Integrated Math 3
Module 8 Honors
Limits &
Introduction to Derivatives
Ready, Set Go Homework
Solutions

Adapted from

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Name **Limits & Introduction to Derivatives | 8.1H**

Ready, Set, Go!

Ready

Topic: Simplifying rational expressions

Simplify each rational expression.

1. $\frac{x^2-3x-10}{x^2+x-2}$

$\frac{x-5}{x-1}$

2. $\frac{8-7x-x^2}{x+8} \cdot \frac{x+5}{9x-9}$

$-\frac{x+5}{9}$

3. $\frac{10x^2-20x}{40x^3-80x^2} \div \frac{6x+30}{16x^3+80x^2}$

$\frac{2x}{3}$

4. $\frac{3x^3+6x^2+12x}{x^3-8}$

$\frac{3x}{x-2}$

5. $\frac{\frac{1}{x+1}-1}{x}$

$-\frac{1}{x+1}$

Set

Topic: Rates of change

Cardiff Kook Academy is headed to the University of Utah for a Robotics tournament. The Robotics team has decided to take a train to get to their tournament so they can ensure the safe keeping of their robots. They need to catch an early train since the University of Utah is 750 miles from home.

6. The train leaves at 6:00 a.m. Assume the train ride is exactly 750 miles with no stops. How many miles per hour must the train average for the Cardiff Kook Academy Robotics team to get the Utah by 5:00p.m.? Note: Utah is 1 hour ahead of San Diego.

   75 mph

7. The 6:00a.m. train averages 50 miles per hour for the first two hours. What speed must it average for the rest of the trip for the Cardiff Kook Academy Robotics team to reach the University of Utah by 5:00pm?

   81.25 mph

8. Suppose the train actually averaged 60 miles per hour for the whole trip (which means the trip took 12.5 hours altogether). That doesn't necessarily mean the train traveled at a constant rate of 60 mph.

   Make up a scenario in which the train average 60 mph for the trip but traveled at least two different speeds along the way. Be specific about speeds, times, and distances.

   Answers may vary

Go

Topic: Features of functions and graphing families of functions

Graph each function and identify the indicated features of the function.

9. $f(x) = -\frac{1}{4}(x-2)^2 + 7$

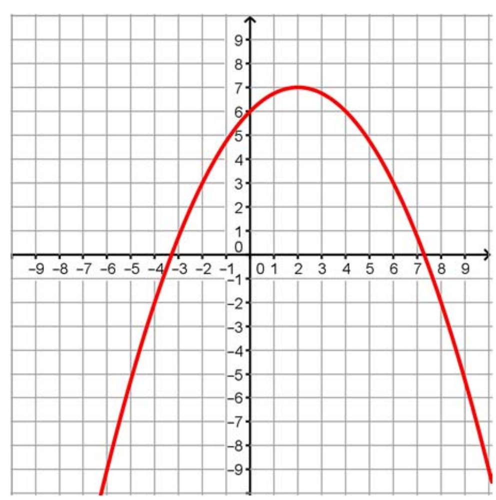
   Domain: $(-\infty, \infty)$

   Range: $(-\infty, \mathbf{7}]$

   Interval(s) of Increase: $(-\infty, \mathbf{2})$

   Interval(s) of Decrease: $(\mathbf{2}, \infty)$

   End Behavior:
   As $x \to -\infty, f(x) \to -\infty$
   As $x \to \infty, f(x) \to -\infty$



10. $f(x) = (x+3)(x+1)(x-2)$

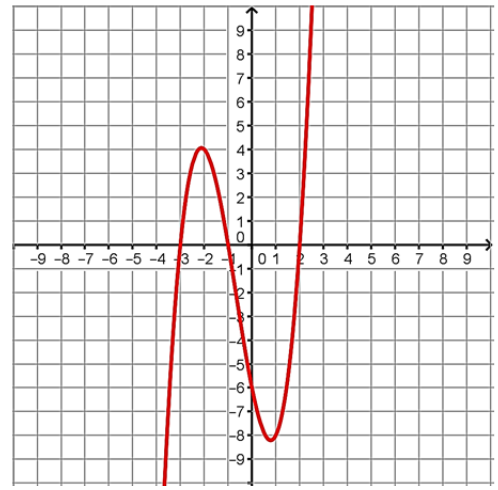
Domain: $(-\infty, \infty)$

Range: $(-\infty, \infty)$

Interval(s) of Increase: **Approximate answers given.**
$(-\infty, -2.1) \cup (0.8, \infty)$

Interval(s) of Decrease: Approximate answers given.
$(-2.1, 0.8)$

End Behavior:
As $x \to -\infty, f(x) \to -\infty$
As $x \to \infty, f(x) \to \infty$



11. $f(x) = 2\sqrt{x+4} - 5$

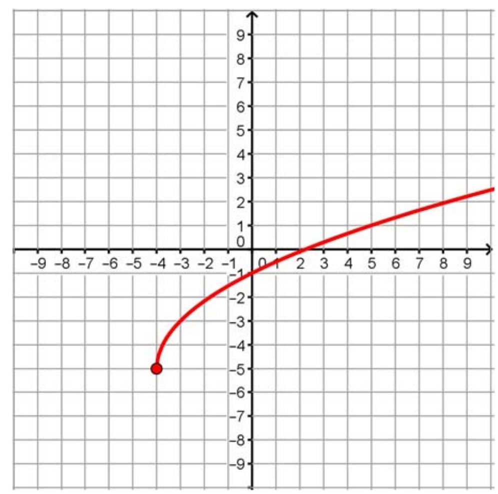
Domain: $[-4, \infty)$

Range: $[-5, \infty)$

Interval(s) of Increase: $(-4, \infty)$

Interval(s) of Decrease: **NA**

End Behavior:
As $x \to -4, f(x) \to -5$
As $x \to \infty, f(x) \to \infty$



12. $f(x) = \frac{x-3}{x+2}$

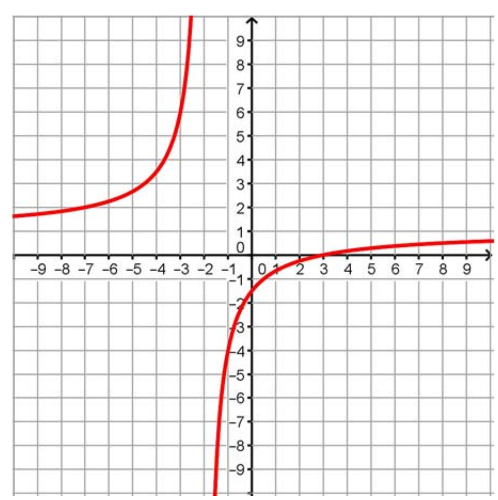
Domain: $(-\infty, -2) \cup (-2, \infty)$

Range: $(-\infty, 1) \cup (1, \infty)$

Interval(s) of Increase: $(-\infty, -2) \cup (-2, \infty)$

Interval(s) of Decrease: **NA**

End Behavior:
As $x \to -\infty, f(x) \to 1$
As $x \to \infty, f(x) \to 1$



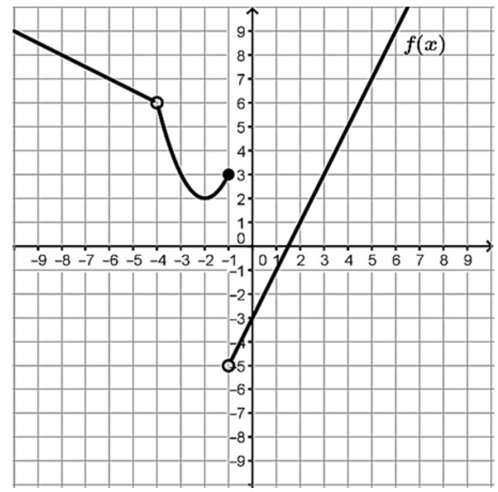
Name **Limits & Introduction to Derivatives | 8.2H**

Ready, Set, Go!

