to explain why 4 is the average of the 12 numbers.

   Answers may vary. Possible answer: the average of the y-coordinates of the points is 4.



Topic: Key features of ellipses

Reminder: The equations of ellipses are $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$ (horizontal ellipse) and $\frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1$ (vertical ellipse). The center has coordinates (h, k), a is the distance from the center to the vertices, and b is the distance from the center to the co-vertices. The distance from the center to the foci is represented by the variable c and can be found using the formula $c^2 = a^2 - b^2$. The eccentricity of ellipses can be found using the ratio $\frac{c}{a}$.

For each ellipse below, identify the center, coordinates of the vertices, co-vertices, and foci, and the value of the eccentricity.

3. $\frac{(x-5)^2}{4} + \frac{(y+3)^2}{16} = 1$

   Center: $(\mathbf{5, -3})$

   Vertices: $(\mathbf{5, 1})$ & $(\mathbf{5, -7})$

   Co-vertices: $(\mathbf{3, -3})$ & $(\mathbf{7, -3})$

   Foci: $\left(\mathbf{5, -3 \pm 2\sqrt{3}}\right)$

   Eccentricity: $\frac{\sqrt{3}}{2} \approx \mathbf{0.866}$

4. $\frac{(x+2)^2}{64} + \frac{(y-6)^2}{9} = 1$

   Center: $(\mathbf{-2, 6})$

   Vertices: $(\mathbf{6, 6})$ & $(\mathbf{-10, 6})$

   Co-vertices: $(\mathbf{-2, 9})$ & $(\mathbf{-2, 3})$

   Foci: $\left(\mathbf{-2 \pm \sqrt{55}, 6}\right)$

   Eccentricity: $\frac{\sqrt{55}}{8} \approx \mathbf{0.927}$

5. $\frac{(x+3)^2}{81}+\frac{(y+2)^2}{16}=1$

Center: $(\mathbf{-3, -2})$

Vertices: $(\mathbf{-12, -2})$ & $(\mathbf{6, -2})$

Co-vertices: $(\mathbf{-3, 2})$ & $(\mathbf{-3, -6})$

Foci: $\left(\mathbf{-3 \pm \sqrt{65}, -2}\right)$

Eccentricity: $\frac{\sqrt{65}}{9} \approx \mathbf{0.896}$

6. $\frac{(x-1)^2}{9}+\frac{(y-3)^2}{25}=1$

Center: $(\mathbf{1, 3})$

Vertices: $(\mathbf{1, 8})$ & $(\mathbf{1, -2})$

Co-vertices: $(\mathbf{4, 3})$ & $(\mathbf{-2, 3})$

Foci: $(\mathbf{1, 7})$ & $(\mathbf{1, -1})$

Eccentricity: $\frac{4}{5} = \mathbf{0.8}$

7. How does the eccentricity of an ellipse compare to the eccentricity of a circle?

The eccentricity of an ellipse is always greater than 0 but less than 1. The eccentricity of a circle is always 0.

Set

Topic: Parametric equations

For each set of parametric equations,

a. Create a table of *x*- and *y*-values

b. Plot the points (x, y) in your table and sketch the graph. (Indicate the direction of the curve.)

c. Find the rectangular equation.

8.

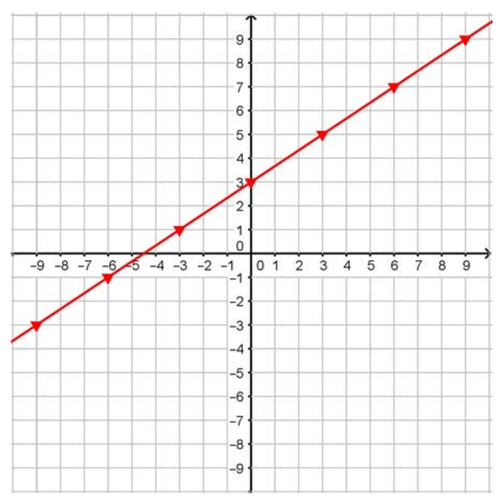
$x = 3t - 3$

$y = 2t + 1$

t	x	y
0	**−3**	**1**
1	**0**	**3**
2	**3**	**5**
3	**6**	**7**
4	**9**	**9**
5	**12**	**11**
6	**15**	**13**

Rectangular Equation:

$\boldsymbol{y = \frac{2}{3}x + 3}$



9.

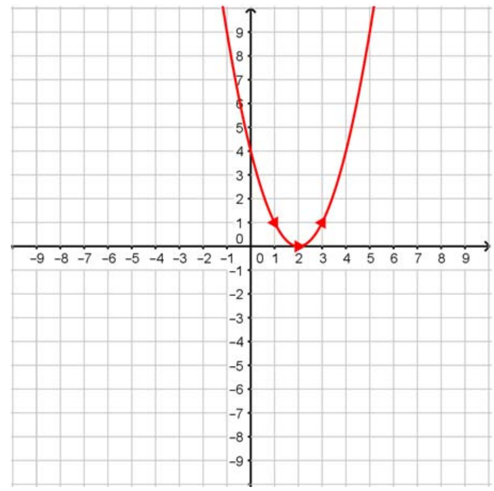
$x = t + 2$

$y = t^2$

t	x	y
-2	0	4
-1	1	1
0	2	0
1	3	1
2	4	4
3	5	9
4	6	16

Rectangular Equation:

$\boldsymbol{y = (x-2)^2}$



10.

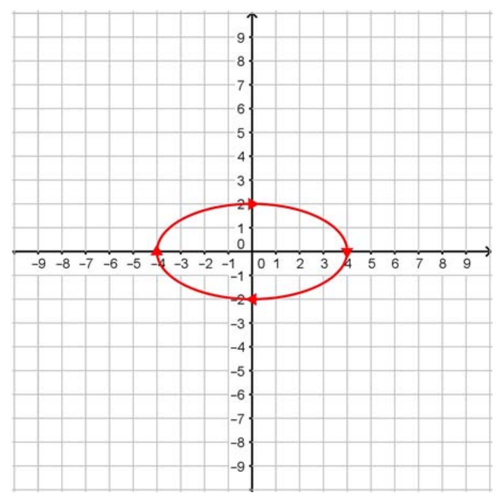
$x = 4\sin 2\theta$

$y = 2\cos 2\theta$

θ	x	y
$-\pi$	0	2
$-\frac{3\pi}{4}$	4	0
$-\frac{\pi}{2}$	0	-2
$-\frac{\pi}{4}$	-4	0
0	0	2
$\frac{\pi}{4}$	4	0
$\frac{\pi}{2}$	0	-2
$\frac{3\pi}{4}$	-4	0
π	0	2

Rectangular Equation:

$\boldsymbol{\frac{x^2}{16} + \frac{y^2}{4} = 1}$



Match the parametric equations with their graphs. (You may use a calculator.)

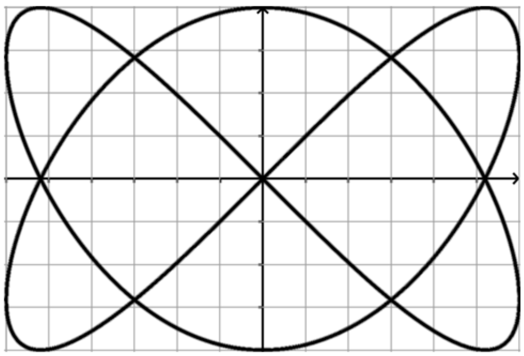
11. $x = 5\cos t$ and $y = \sin 3t$
    D

12. $x = 6\sin(4t)$ and $y = 4\sin(6t)$
    A

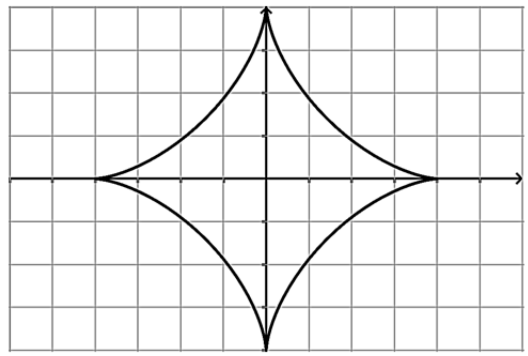
13. $x = 4\cos^3 t$ and $y = 4\sin^3 t$
    B

14. $x = \frac{1}{2}(\cos t + t \cdot \sin t)$ and $y = \frac{1}{2}(\sin t - t \cdot \cos t)$
    C

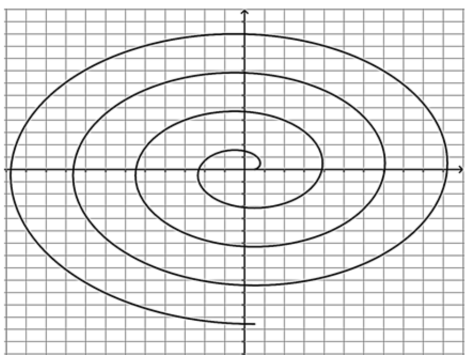
a.