Ready

Topic: Reading function values from a graph

Use the graph at the right to answer each question.



1. $f(-2) =$ **2**

2. $f(1) =$ **−1**

3. $f(-4) =$ **undefined**

4. $f(-1) =$ **3**

5. Find the value(s) of x when $f(x) = 7$.

   $\boldsymbol{x = -6}$ & $\boldsymbol{5}$

6. Find the value(s) of x when $f(x) = 1$.

   $\boldsymbol{x = 2}$

7. Find the value(s) of x when $f(x) = 3$.

   $\boldsymbol{x = -3, -1, 3}$

Set

Topic: Average speed

8. Roger is an adventure seeker who loves the thrill of cliff diving. His most famous dive is off of a cliff into Lake Champlain in Red Rocks Park, Vermont.

   Unfortunately, Roger has sprained his wrist. Roger's doctor is concerned that he might do further damage to his wrist if he hits the water at speeds greater than 58 meters per second.

   The cliff at Red Rocks Park is 89 meters high. Roger always begins his dive with a jump, so he actually starts his fall from a height of 90 meters. Therefore, his height above the lake is given by the formula $h(t) = 90 - 10t^2$ where t is the time (in seconds) from when he begins to fall towards the water and $h(t)$ is the height (in meters) above the lake.

   a. How high above the lake is Roger 1 second after he begins his fall?

      80 meters

   b. What is the value of t when Roger hits the water?

      3 seconds

   c. What is Roger's average speed during the **final second** of his dive?

      50 meters per second

   d. What is Roger's average speed during the **final half-second** of his dive?

      55 meters per second

   e. Can Roger perform his famous dive without violating hid doctor's instructions? Explain.

      No because he would be going close to 60 meters per second when he hits the water.

9. Cindy is the star runner of her school's track team. Among other events, she runs that last 400 meters of the 1600-meter relay race.

   Cindy's coach studied the video of one of her races. He came up with the formula $m(t) = 0.1t^2 + 3t$ to describe the distance Cindy had run at given times in the race.

   In this formula, $m(t)$ gives the number of meters Cindy had run after t seconds, with time and distance measured from the beginning on her 400-meter segment of the race. (Adapted from Interactive Mathematics Program, Year 3)

   a. How long did it take Cindy to finish her leg of the relay race? Explain how you found your answer.
   50 seconds, find $t > 0$ where $m(t) = 400$

   b. The coach photographed Cindy at the instant she crossed the finish line. The photo is slightly blurred, so you know Cindy was going pretty fast, but you can't tell her exact speed at the instant the photo was taken. Find Cindy's speed at that instant.

   Approximately 13 meters per second

   c. Find Cindy's speed at three other instants during the race.

   Answers will vary. Students should note that Cindy's speed increases throughout the race.

   d. Was there an instant when Cindy was going exactly 10 meters per second? If so, when was that instant?

   She will run exactly 10 meters per second 35 seconds into the race.

Go

Topic: Solving trigonometric equations

Solve each trigonometric equation over the domain $[0, 2\pi)$.

10. $4\sin^2 x + 5 = 6$

$x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$

11. $2\cos(2x) - 1 = 1$

$x = 0, \pi$

12. $\cos(2x) - 1 = 3\cos x$

$x = \frac{2\pi}{3}, \frac{4\pi}{3}$

13. $5\sin(2x) - 6\sin x = 0$

$x = 0, \pi, 0.927, 5.356$

14. $2\sec^2 x - 4 = 0$

$x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

15. $\csc\left(\frac{3x}{2}\right) + 2 = 0$

$x = \frac{7\pi}{9}, \frac{11\pi}{9}$

16. $\cot x \sec x + \cot x = 0$

$x = \pi$

17. $\sec x \csc x = 2\csc x$

$x = \frac{\pi}{3}, \frac{5\pi}{3}$

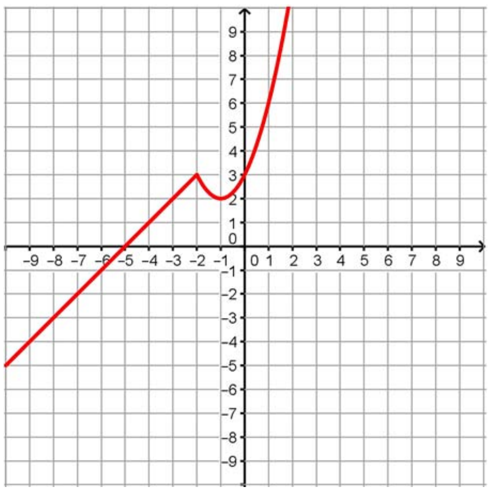
Ready, Set, Go!

Ready

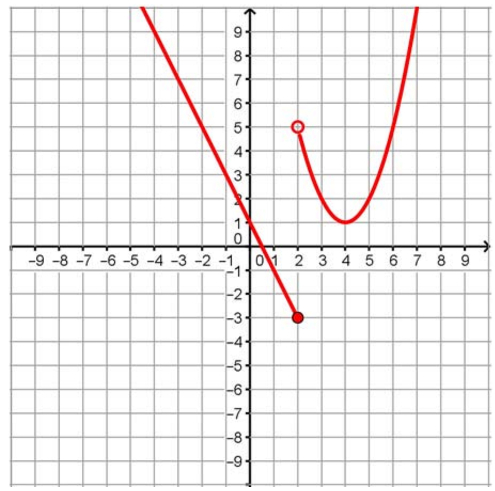
Topic: Graphing piecewise functions

Graph each piecewise function.

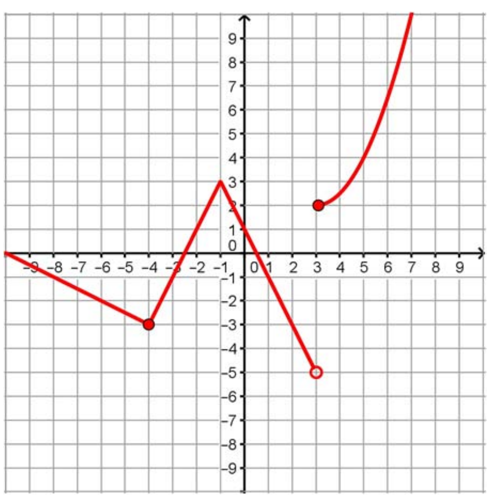
1. $f(x) = \begin{cases} x+5, & x < -2 \\ x^2 + 2x + 3, & x \geq -2 \end{cases}$



2. $f(x) = \begin{cases} -2x+1, & x \leq 2 \\ (x+4)^2 + 1, & x > 2 \end{cases}$



3. $f(x) = \begin{cases} -\frac{1}{2}(x+4) - 3, & x < -4 \\ -2|x+1| + 3, & -4 \leq x < 3 \\ \frac{1}{2}(x-3)^2 + 2, & x \geq 3 \end{cases}$