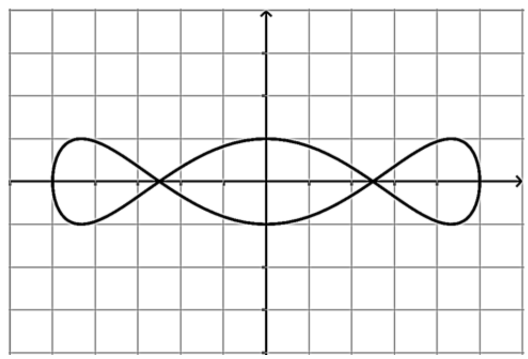
b.



c.



d.



Go

Topic: Composition of functions

15. Given $f(x) = 3x + 2$ and $g(x) = 4 - x^2$, find the following.

a. $(f \circ g)(x)$
$\mathbf{-3x^2 + 14}$

b. $(g \circ f)(x)$
$\mathbf{-9x^2 - 12x}$

c. $(f \circ g)(-4)$
$\mathbf{-34}$

d. $(g \circ f)(-2)$
$\mathbf{-12}$

16. Is there a restriction on the domain in any of the exercises (a–d) in question 15? Explain.
No because there are no denominators or square roots.

17. Given $f(x) = \frac{3x+2}{x-1}$ and $g(x) = 3x + 3$, find the following.

a. $(f \circ g)(x)$
$\mathbf{\frac{9x+11}{3x+2}}$

b. $(g \circ f)(x)$
$\mathbf{\frac{12x+3}{x-1}}$

c. $(f \circ g)(10)$
$\mathbf{\frac{101}{32}}$

d. $(g \circ f)(4)$
$\mathbf{17}$

18. Is there a restriction on the domain in any of the exercises (a–d) in question 17? Explain.
a: $\mathbf{x \neq -\frac{2}{3}}$
b: $\mathbf{x \neq 1}$

Topic: Composition of inverse trigonometric functions

Evaluate each expression.

19. $\cos^{-1}\left(\cos\frac{5\pi}{3}\right)$
$\mathbf{\frac{\pi}{3}}$

20. $\sin\left(\cos^{-1}\left(\frac{3}{5}\right)\right)$ Hint: draw a picture
$\mathbf{\frac{4}{5}}$

Ready, Set, Go!

Ready

Topic: Key features of parabolas

Reminder: The equations of parabolas are $4p(y-k)=(x-h)^2$ and $4p(x-h)=(y-k)^2$ where p is the distance from the vertex to the focus and the distance from the vertex to the directrix. The eccentricity of a parabola is the ratio between the distances from the focus and any point on the parabola and the same point and the directrix.

For each parabola below, identify the vertex, the coordinates of the focus, the equation of the directrix, and the value of the eccentricity.

1. $8(x-5)=(y-3)^2$

   Vertex: $(5,3)$

   Focus: $(7,3)$

   Directrix: $x=3$

   Eccentricity: **1**

2. $-20(y-1)=(x+3)^2$

   Vertex: $(-3,1)$

   Focus: $(-3,-4)$

   Directrix: $y=6$

   Eccentricity: **1**

3. $-1x=(y+5)^2$

   Vertex: $(0,-5)$

   Focus: $\left(-\frac{1}{4},-5\right)$

   Directrix: $x=\frac{1}{4}$

   Eccentricity: **1**

4. $16(y-1)=(x-2)^2$

   Vertex: $(2,1)$

   Focus: $(2,5)$

   Directrix: $y=-3$

   Eccentricity: **1**

5. How does the eccentricity of parabolas compare to the eccentricity of circles and ellipses?

   The eccentricity of parabolas is always 1 whereas the eccentricity of circles is always 0 and the eccentricity of ellipses is greater than 0 but less than 1.

Set

Topic: Applying parametric functions

6. Each year Sam looks forward to the ring toss at the San Diego County Fair. Sam is not happy to hit just any peg, but instead prides himself on hitting the middle peg in the front row. Suppose Sam tosses the ring with an initial horizontal velocity of 8 feet/second and an initial vertical velocity of 3 feet/second and that he'll release the ring when the ring is 4 feet above the ground. Sam lines up directly in front of the middle ring at a distance of 4 feet from the ring platform. The Ring Toss game is shown below. It sits on a short stool measuring 1 foot in height. The pegs are two inches high and the bottom of the first row of pegs is 4 inches from the bottom of the game.

a. Write an equation to model the horizontal position of the ring.

$x(t) = 8t$

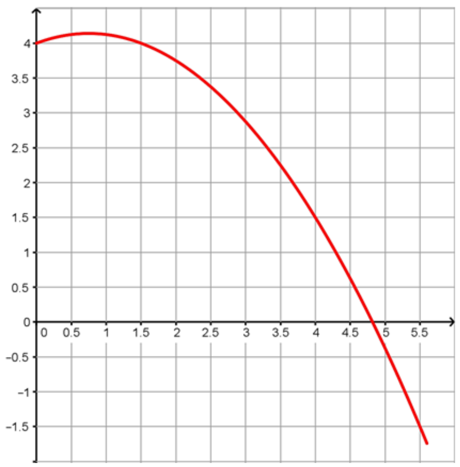
b. The vertical position of the ring will be given by $y(t) = -\frac{1}{2}gt^2 + v_0t + y_0$, where g is the acceleration due to gravity ($32\ ft/sec^2$), v_0 is the initial velocity and y_0 is the initial height of the ring when Sam releases it. Write the equation for the vertical position of the ring.

$y(t) = -16t^2 + 3t + 4$

c. Complete the table below for the horizontal and vertical positions of the ring.

Time (t)	Horizontal Position $x(t)$	Vertical Position $y(t)$
0		
0.1		
0.2		
0.3		
0.4		
0.5		
0.6		
0.7		

d. Sketch a graph of the ring's position for the values in your table. Label your axes carefully.



e. Using your parametric equations, determine whether or not Sam will hit his target peg. Explain your reasoning.

In order to find out whether or not Sam hits his target, we need to calculate the position of the top of the target peg. The game is 4 feet from Sam and the game is on the one foot stool. The peg measures 2 inches and is located 4 inches from the bottom of the game. This information indicates that the top of the peg should be at the coordinates $(4, 1.5)$. If we trace on our graph to the point where $x = 4$, we see that the y-value is 1.5 when $t = 0.5$, so Sam has hit his mark again!

7. A baseball is hit and follows the motion where x and y are measured in feet and t is measured in seconds.

$x(t) = 27t$
$y(t) = -9t^2 + 108t$

a. How high is the ball after 4 seconds?

288 feet

b. When does the ball hit the ground?

12 seconds

c. How far is the baseball when it hits the ground?

324 feet

d. When is the baseball at its highest?

6 seconds

Topic: Converting between rectangular and parametric forms

Convert each given rectangular equation to parametric form and each set of parametric equations to rectangular form.

8. $(x+2)^2 = 3y$

$\boldsymbol{x = t - 2, y = \frac{t^2}{3}}$
Answers may vary

9. $x = t - 3, y = t^2 + 5$

$\boldsymbol{y = x^2 + 6x + 14}$

Go

Topic: Solving trigonometric equations.

Solve each equation in the domain $[0, 2\pi]$.

10. $2\sin(3x) - 1 = 0$

$x = \frac{\pi}{18}, \frac{5\pi}{18}, \frac{13\pi}{18}, \frac{17\pi}{18}, \frac{25\pi}{18}, \frac{29\pi}{18}$

11. $\cot x \cos^2 x = 2\cot x$

$x = \frac{\pi}{2}, \frac{3\pi}{2}$

12. $2\sin^2 x - \sin x - 1 = 0$

$x = \frac{\pi}{2}, \frac{7\pi}{6}, \frac{11\pi}{6}$

13. $\sec^2 x - 2\tan x = 4$

$x = \frac{3\pi}{4}, \frac{7\pi}{4}$ & $x \approx 1.249, 4.3906$

Topic: Operations on rational expressions.

Simplify each rational expression.

14. $\frac{5n+5}{5n^2+35n-40} + \frac{7n}{3n}$

$\frac{7n^2+52n-53}{3(n+8)(n-1)}$

15. $\frac{20n^3+4n^2}{50n^2+10n} \div \frac{4n^2}{2n^2}$

$\frac{n}{5}$

16. $\frac{27n-45}{9n^2-27n+20} \cdot \frac{30n^3-40n^2}{9n-72}$

$\frac{10n^2}{n-8}$

17. $\frac{x+2}{3x^2+7x-20} - \frac{x+1}{9x^2-25}$

$\frac{2(x^2+3x+3)}{(3x-5)(3x+5)(x+4)}$

Name **Trigonometric Functions** | **6.4H**

Ready, Set, Go!

Ready

Topic: Distance formula

Find the distance from the origin to the given point.

1. $A\ (8,\ 6)$
   $\mathbf{10}$

2. $P\ (-5,-6)$
   $\sqrt{61} \approx 7.81$

3. $F\ (-7,7)$
   $7\sqrt{2} \approx 9.899$

Topic: Key features of hyperbola

Reminder:

The equations of hyperbolas are $\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$ (horizontal hyperbola) and $\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$ (vertical hyperbola). The center has coordinates is (h,k), a is the distance from the center to the vertices, and the slopes of the asymptotes are $\pm\frac{b}{a}$ (horizontal hyperbolas) or $\pm\frac{a}{b}$ (vertical hyperbolas). The distance from the center to the foci is represented by the variable c and can be found using the formula $c^2 = a^2 + b^2$. The <u>eccentricity</u> of hyperbolas can be found using the ratio $\frac{c}{a}$.

For each hyperbola below, identify the center, coordinates of the vertices, slope of asymptotes, and coordinates of foci, and the value of the eccentricity.

4. $\frac{(x-5)^2}{4} - \frac{(y+3)^2}{16} = 1$

   Center: $(5,-3)$

   Vertices: $(3,-3)$ **and** $(7,-3)$

   Slope of Asymptotes: ± 2

   Foci: $\left(5 \pm \sqrt{20}, -3\right)$

   Eccentricity: $\frac{\sqrt{20}}{2} \approx 2.236$

5. $\frac{(y-6)^2}{9} - \frac{(x+2)^2}{64} = 1$

   Center: $(-2,6)$

   Vertices: $(-2,3)$ **and** $(-2,9)$

   Slope of Asymptotes: $\pm\frac{3}{8}$

   Foci: $\left(-2, 6 \pm \sqrt{73}\right)$

   Eccentricity: $\frac{\sqrt{73}}{3} \approx 2.848$

6. $\frac{(y+3)^2}{81} - \frac{(x+2)^2}{16} = 1$

Center: $(-2, -3)$

Vertices: $(-2, -12)$ **and** $(-2, 6)$

Slope of Asymptotes: $\pm\frac{9}{4}$

Foci: $\left(-2, -3 \pm \sqrt{97}\right)$

Eccentricity: $\frac{\sqrt{97}}{9} \approx 1.094$

7. $\frac{(x-1)^2}{9} + \frac{(y-3)^2}{25} = 1$

Center: $(1, -3)$

Vertices: $(-2, -3)$ **and** $(4, -3)$

Slope of Asymptotes: $\pm\frac{5}{3}$

Foci: $\left(1 \pm \sqrt{34}, -3\right)$

Eccentricity: $\frac{\sqrt{34}}{3} \approx 1.944$

8. How does the eccentricity of a hyperbola compare to the eccentricity of other conic sections?

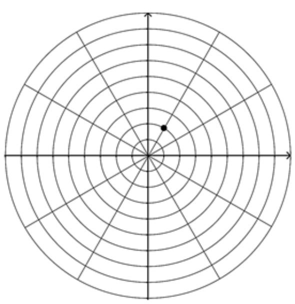
The eccentricity of hyperbolas is greater than one, whereas all other conic sections have eccentricity less than one.

Set

Topic: Converting between polar and rectangular coordinates

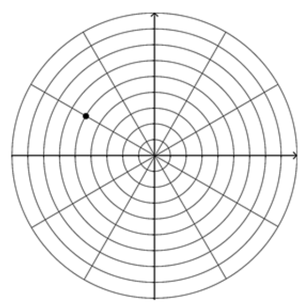
The given polar point can be described using many different pairs of polar coordinates. Give three different polar coordinates that, when plotted, are equivalent to the given point.

9.



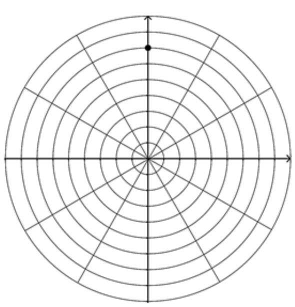
Polar Coordinates: **Answers will vary, possible answers:** $\left(2, \frac{\pi}{3}\right), \left(2, -\frac{5\pi}{3}\right)$

10.



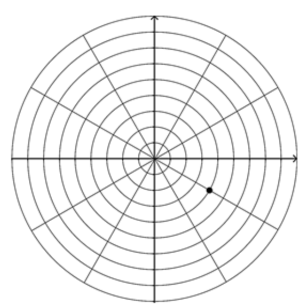
Polar Coordinates: **Answers will vary, possible answers:** $\left(5, \frac{5\pi}{6}\right), \left(5, -\frac{7\pi}{6}\right)$

11.



Polar Coordinates: **Answers will vary, possible answers:** $\left(7, \frac{\pi}{2}\right), \left(7, -\frac{3\pi}{2}\right)$

12.



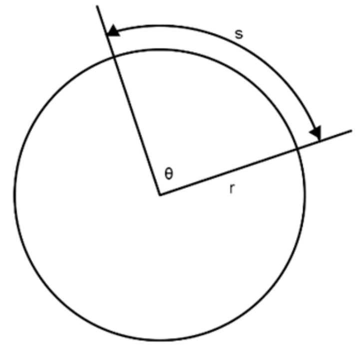
Polar Coordinates: **Answers will vary, possible answers:** $\left(4, \frac{11\pi}{6}\right), \left(4, -\frac{\pi}{6}\right)$

13. Find the rectangular coordinates that, when plotted, are in the same location as the point in polar form given in question 12. (Hint: Draw a triangle)

$\left(2\sqrt{3}, -2\right)$

Go

Topic: Arc length



Recall the formula for arc length: $s = r\theta$, where θ is always in radians. Write your answers with π in it. Then use your calculator to find the approximate length of the arc to 2 decimal places.

14. Find the length of an arc given that $r = 10$ in and $\theta = \frac{\pi}{4}$

    7.85 in

15. Find the arc length given $r = 4$ cm and $\theta = \frac{5\pi}{6}$

    10.47 cm

16. Find the arc length given $r = 72.0$ ft and $\theta = \frac{\pi}{8}$

    28.27 ft

17. Find the radius given $s = 3.08$ mm. and $\theta = \frac{11\pi}{10}$

    0.892 mm

Name **Trigonometric Functions | 6.5H**

Ready, Set, Go!