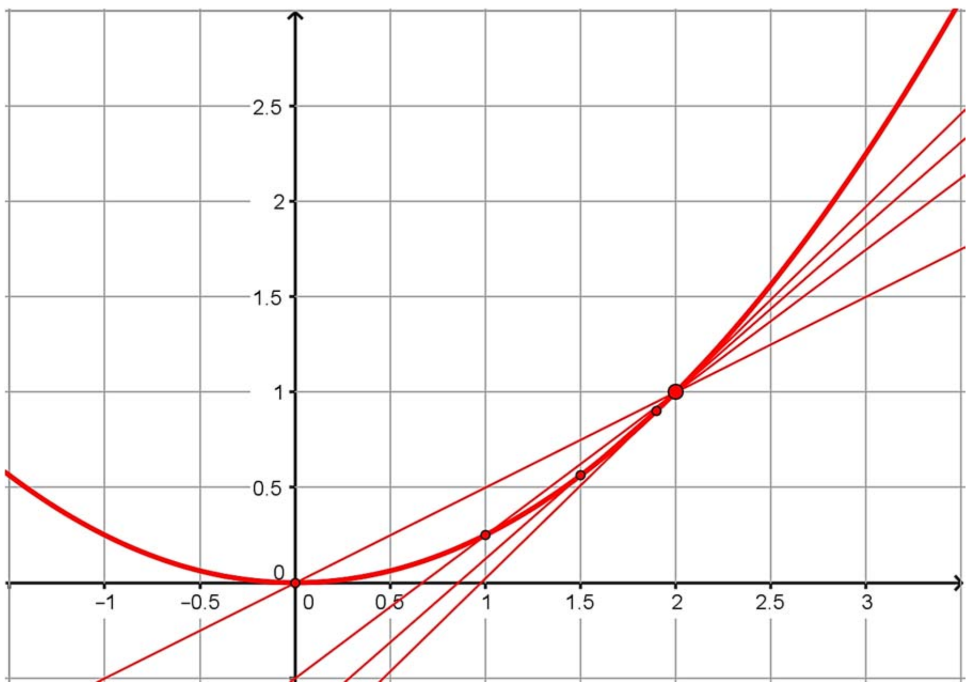
Set

Topic: Using secants to find the derivative of a function at a point.

A **secant line** for the graph of a function is the line (or line segment) connecting two points on the graph. A **tangent line** is a line that "just touches" the graph at a point (some tangent lines may cross the curve elsewhere). In the following problems, you will explore these two geometric concepts and their connections with derivatives.

4. Consider the function f defined by the equation $f(x) = 0.25x^2$.

a. Sketch the graph of this function. Label the point $(2, 1)$ on your graph.



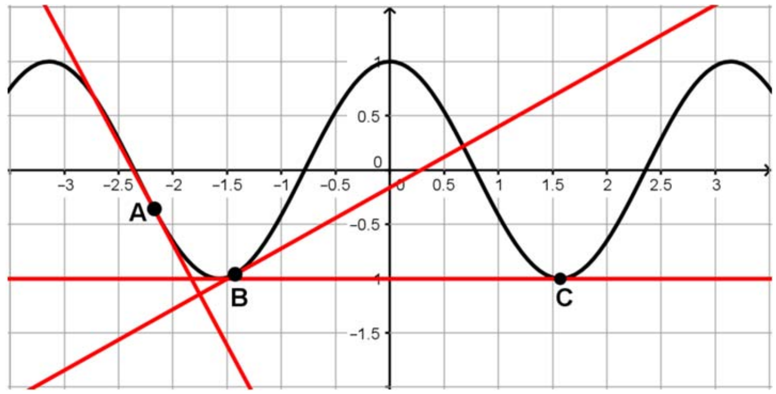
b. The points listed below are also on your graph. In each case, use a straight edge to draw the secant line connecting the point to $(2, 1)$ and find the slope of that secant line.

i. $(0, 0)$	ii. $(1, 0.25)$	iii. $(1.5, 0.5625)$	iv. $(1.9, 0.925)$
0.5	**0.75**	**0.875**	**0.975**

c. Draw the line that is tangent to your graph at $(2, 1)$. Estimate the slope of that tangent line and explain your reasoning.
The slope appears to be 1. The slope of the secant line, as it approaches the point $(2, 1)$, is approaching the value of 1.

d. Find the derivative of the function f at the point $(2, 1)$.
1

5. Consider the graph of the function at the right.



   a. Draw the tangent lines to the graph at the points A, B, and C.

   b. Use your tangent lines to estimate the derivative of the function at each of the points.

   A: approximately -2

   B: approximately 0.5

   C: 0

Go

Topic: Solving exponential and logarithmic equations

Solve each equation.

6. $3^{x-1} = 81$

   $\boldsymbol{x = 5}$

7. $2e^x + 4 = 5$

   $\boldsymbol{x = \ln\frac{1}{2} \approx -0.6931}$

8. $\log(3x+1) = 2$

   $\boldsymbol{x = 33}$

9. $\log_3(2x+1) = 2$

   $\boldsymbol{x = 4}$

10. $5^{x+2} = 4$

$\boldsymbol{x = -2 + \log_5 4 \approx -1.1386}$

11. $\log_2(x+5) - \log_2(x-2) = 3$

$\boldsymbol{x = 3}$

12. $4\ln(2x+3) = 11$

$\boldsymbol{x = -\frac{3}{2} + \frac{1}{2}e^{\frac{11}{4}} \approx 6.3213}$

13. $2e^{2x} - 5e^{x} - 3 = 0$

$\boldsymbol{x = \ln 3 \approx 1.0986}$

Ready, Set, Go!

Ready

Topic: Domain and range

Write the domain and range of each function in interval notation.

1. $f(x) = \frac{2x-3}{x+5}$

   Domain: $(-\infty, -\mathbf{5}) \cup (-\mathbf{5}, \infty)$

   Range: $(-\infty, \mathbf{2}) \cup (\mathbf{2}, \infty)$

2. $f(x) = \sqrt{x-2} + 5$

   Domain: $[\mathbf{2}, \infty)$

   Range: $[\mathbf{5}, \infty)$

3. $f(x) = \frac{x-1}{(x+2)(x-6)}$

   Domain: $(-\infty, -\mathbf{2}) \cup (-\mathbf{2}, \mathbf{6}) \cup (\mathbf{6}, \infty)$

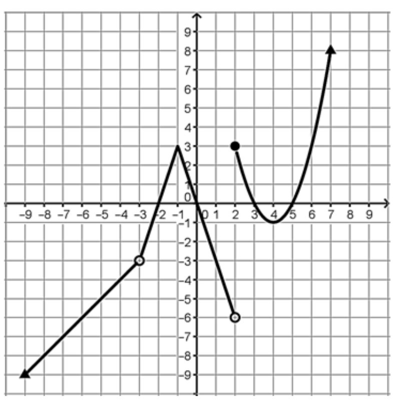
   Range: $(-\infty, \infty)$

4. $f(x) = 2(x-5)^2 - 8$

   Domain: $(-\infty, \infty)$

   Range: $[-\mathbf{8}, \infty)$

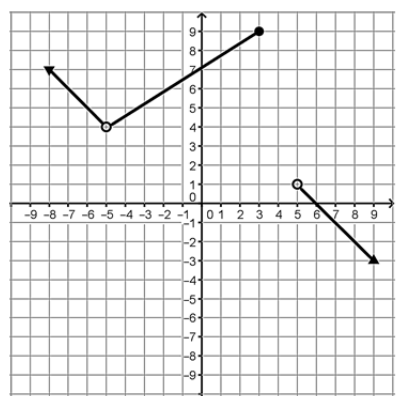
5.



   Domain: $(-\infty, -\mathbf{3}) \cup (-\mathbf{3}, \infty)$

   Range: $(-\infty, \infty)$

6.