Ready

Topic: Finding the central angle when given arc length and radius

Find the radian measure of the central angle of a circle of radius *r* that intercepts an arc of length *s*. ($s = r\theta$)
Round answers to 4 decimal places.

Radius	*Arc Length*	*Radian Measure*
1. 35 mm	11 mm	**0.3143**
2. 14 feet	9 feet	**0.6429**
3. 16.5 m	28 m	**1.6970**
4. 45 miles	90 miles	**2**

Topic: Determining the type of conic section based upon the value of the eccentricity.

Determine the type of conic section represented by the given values of eccentricity.

5. 0.75
ellipse

6. 0
circle

7. 2.43
hyperbola

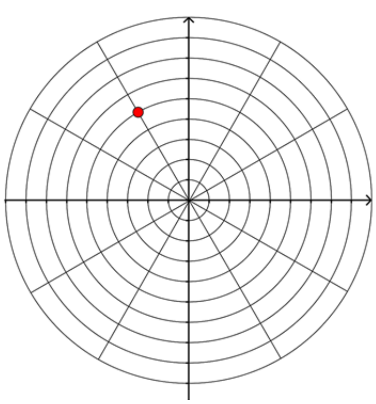
8. 1
parabola

Set

Topic: Polar coordinates

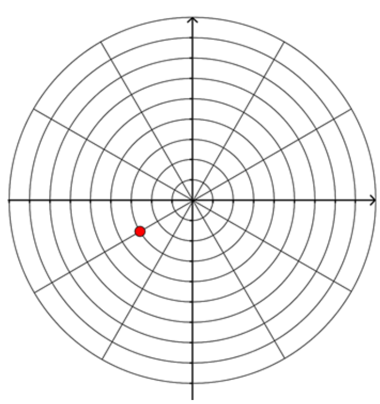
Plot the given point (r, θ), in the polar coordinate system. Give an additional coordinate in polar form that plots the same point.

9. $(r, \theta) = \left(5, \frac{2\pi}{3}\right)$



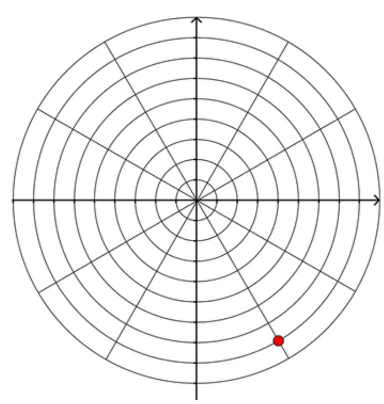
Answers vary: $\left(-5, -\frac{\pi}{3}\right)$

10. $(r, \theta) = \left(3, \frac{7\pi}{6}\right)$



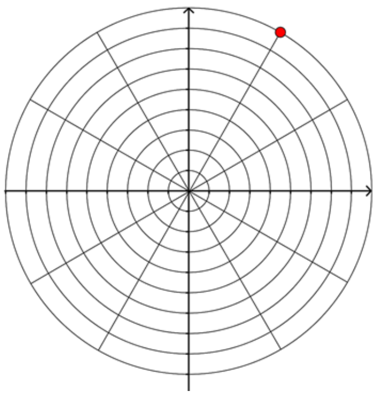
Answers vary: $\left(-3, \frac{\pi}{6}\right)$

11. $(r, \theta) = \left(8, \frac{5\pi}{3}\right)$



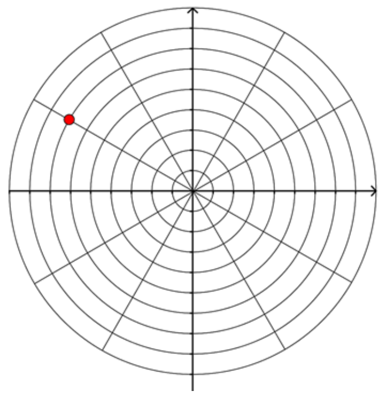
Answers vary: $\left(-8, \frac{2\pi}{3}\right)$

12. $(r, \theta) = \left(9, \frac{\pi}{3}\right)$



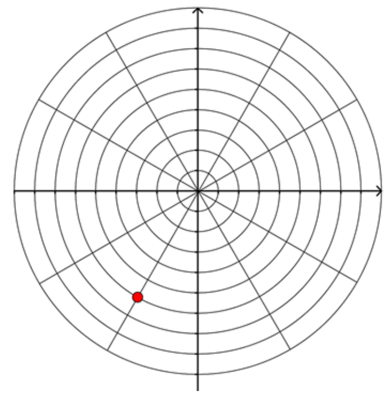
Answers vary: $\left(-9, \frac{4\pi}{3}\right)$

13. $(r, \theta) = \left(7, \frac{5\pi}{6}\right)$



Answers vary: $\left(7, -\frac{7\pi}{6}\right)$

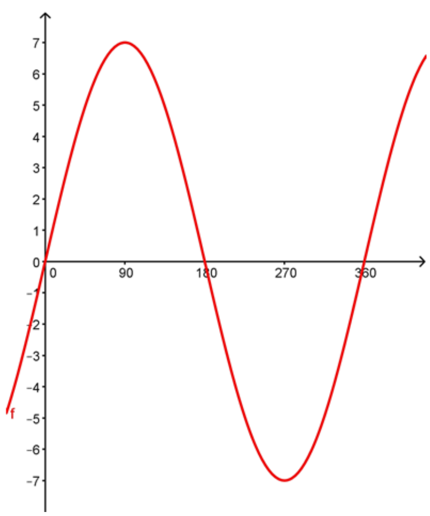
14. $(r, \theta) = \left(-6, \frac{\pi}{3}\right)$



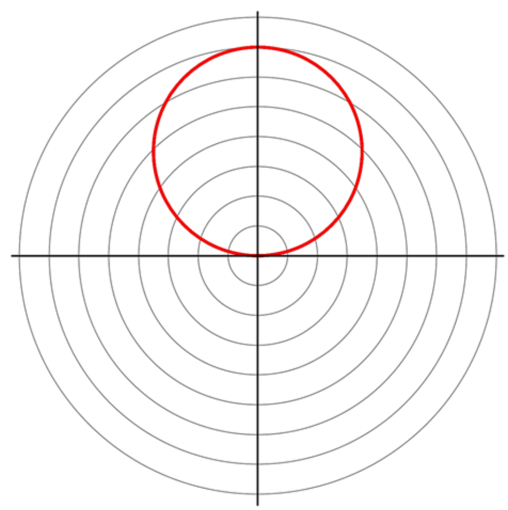
Answers vary: $\left(6, -\frac{2\pi}{3}\right)$

Sketch the indicated graphs.