   Domain: $(-\infty, -\mathbf{5}) \cup (-\mathbf{5}, \mathbf{3}] \cup (\mathbf{5}, \infty)$

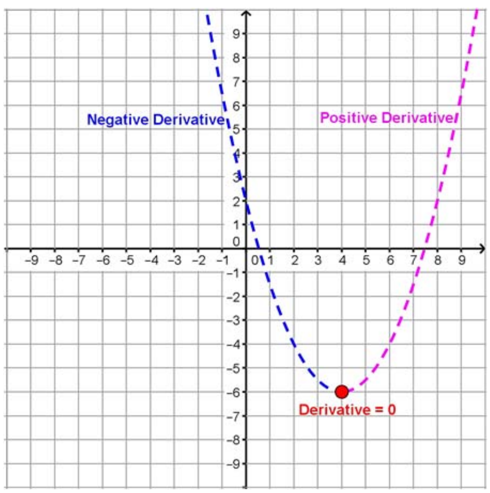
   Range: $(-\infty, \mathbf{1}) \cup (\mathbf{4}, \infty)$

Topic: Using graphs to determine the signs of the derivative functions.

When you are graphing a function, knowing the signs of the coordinates of the points can be helpful. For instance, those signs tell you which quadrant a point is in. And if one of the coordinates is 0, you know that the point is on a coordinate axis.

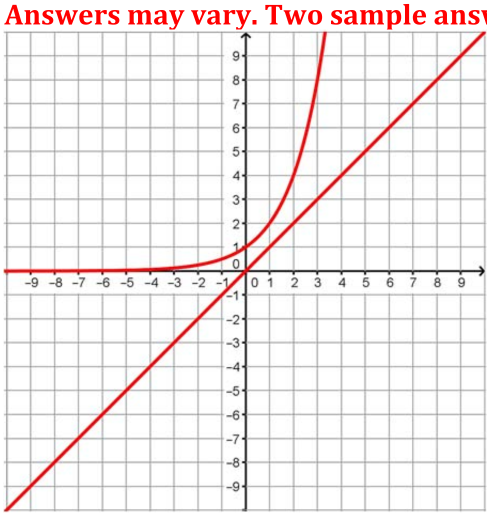
In the following questions, you'll explore similar issues concerning the ***sign of the derivative of a function***.

7. Using three different colors, identify where on the graph the function's derivative is positive, where the derivative is negative, and where the derivative is 0. ***Remember that the slope of a tangent line at a point on the graph is the same as the derivative at the point.***



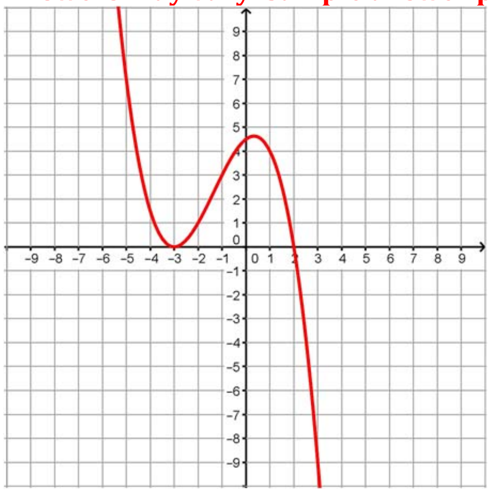
8. Sketch the graph of a function for which the derivative is positive for all values of *x*.

Answers may vary. Two sample answers provided.



9. Sketch the graph of a function for which there are exactly two points where the derivative is 0.

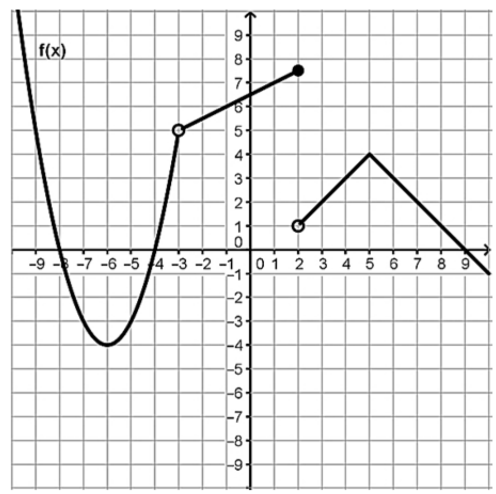
Answers may vary. Sample answer provided.



Set

Topic: Finding limits using tables and graphs

Use the graph at the right to answer the following questions about $f(x)$.



10. $\lim_{x\to -5} f(x) = \mathbf{-3}$

11. $\lim_{x\to -3} f(x) = \mathbf{5}$

12. $\lim_{x\to 2} f(x) =$ **does not exist**

13. $f(-5) = \mathbf{-3}$

14. $f(-3) =$ **undefined**

15. $f(2) = \mathbf{7.5}$

Complete the tables of values for each function to determine the value of the limit.

16. $f(x) = \frac{x-2}{x^2-4}$

$\lim_{x \to 2} \frac{x-2}{x^2-4} = \mathbf{0.25}$

x	$f(x)$
1.5	**0.28571**
1.9	**0.25641**
1.99	**0.25063**
2	**Undefined**
2.01	**0.24938**
2.1	**0.2439**
2.5	**0.2222**

17. $g(x) = \frac{\frac{1}{x+1} - \frac{1}{4}}{x-3}$

$\lim_{x \to 3} \frac{\frac{1}{x+1} - \frac{1}{4}}{x-3} = \mathbf{-0.0625}$

x	$g(x)$
2.5	**−0.0714**
2.9	**−0.0641**
2.99	**−0.0627**
3	**Undefined**
3.01	**−0.0623**
3.1	**−0.061**
3.5	**−0.0556**

Go

Topic: Factoring polynomial expressions

Factor each expression completely.

18. $x^2 - 9x + 8$

$\mathbf{(x-8)(x-1)}$

19. $2x^2 + 17x + 21$

$\mathbf{(2x+3)(x+7)}$

20. $28x^4 + 16x^3 - 80x^2$

$\mathbf{4x^2(7x-10)(x+2)}$

21. $5x^2 - x - 18$

$\mathbf{(5x+9)(x-2)}$

22. $64x^2 - 81$

$\mathbf{(8x-9)(8x+9)}$

23. $16x^2 - 24x + 9$

$\mathbf{(4x-3)^2}$

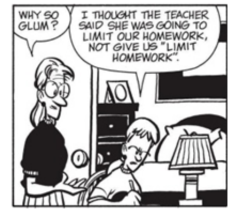
24. $3x^3 - 5x^2 + 2x$

$\mathbf{x(3x-2)(x-1)}$

25. $12x^3 + 2x^2 - 2x$

$\mathbf{2x(3x-1)(2x+1)}$

Name

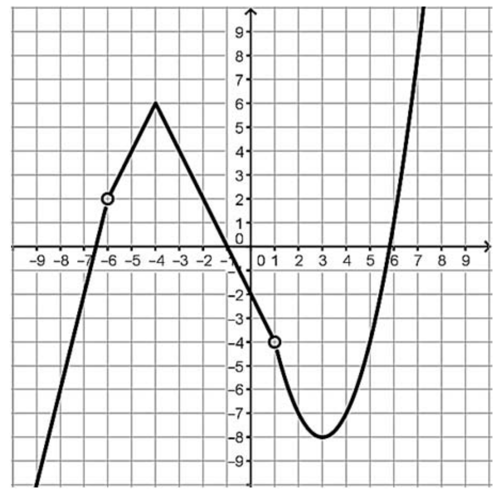


Ready, Set, Go!

Ready

Topic: Finding limits using graphs and tables

Use the graph of $f(x)$ at the right to find the following limits and function values.



1. $\lim_{x\to -8} f(x) = $ **−6**

2. $\lim_{x\to -6} f(x) = $ **2**

3. $\lim_{x\to 1} f(x) = $ **−4**

4. $f(-6) = $ **undefined**

5. $f(-3) = $ **4**

6. $f(5) = $ **−4**

Find the limit of each function represented by the tables below.