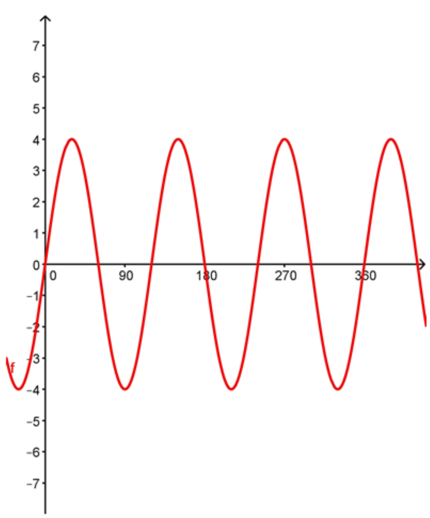
15. $y = 7 \sin x$



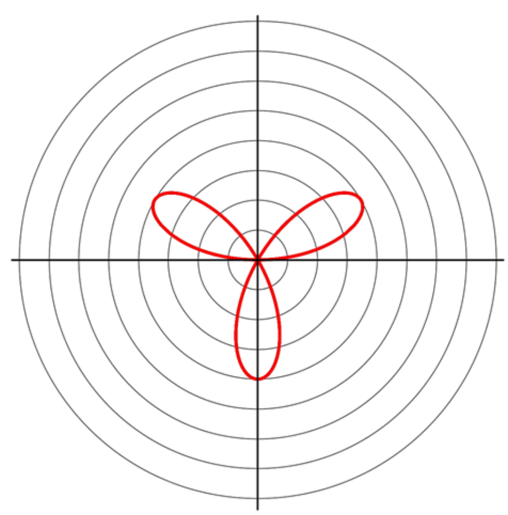
16. $r = 7 \sin \theta$



17. $y = 4 \sin(3x)$



18. $r = 4 \sin(3\theta)$

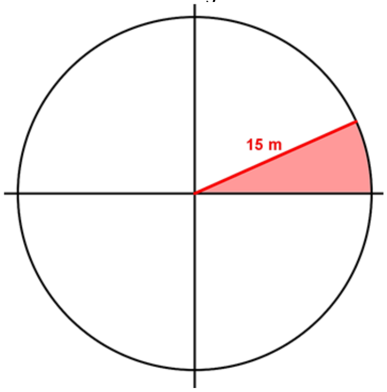


Go

Topic: The area of a sector

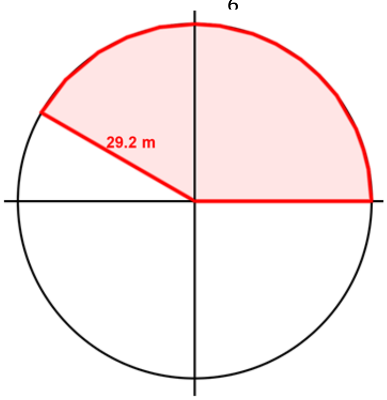
Find the area of a sector of a circle having radius r and central angle θ. $\left(A=\frac{1}{2}r^2\theta\right)$. Make a sketch and shade in the sector. Round answers to 2 decimals.

19. $r = 15\ m;\ \theta = \frac{2\pi}{5}$ radians



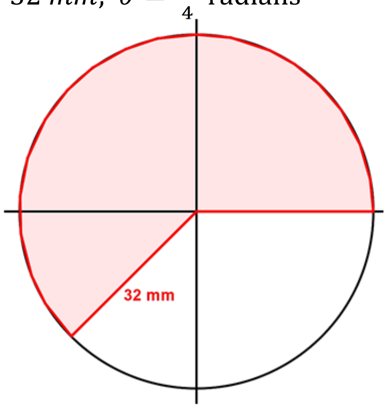
$A = 141.27\ m^2$

20. $r = 29.2\ m;\ \theta = \frac{5\pi}{6}$ radians



$A = 1116.10\ m^2$

21. $r = 32\ mm;\ \theta = \frac{5\pi}{4}$ radians



$A = 2010.62\ mm^2$

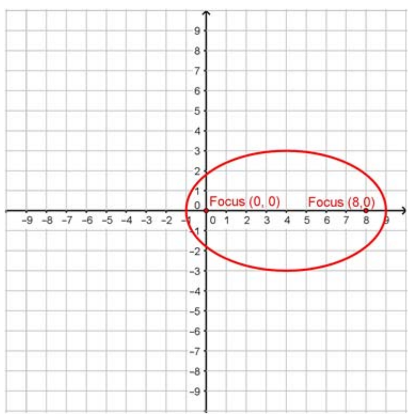
Ready, Set, Go!

Ready

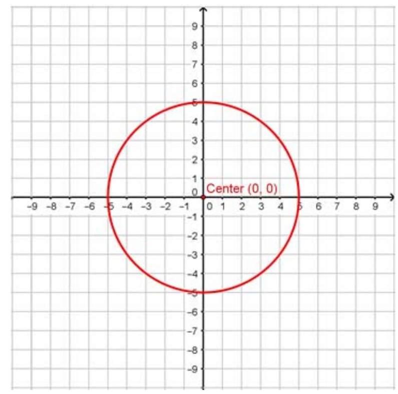
Topic: Graphing conic sections

Graph each conic section. Label all key features of each including coordinates of foci.

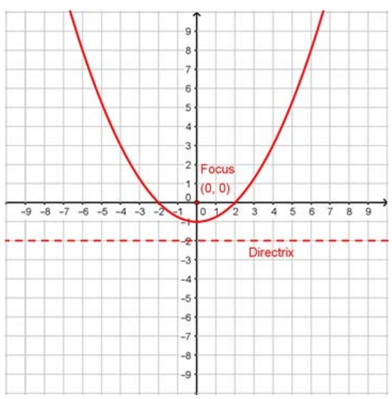
1. $\frac{(x-4)^2}{25}+\frac{y^2}{9}=1$



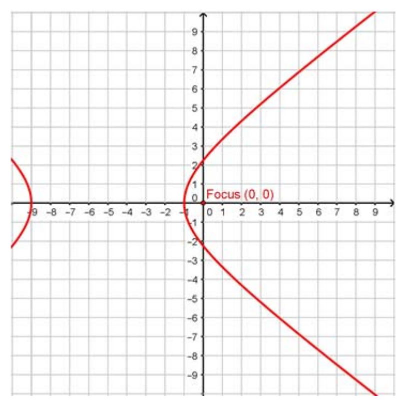
2. $x^2+y^2=16$



3. $x^2=4(y+1)$



4. $\frac{(x+5)^2}{16}-\frac{y^2}{9}=1$



Topic: Rewriting equations in polar form

Recall: $x=r\cos\theta$, $y=r\sin\theta$, $r^2=x^2+y^2$, and $\theta=\tan^{-1}\left(\frac{y}{x}\right)$.
Rewrite each equation using polar coordinates (r,θ). Express answers in as $r=$.

5. $x=-4$
   $r=$______________

   $r=-\frac{4}{\cos\theta}$

6. $x^2+y^2=16$
   $r=$______________

   $r=4$

7. $x^2=4y$
   $r=$______________

   $r=\frac{4\sin\theta}{\cos^2\theta}=4\tan\theta\sec\theta$

Set

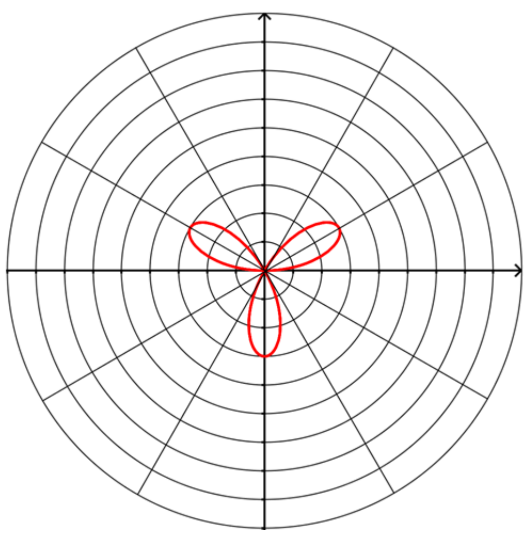
Topic: Graphing polar functions

Complete the table and sketch the graph of each polar function.