7. $\lim_{x\to 1} f(x) = $ **5**

x	$f(x)$
0.5	4.5
0.8	4.8
0.9	4.9
0.999	4.99
1	Undefined
1.001	5.001
1.1	5.1
1.2	5.2
1.5	5.5

8. $\lim_{x\to -4} g(x) = $ **1**

x	$g(x)$
−4.5	1.0909
−4.2	1.0385
−4.1	1.0196
−4.001	1.0002
−4	Undefined
−3.999	0.9998
−3.9	0.9796
−3.8	0.9583
−3.5	0.8889

Set

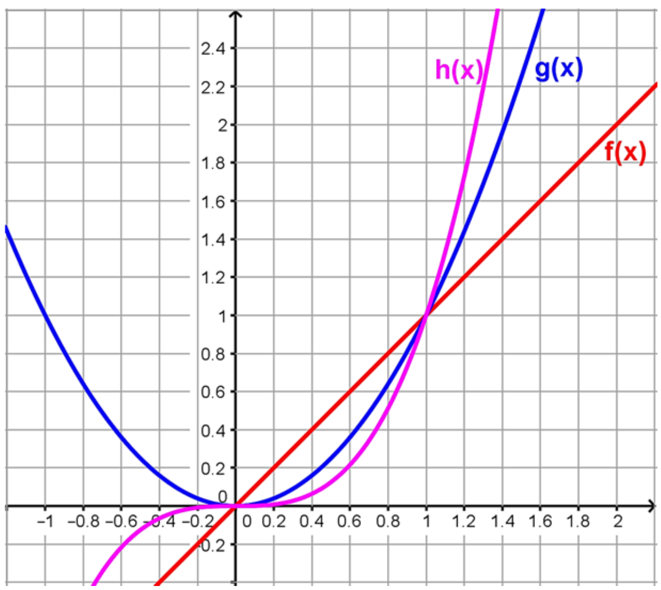
Topic: Comparing derivatives at points

Below are the equations for three functions whose graphs all past through the points $(0, 0)$ and $(1, 1)$.

$$f(x) = x$$
$$g(x) = x^2$$
$$h(x) = x^3$$

The following questions will have you investigate whether or not these functions have the same derivatives at $(0, 0)$ and $(1, 1)$.

9. Draw graphs of all three functions on the same set of axes. Plot enough points for each function (including non-integer values of x) to get accurate graphs. Take particular care in plotting values of x between 0 and 1.



10. **Based on your graphs, answer each of the following questions and explain your reasoning.**
    a. Which of the three functions has the greatest derivative at the point $(0, 0)$?

    $f(x)$

    b. Which of the three functions has the greatest derivative at the point $(1, 1)$?

    $h(x)$

11. Find the actual derivative of each function at the points $(0, 0)$ and $(1, 1)$. How do they compare with your answers to question 10? (Reminder: the derivative, $f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$)

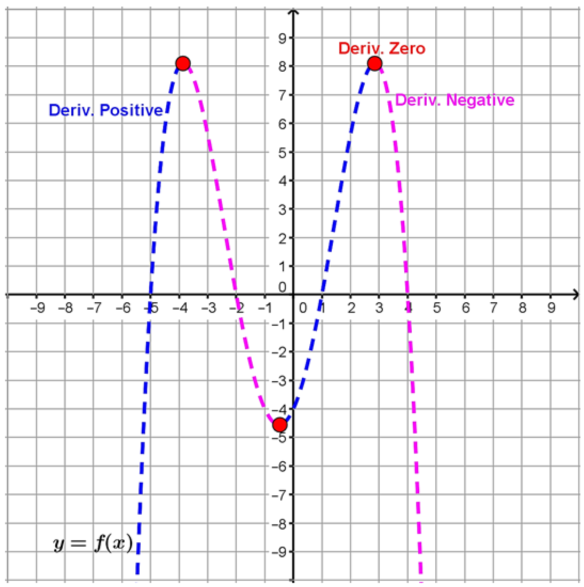
Derivatives at $(0, 0)$:
$f(x)$: **1**
$g(x)$: **0**
$h(x)$: **0**

Derivatives at $(1, 1)$:
$f(x)$: **1**
$g(x)$: **2**
$h(x)$: **3**

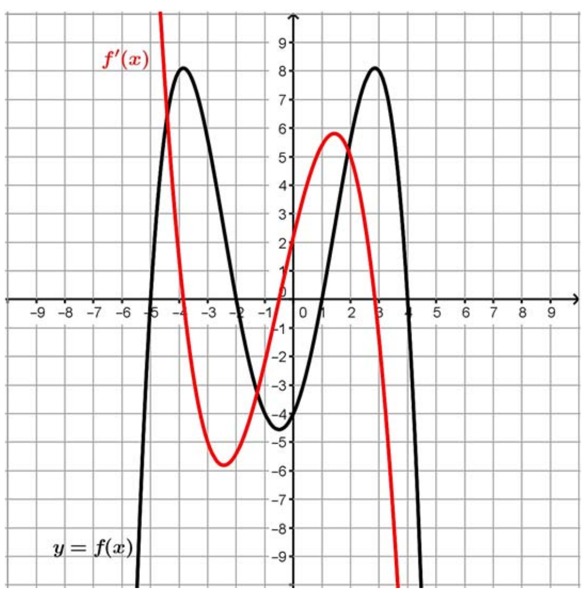
Topic: Graphing a derivative using a given graph.

Let the graph below represents the function defined by the equation $y = f(x)$.

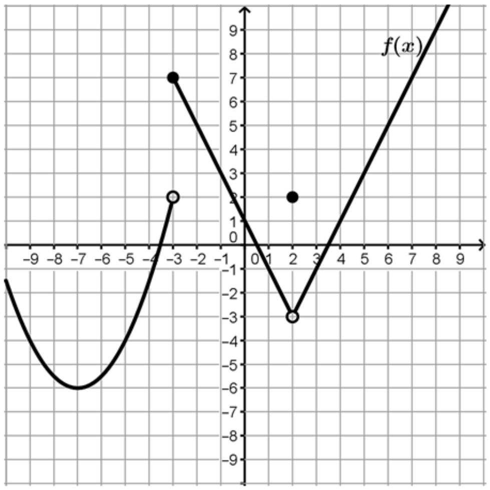
12. Using three different colors, identify where on the graph the function's derivative is positive, where the derivative is negative, and where the derivative is 0.



13. Sketch the graph of the derivative of f.



14. Use the graph below to discuss the continuity of the function. State the type of discontinuity (removable or nonremovable) and explain why one (or more) of the conditions for continuity is not met.



At $x = -3$, there is a nonremovable disconituity because $\lim_{x \to -3} f(x)$ does not exist.
At $x = 2$, there is a removable discontinuity because $\lim_{x \to 2} f(x) \neq f(2)$.

Go

Topic: Solving radical, rational, and quadratic equations

Solve each equation.

15. $-8 + \sqrt{5x - 5} = -3$

$\boldsymbol{x = 6}$

16. $-12 = -6\sqrt{x + 4}$

$\boldsymbol{x = 0}$

17. $\sqrt{3x} = \sqrt{4x - 1}$

$\boldsymbol{x = 1}$

18. $x = \sqrt{42 - x}$

$\boldsymbol{x = 6}$
$x = -7$ is extraneous

19. $\frac{1}{x^2-7x+10}+\frac{1}{x-2}=\frac{2}{x^2-7x+10}$

$\boldsymbol{x=6}$

20. $\frac{3}{2x}-\frac{2x}{x+1}=-2$

$\boldsymbol{x=-\frac{3}{7}}$

21. $x-\frac{2}{x-3}=\frac{x-1}{3-x}$

$\boldsymbol{x=-1}$
$\boldsymbol{x=3}$ **is extraneous**

22. $x^2+6x+13=0$

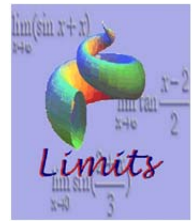
$\boldsymbol{x=-3\pm 2i}$

23. $x^4+13x^2+36=0$

$\boldsymbol{x=\pm 3i, \pm 2i}$

24. $3x^2-16x-7=5$

$\boldsymbol{x=-\frac{2}{3}, 6}$



Ready, Set, Go!