8. $r = 3\sin 3\theta$

$\boldsymbol{\theta}$	$\boldsymbol{r}$
0	**0**
$\frac{\pi}{6}$	**3**
$\frac{\pi}{3}$	**0**
$\frac{\pi}{2}$	**−3**
$\frac{2\pi}{3}$	**0**
$\frac{5\pi}{6}$	**3**
π	**0**
$\frac{7\pi}{6}$	**−3**
$\frac{4\pi}{3}$	**0**
$\frac{3\pi}{2}$	**3**
$\frac{5\pi}{3}$	**0**
$\frac{11\pi}{6}$	**−3**
2π	**0**

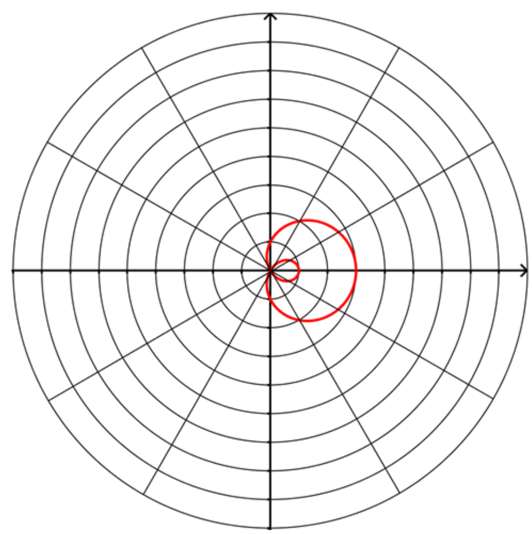
Sketch:



9. $r = 1 + 2\cos\theta$

$\boldsymbol{\theta}$	$\boldsymbol{r}$
0	**3**
$\frac{\pi}{6}$	$\mathbf{1+\sqrt{3}}$
$\frac{\pi}{3}$	**2**
$\frac{\pi}{2}$	**1**
$\frac{2\pi}{3}$	**0**
$\frac{5\pi}{6}$	$\mathbf{1-\sqrt{3}}$
π	**−1**
$\frac{7\pi}{6}$	$\mathbf{1-\sqrt{3}}$
$\frac{4\pi}{3}$	**0**
$\frac{3\pi}{2}$	**1**
$\frac{5\pi}{3}$	**2**
$\frac{11\pi}{6}$	$\mathbf{1+\sqrt{3}}$
2π	**3**

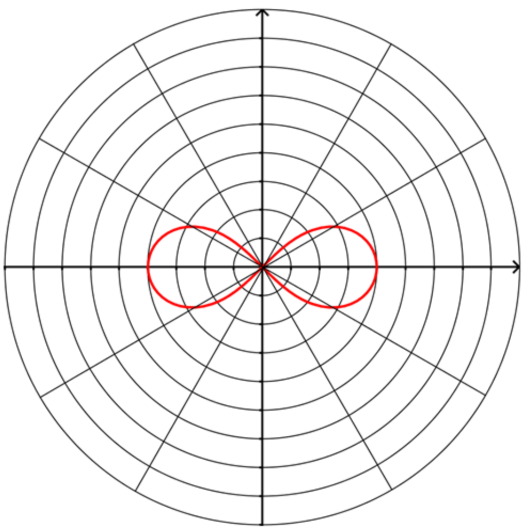
Sketch:



10. $r^2 = 16\cos 2\theta$

$\boldsymbol{\theta}$	$\boldsymbol{r}$
0	± 4
$\frac{\pi}{6}$	$\pm\sqrt{8}$
$\frac{\pi}{3}$	non real answer
$\frac{\pi}{2}$	non real answer
$\frac{2\pi}{3}$	non real answer
$\frac{5\pi}{6}$	$\pm\sqrt{8}$
π	± 4
$\frac{7\pi}{6}$	$\pm\sqrt{8}$
$\frac{4\pi}{3}$	non real answer
$\frac{3\pi}{2}$	non real answer
$\frac{5\pi}{3}$	non real answer
$\frac{11\pi}{6}$	$\pm\sqrt{8}$
2π	± 4

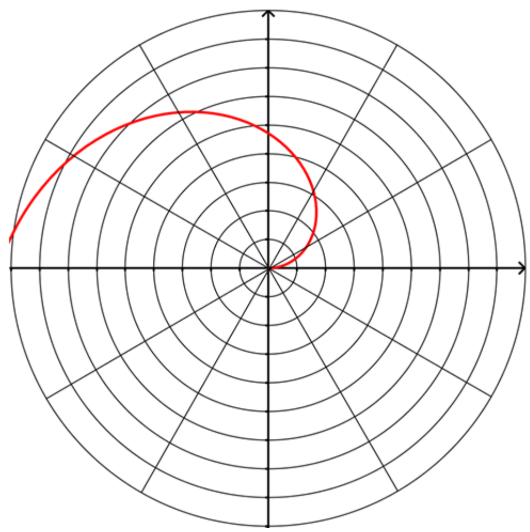
Sketch:



11. $r = 3\theta$

$\boldsymbol{\theta}$	$\boldsymbol{r}$
0	0
$\frac{\pi}{6}$	$\frac{\pi}{2}$
$\frac{\pi}{3}$	π
$\frac{\pi}{2}$	$\frac{3\pi}{2}$
$\frac{2\pi}{3}$	2π
$\frac{5\pi}{6}$	$\frac{5\pi}{2}$
π	3π
$\frac{7\pi}{6}$	$\frac{7\pi}{2}$
$\frac{4\pi}{3}$	4π
$\frac{3\pi}{2}$	$\frac{9\pi}{2}$
$\frac{5\pi}{3}$	5π
$\frac{11\pi}{6}$	$\frac{11\pi}{2}$
2π	6π

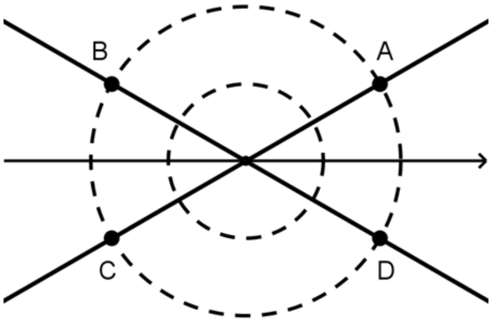
Sketch:



Go

Topic: Plotting polar coordinates

Match each of the points in polar coordinates with either A, B, C, or D on the graph.



12. $\left(2,\ -\frac{11\pi}{6}\right)$ **A**

13. $\left(-2,\ -\frac{\pi}{6}\right)$ **B**

14. $\left(-2,\ \frac{\pi}{6}\right)$ **C**

15. $\left(2,\ \frac{7\pi}{6}\right)$ **C**

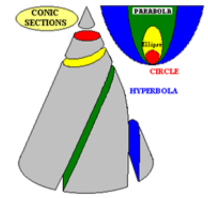
16. $\left(2,\ \frac{5\pi}{6}\right)$ **B**

17. $\left(-2,\ \frac{5\pi}{6}\right)$ **D**

18. $\left(-2,\ \frac{7\pi}{6}\right)$ **A**

19. $\left(2,\ \frac{11\pi}{6}\right)$ **D**

Name **Trigonometric Functions | 6.7H**



Ready, Set, Go!

Ready

Topic: Operations on complex numbers

Perform each operation and write the answer in the form $a + bi$.

1. $(4 - 7i) + (8 + 3i)$

   $\mathbf{12 - 4i}$

2. $2(3 + 5i) - 3(7 + i)$

   $\mathbf{-15 + 7i}$

3. $(2 - 9i)(3 + 6i)$

   $\mathbf{60 - 6i}$

4. $\frac{4}{3+2i}$

   $\mathbf{\frac{12}{13} - \frac{8}{13}i}$

5. $(3 - 5i)^2$

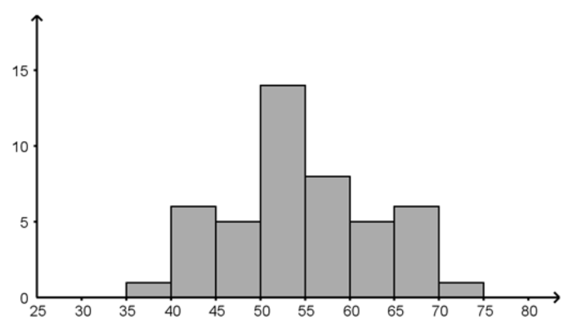
   $\mathbf{-16 - 30i}$

Topic: Describing center and spread

Using the following data set and graph, identify the mean, median, mode, and describe the spread of the data. Then calculate the standard deviation.

6. The data set below gives the ages of the first 46 vice presidents of the United States when they first took office.

   53, 53, 45, 65, 68, 42, 42, 50, 56, 50, 52, 49, 66, 36, 51, 56, 45, 61, 57, 51, 65, 64, 57, 52, 42, 52, 53, 58, 48, 59, 69, 64, 52, 60, 71, 40, 52, 53, 51, 60, 66, 49, 56, 41, 44, 59



Mean: 54.02, Median: 53, Mode: 52
Data appears to be normally distributed,
Standard Deviation: $\sigma = 8.35$

Set

Topic: Identifying and graphing conic sections in polar form

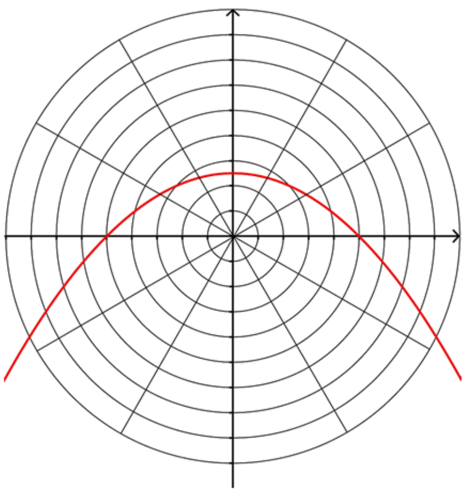
For each conic section below, write the equation in the form $r = \frac{ep}{1 \pm e\cos\theta}$ or $= \frac{ep}{1 \pm e\sin\theta}$, if necessary. Identify the eccentricity of the conic section and the type of conic section. Then graph the conic section.

7. $r = \frac{5}{1+\sin\theta}$

Re-written Equation: $r = \frac{5}{1+\sin\theta}$

Eccentricity: **1**

Type of Conic Section: **Parabola**

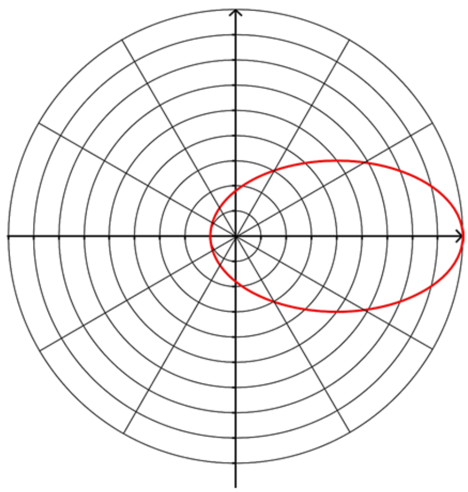


8. $r = \frac{9}{5-4\cos\theta}$

Re-written Equation: $r = \frac{\frac{9}{5}}{1-\frac{4}{5}\cos\theta}$

Eccentricity: $\frac{4}{5}$

Type of Conic Section: **Ellipse**

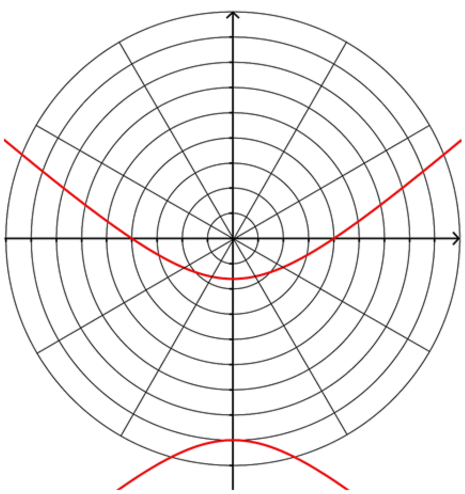


9. $r = \frac{8}{2-3\sin\theta}$

Re-written Equation: $r = \frac{4}{1-\frac{3}{2}\sin\theta}$

Eccentricity: $\frac{3}{2}$

Type of Conic Section: **Hyperbola**



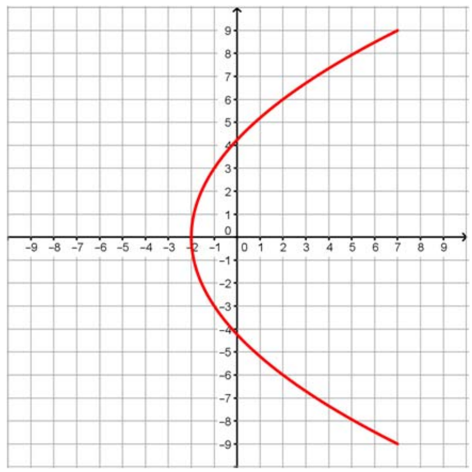
Go

Topic: Graphing parametric functions

Graph each parametric function.

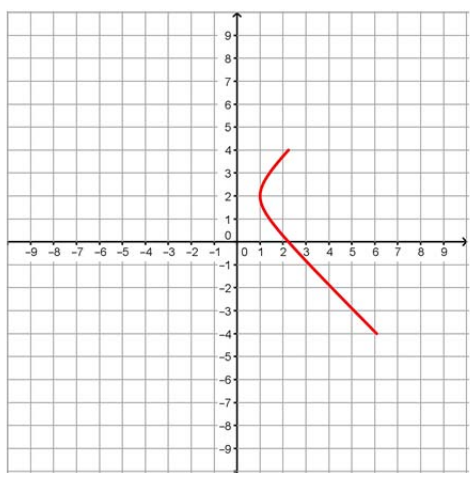
10. $x(t) = t^2 - 2$
$y(t) = 3t$

t	$x(t)$	$y(t)$
-3	**7**	**-9**
-2	**2**	**-6**
-1	**-1**	**-3**
0	**-2**	**0**
1	**-1**	**3**
2	**2**	**6**
3	**7**	**9**



11. $x(t) = \sqrt{t^2 + 1}$
$y(t) = 2 - t$

t	$x(t)$	$y(t)$
-2	**2.2361**	**4**
-1	**1.4142**	**3**
0	**1**	**2**
1	**1.4142**	**1**
2	**2.2361**	**0**
3	**3.1623**	**-1**
4	**4.1231**	**-2**
5	**5.099**	**-3**
6	**6.0928**	**-4**



Topic: Converting equations and coordinates in rectangular form to polar form

Write each of the following equations in polar form.

12. $5x + 7y = 12$

$\boldsymbol{r = \frac{12}{5\cos\theta + 7\sin\theta}}$

13. $(x-3)^2 + y^2 = 9$

$\boldsymbol{r = 6\cos\theta}$

Write each ordered pair in polar form (r, θ).

14. $\left(-6\sqrt{3}, 18\right)$

$\boldsymbol{\left(12, \frac{2\pi}{3}\right)}$

15. $(3, 4)$

$\boldsymbol{(5, 0.9273)}$

Ready, Set, Go!

Ready

Topic: The complex plane

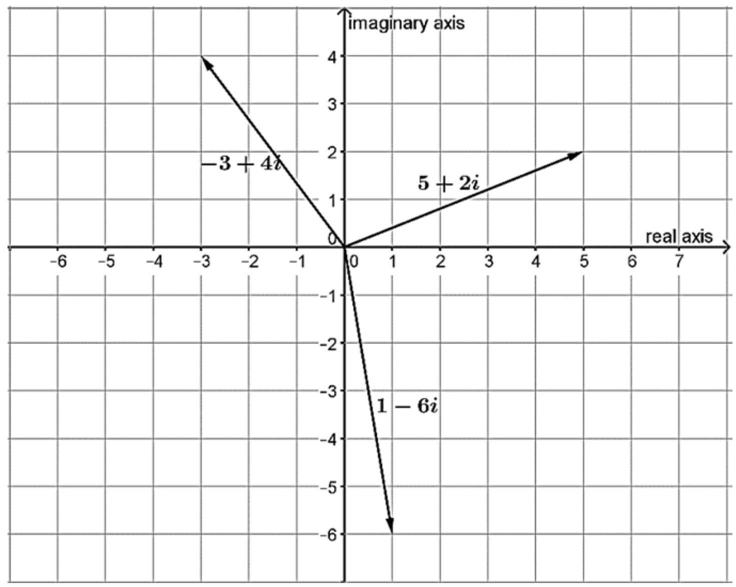
Recall from Integrated Math 1H, (Module 9), that just as real numbers can be represented by points on the real number line, you can represent a complex number $z = a + bi$ as the ordered pair (a, b) in a coordinate plane called the **complex plane**. The horizontal axis is called the **real axis** and the vertical axis is called the **imaginary axis**. A complex number $a + bi$ can also be represented by a position vector with its tail located at the point $(0, 0)$ and its head located at the point (a, b), as shown in the diagram. It will be useful to be able to move back and forth between both geometric representations of a complex number in the complex plane - sometimes representing the complex number as a single point and sometimes as a vector.