Ready

Topic: End behavior

Describe the end behavior of each function.

1. $f(x) = \frac{1}{2}(x-4)^2 - 8$

   as $x \to -\infty, f(x) \to \infty$

   as $x \to \infty, f(x) \to \infty$

2. $f(x) = \frac{2x-1}{6x+5}$

   as $x \to -\infty, f(x) \to \frac{1}{3}$

   as $x \to \infty, f(x) \to \frac{1}{3}$

3. $f(x) = \sqrt{x+9} - 6$

   as $x \to -9, f(x) \to -6$

   as $x \to \infty, f(x) \to \infty$

4. $f(x) = \frac{2x}{(x-4)(x+2)}$

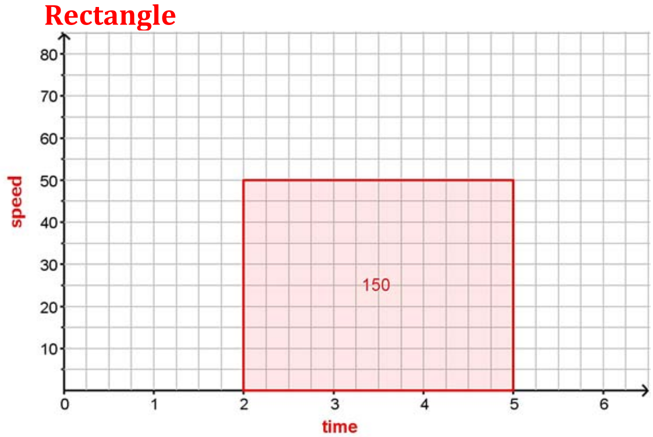
   as $x \to -\infty, f(x) \to 0$

   as $x \to \infty, f(x) \to 0$

Topic: Using areas to connect distance, rate, and time

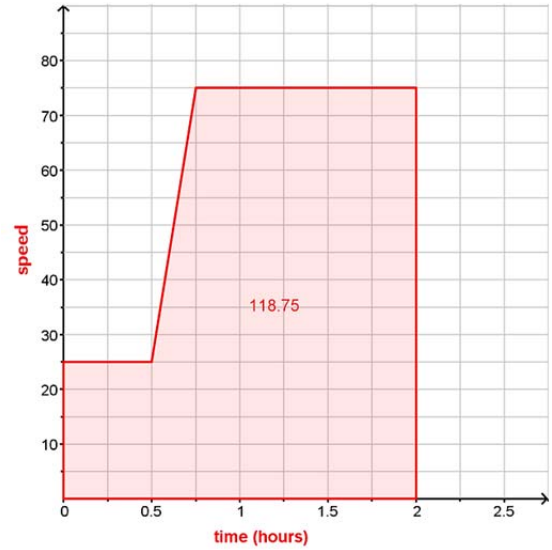
5. Burton travels on cruise control at 50 mph from 2:00 PM until 5:00 PM.
   a. How far has he traveled? **150 miles**

   b. Sketch a graph of the previous information using speed in mph and time in hours. Shade the area under the graph in the first quadrant. What shape is the shaded figure?
   Rectangle



   c. Explain why the shaded area represents a distance of 150 miles.
   Distance is the product of time (hours) and velocity (miles per hour)

6. Erin started driving from Sacramento to San Francisco. For the first half hour she drove through residential neighborhood and traveled at a constant speed of 25 miles per hour. She then got on the freeway, only to encounter heavy traffic. She was, however, able to slowly increase her speed at a constant rate until she reached a speed of 75 miles per hour, 45 minutes into her trip. She continued at that speed until she got to her destination after a total of 2 hours of driving. Assume she made no stops along the way.



   a. Graph the situation carefully on the grid at right.

   b. Using the units from each axis, what are the resulting units when you find the area?

      Miles

   c. How far did Erin travel?

      118.75 miles

Set

Topic: Finding limits using algebraic methods

Find the value of each limit.

7. $\lim_{x\to 3} \frac{x^2-9}{x-3}$

   6

8. $\lim_{x\to 0} \frac{\sqrt{x^2+4}-2}{x^2}$

   $\mathbf{\frac{1}{4}}$

9. $\lim_{x\to 1} \frac{x^4-1}{x^2-1}$

   2

10. $\lim_{x\to 4} \frac{\sqrt{x}-2}{x-4}$

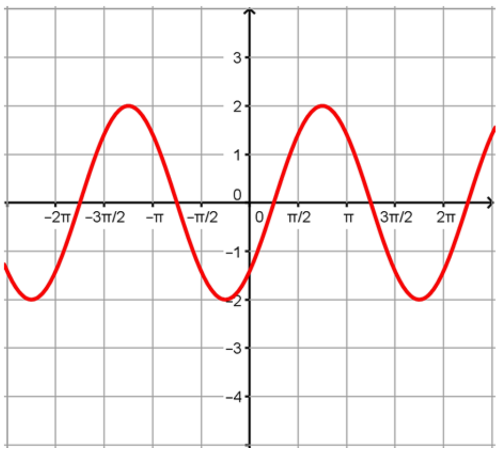
   $\mathbf{\frac{1}{4}}$

Go

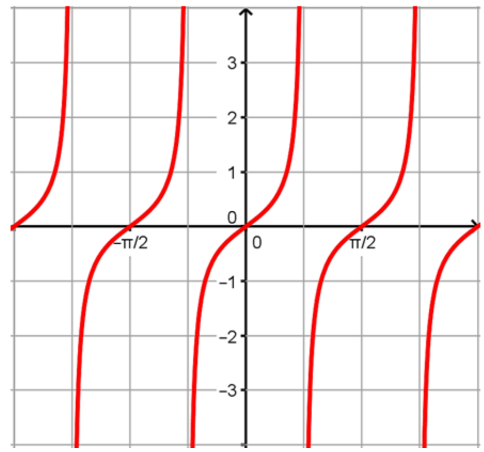
Topic: Graphing trigonometric functions

Graph at least two periods of each trigonometric function.

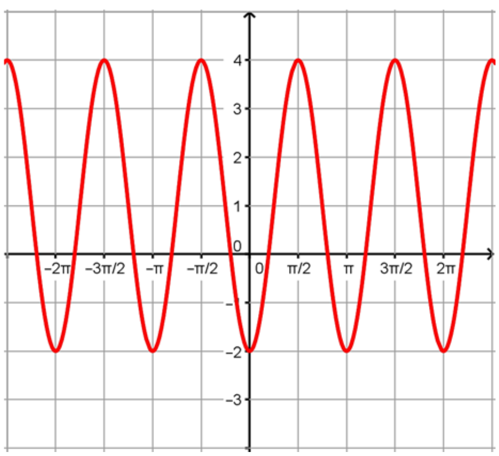
11. $y = 2\sin\left(x - \frac{\pi}{4}\right)$