In the diagram, 3 complex numbers have been graphed as vectors. Rewrite each complex number as a point in the form (a, b).

1. $-3 + 4i$ graphs as (__-3__, __4__)

2. $5 + 2i$ graphs as (__5__, __2__)

3. $1 - 6i$ graphs as (__1__, __-6__)



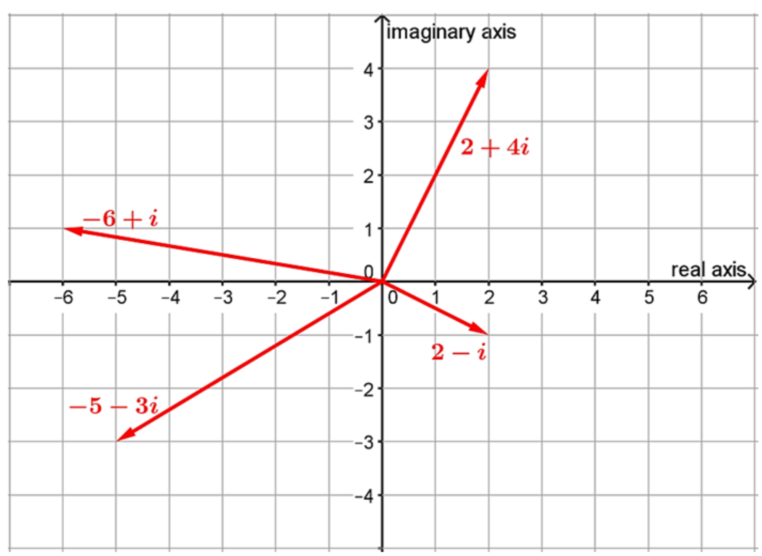
On the diagram below, graph the following complex numbers as vectors.

4. $-5 - 3i$

5. $2 + 4i$

6. $-6 + i$

7. $2 - i$



Topic: The Modulus of a complex number

We can compare the relative magnitudes of complex numbers by determining how far they lie away from the origin in the complex plane. We refer to the magnitude of a complex number as its **modulus** and symbolize this length with the notation $|a+bi|$ where $|a+bi| = \sqrt{a^2+b^2}$.

8. Find the modulus of each of the complex numbers listed below.

a. $-3+4i$ **5**

b. $5+2i$ $\sqrt{29}$

c. $1-6i$ $\sqrt{37}$

d. $-5-3i$ $\sqrt{34}$

e. $2+4i$ $2\sqrt{5}$

f. $-6+i$ $\sqrt{37}$

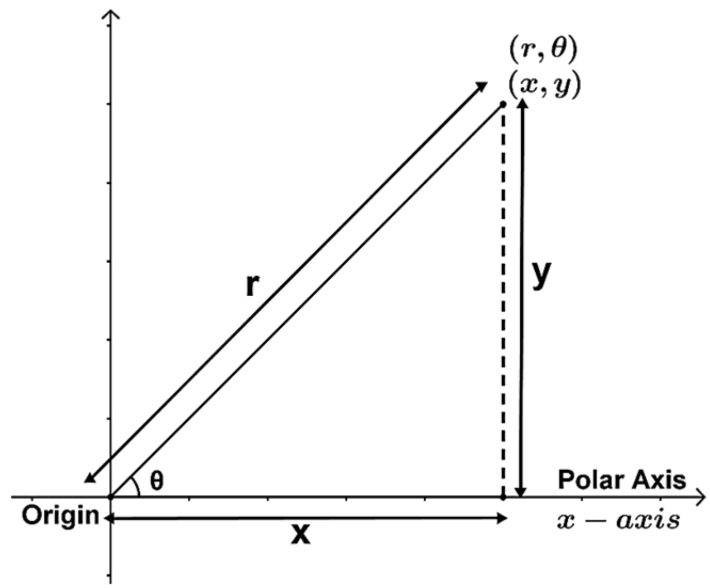
g. $2-i$ $\sqrt{5}$

Set

Topic: The relationship between polar coordinates and rectangular coordinates

Coordinate Conversion: The polar coordinates (r,θ) are related to the rectangular coordinates (x,y) as follows:

$$x = r\cos\theta \qquad \tan\theta = \frac{y}{x}$$
$$y = r\sin\theta \qquad r^2 = x^2+y^2$$



Convert the points from polar to rectangular coordinates.

9. $\left(4,\frac{\pi}{2}\right)$

$(0,4)$

10. $\left(\sqrt{3},\frac{5\pi}{6}\right)$

$\left(-\frac{3}{2},\frac{\sqrt{3}}{2}\right)$

11. $(2,\pi)$

$(-2,0)$

12. $\left(5\sqrt{2},\frac{7\pi}{4}\right)$

$(5,-5)$

Convert the points from rectangular to polar coordinates.

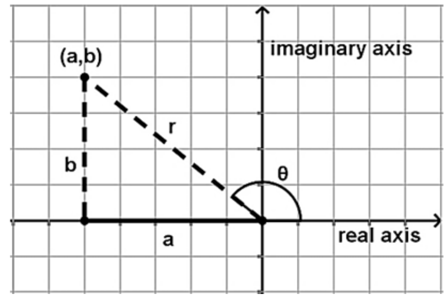
13. $(-3,3)$ $\left(3\sqrt{2},\frac{3\pi}{4}\right)$

14. $(-6,0)$ $(6,\pi)$

15. $(1,\sqrt{3})$ $\left(2,\frac{\pi}{3}\right)$

16. $(-4\sqrt{3},-4)$ $\left(8,\frac{7\pi}{6}\right)$

Topic: Polar form of a complex number



Consider the complex number $a + bi$. By letting θ be an angle in standard form as soon in the diagram at the right, you can write $a = r\cos\theta$ and $b = r\sin\theta$, where $r = \sqrt{a^2 + b^2}$. By replacing a and b, you have $a + bi = (r\cos\theta) + (r\sin\theta)i$. We can factor out the r, obtaining the **polar form of a complex number.**

If $z = a + bi$ then the polar form is $z = r(\cos\theta + i\sin\theta)$.

Write the complex numbers in polar form $z = r(\cos\theta + i\sin\theta)$

17. $-3 - i$

    $\sqrt{10}(\cos 3.46 + i \cdot \sin 3.46)$

18. $3 - 3i$

    $4\sqrt{2}\left(\cos\frac{3\pi}{4} + i \cdot \sin\frac{3\pi}{4}\right)$

19. $-4 + 4i$

    $\sqrt{65}(\cos 2.62 + i \cdot \sin 2.62)$

20. $\sqrt{3} + i$

    $2\left(\cos\frac{\pi}{6} + i \cdot \sin\frac{\pi}{6}\right)$

Write the complex number in standard form, $a + bi$.

21. $3(\cos 120° + i\sin 120°)$

    $-\frac{3}{2} + \frac{3\sqrt{3}}{2}i$

22. $5(\cos 135° + i\sin 135°)$

    $-\frac{5\sqrt{2}}{2} + \frac{5\sqrt{2}}{2}i$

23. $\sqrt{8}\left(\cos\frac{7\pi}{4} + i\sin\frac{7\pi}{4}\right)$

    $2 - 2i$

24. $8\left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right)$

    $8i$

Go

Topic: The arithmetic of complex numbers

Perform the indicated operation. Write your answers in standard form, $a + bi$.

25. $(4 + 7i) + (12 - 2i)$

    $16 + 5i$

26. $(11 - 8i) + (-4 - 3i)$

    $7 - 11i$

27. $(10 + 6i) - (16 - 3i)$

    $-6 + 9i$

28. $(-7 - i) - (9 + i)$

    $-16 - 2i$

29. $(1 + i)(4 - 2i)$

    $6 + 2i$

30. $(5 + 6i)(5 - 6i)$

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