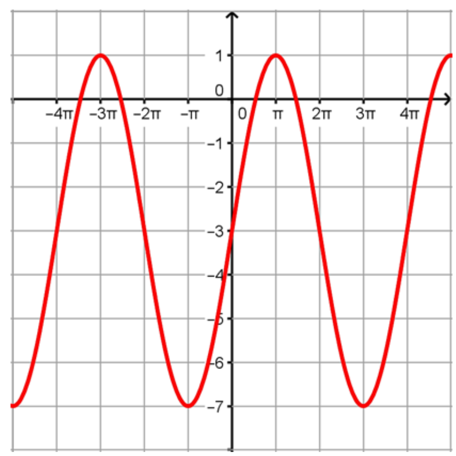
12. $y = \frac{1}{2}\tan 2x$



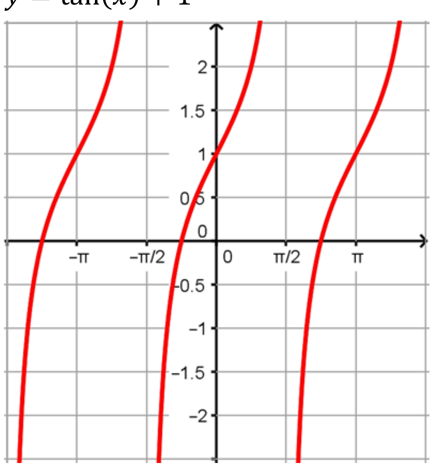
13. $y = -3\cos(2x) + 1$



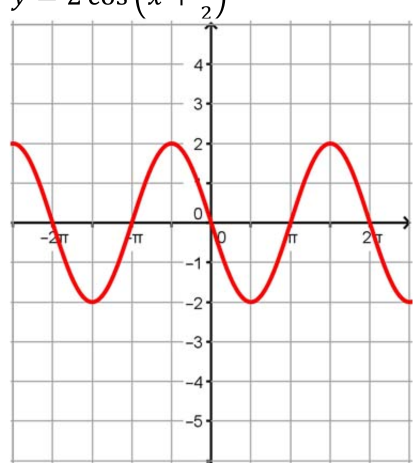
14. $y = 4\sin\left(\frac{1}{2}x\right) - 3$



15. $y = \tan(x) + 1$



16. $y = 2\cos\left(x + \frac{\pi}{2}\right)$



Ready, Set, Go!

Ready

Topic: Evaluating functions

For each function below, find the value of $\frac{f(x+h)-f(x)}{h}$. An example is provided for you.

Example: If $f(x)=3x+7$, then $\frac{f(x+h)-f(x)}{h}=\frac{3(x+h)+7-(x+7)}{h}=\frac{3x+3h+7-x-7}{h}=\frac{3h}{h}=3$

1. $f(x)=x^2$

   $\mathbf{2x+h}$

2. $f(x)=\sqrt{x}$

   $\frac{1}{\sqrt{x+h}+\sqrt{x}}$

3. $f(x)=\frac{1}{x}$

   $-\frac{1}{x(x+h)}$

4. $f(x)=2x^2+3$

   $\mathbf{4x+2h}$

Set

Topic: Finding limits

Find the values of the following limits.

5. $\lim_{x\to 6}\frac{x^2-6x}{x^2-7x+6}$

   $\frac{6}{5}$

6. $\lim_{x\to 5}\frac{\sqrt{x+4}-3}{x-5}$

   $\frac{1}{6}$

7. $\lim_{x \to \frac{3}{2}} \frac{2x^2+3x-9}{2x-3}$

$\frac{9}{2}$

8. $\lim_{x \to 0} \frac{\sqrt{x}-1}{x-1}$

1

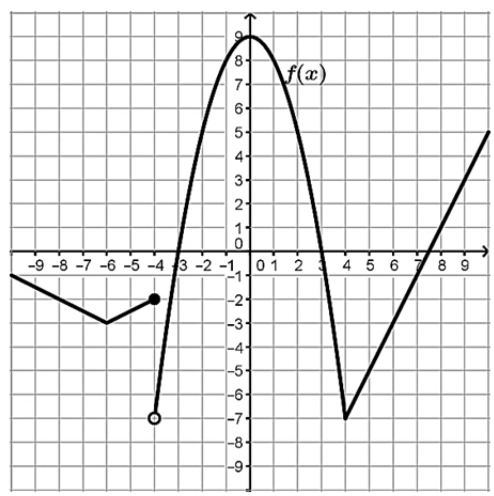
9. $\lim_{x \to -4} \frac{\frac{8}{x+6}-4}{x+4}$

-2

10. $\lim_{x \to 2} \frac{2-x}{\frac{2}{x}-1}$

2

Use the graph at the right to find the value of each limit.



11. $\lim_{x \to -6} f(x) = -3$

12. $\lim_{x \to -4^-} f(x) = -2$

13. $\lim_{x \to -4^+} f(x) = -7$

14. $\lim_{x \to -4} f(x) =$ **does not exist**

15. $\lim_{x \to 0} f(x) = 9$

16. $\lim_{x \to 3^-} f(x) = 0$

17. $\lim_{x \to 3^+} f(x) = 0$

18. $\lim_{x \to 3} f(x) = 0$

19. $\lim_{x \to -\infty} f(x) = \infty$

20. $\lim_{x \to \infty} f(x) = \infty$

Go

Topic: Simplifying rational expressions

Simplify each expression as much as possible.

21. $\frac{x^2+3x-10}{3x^2+24x+45}$

$\frac{x-2}{3(x+3)}$

22. $\frac{x^2-16}{2x^2+7x-4}$

$\frac{x-4}{2x-1}$

23. $\frac{4x^2+12x+9}{8x^2+6x-9}$

$\frac{2x+3}{4x-3}$

24. $\frac{3x^2-13x-30}{2x^2+4x-96}$

$\frac{3x+5}{2(x+8)}$

25. $\frac{8x^2+10x-3}{2x^2-11x-21}$

$\frac{4x-1}{x-7}$

26. $\frac{9x^2-25}{3x^2+23x+30}$

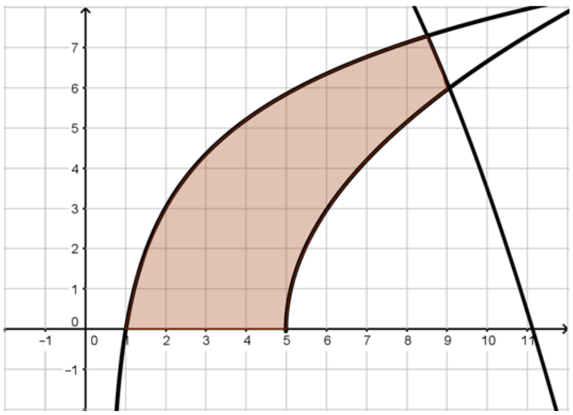
$\frac{3x-5}{x+6}$

Ready, Set, Go!

Ready

Topic: Estimating areas

1. The shaded region below has sides made up of three curves and the x-axis. Estimate the area of the region.



Answers will vary. Approximately 27 square units.

Topic: Using sigma notation

Recall:

$$\sum_{i=1}^{4} 2i + 4 = [2(1) + 4] + [2(2) + 4] + [2(3) + 4] + [2(4) + 4] = 36$$

2. Find the sum:

$$\sum_{i=1}^{3} 4i - 6 =$$

6

3. Rewrite each in sigma (summation) notation:

a. $14 + 20 + 26 + 32 + 38$

$$\sum_{i=1}^{5} 14 + 6(i - 1)$$

or

$$\sum_{i=1}^{5} 8 + 6i$$

b. $[(14)^2+3]+[(20)^2+3]+[(26)^2+3]+[(32)^2+3]+[(38)^2+3]$

$$\sum_{i=1}^{5}[14+6(i-1)]^2+3$$

or

$$\sum_{i=1}^{5}(8+6i)^2+3$$

Set

Topic: Finding derivatives of functions using the limit process

Find the derivative of each function using the limit definition:

$$\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

4. $f(x)=3x^2-2x$

$6x-2$

5. $f(x)=\sqrt{x-4}$

$\frac{1}{2\sqrt{x-4}}$

6. $f(x)=\frac{2}{x}$

$-\frac{2}{x^2}$

7. $f(x)=-6x+5$

-6

8. $f(x)=2x^2+5$

$4x$

9. $f(x)=\sqrt{2x+1}$

$\frac{1}{\sqrt{2x+1}}$

10. Use the derivative of $f(x)=3x^2-2x$ (question 4) to find when $f(x)$ has a slope of 0. What feature of the graph of $f(x)$ is at this location?

$x=\frac{1}{3}$, **this occurs at the vertex of the parabola**

Go

Topic: Solving quadratic and rational inequalities

Solve each quadratic and rational inequality. Write your answers in interval notation.

11. $x^2 + 4x + 3 \leq 0$

$[\mathbf{-3}, \mathbf{-1}]$

12. $5x^2 + 10 \geq 27x$

$\left(-\infty, \frac{2}{5}\right] \cup [\mathbf{5}, \infty)$

13. $-2x^2 + 5x + 12 < 0$

$\left(-\infty, -\frac{3}{2}\right) \cup (\mathbf{4}, \infty)$

14. $4x^2 > 9$

$\left(-\infty, -\frac{3}{2}\right) \cup \left(\frac{3}{2}, \infty\right)$

15. $\frac{3x-2}{x+4} < 2$

$(\mathbf{-4}, \mathbf{10})$

16. $\frac{2x-8}{x-2} \geq 0$

$(-\infty, \mathbf{2}) \cup [\mathbf{4}, \infty)$

17. $\frac{x^2-2x-8}{x+4} \geq 0$

$(\mathbf{-4}, \mathbf{2}] \cup [\mathbf{4}, \infty)$

18. $\frac{4x-8}{x-7} < 0$

$[\mathbf{2}, \mathbf{7})$

Ready, Set, Go!

Ready

Topic: Infinite limits

1. **Find each limit as $n \to \infty$ (Reminder, the limit as $n \to \infty$ is similar to finding end behavior):**

a. $\frac{5n^2+3n+1}{n^2+5}$

5

b. $\frac{3n(n+1)}{2n^2}$

$\frac{3}{2}$

c. $\frac{3n}{n} + \frac{n(n+1)}{2} \cdot \frac{1}{n^2}$

$\frac{7}{2}$

d. $10 - \frac{5n(n+1)}{2n^2}$

$\frac{15}{2}$

e. $\left[6n + \frac{4n(n+1)}{2n} - \frac{4n(n+1)(2n+1)}{6n^2}\right] \cdot \frac{1}{n}$

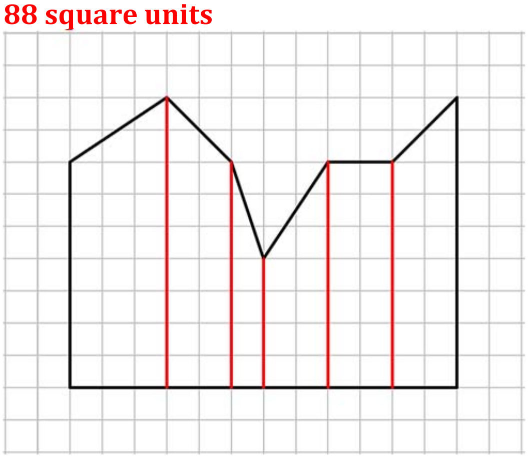
$\frac{20}{3}$

Topic: Area of composite regions

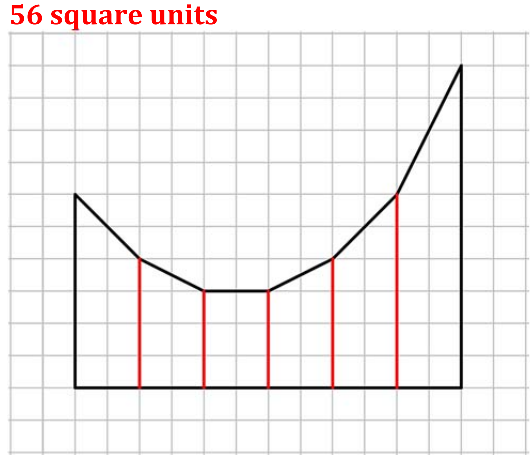
Find the area of the entire shape.
Use only vertical segments and quadrilaterals in your figure dissections.

2.

a. **88 square units**



b. **56 square units**



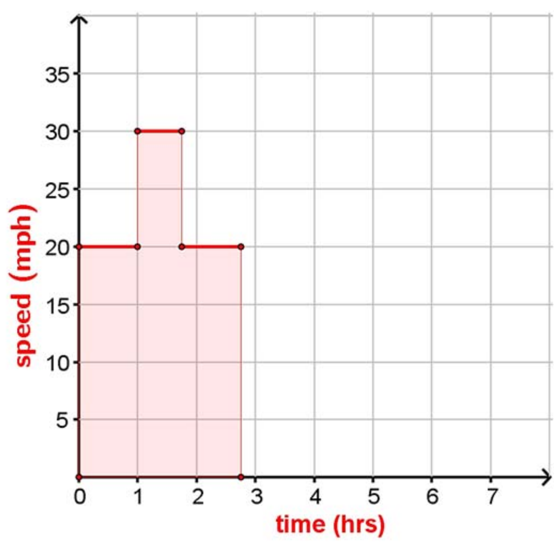
Set

Topic: Distance as an area

Michelle was training for her next triathlon. According to her training schedule she needed to ride 75 miles on her bike this upcoming weekend Unfortunately, the weather report is calling for heavy rain so Michelle will have to do her bike ride on a stationary bike. The stationary bike Michelle uses only shows the "speed" of the bike.

3. Michelle noticed that she was able to keep a steady pace of 20 mph for the first hour she was on the stationary bike. She was able to increase her speed to 30 mph for the next 45 minutes. She then slowed to 20 mph for the next hour.

   a. Draw a graph of this situation. Be sure to label and scale the axes appropriately.



   b. Find the area under the curve of your graph.

   62.5 square units

   c. How far has Michelle gone?

   62.5 miles

   d. If she wants to be done with her work out in 15 minute, how fast should she go in order to complete her 75 mile training ride? Does this seem reasonable?

   50 mph, this does not seem reasonable

Go

Topic: Limits

Find the values of the following limits.

4. $\lim_{x \to 2} \frac{3x^2 - x - 10}{x^2 - 4}$

$\frac{11}{4}$

5. $\lim_{x \to 4} \frac{3 - \sqrt{x+5}}{x - 4}$

$-\frac{1}{6}$

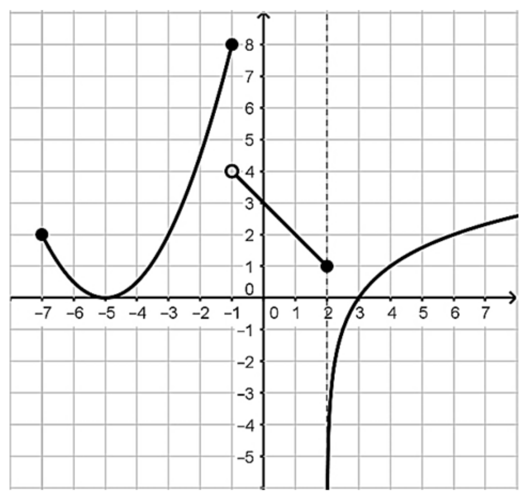
6. $\lim_{x \to 3} \frac{x^4 - 81}{2x^2 - 5x - 3}$

$\frac{108}{7}$

7. $\lim_{h \to 0} \frac{\sqrt{3h+1} - 1}{h}$

$\frac{3}{2}$

Use the graph at the right to find the value of each limit.



8. $\lim_{x \to -5} f(x) = \mathbf{0}$

9. $\lim_{x \to -7^+} f(x) = \mathbf{2}$

10. $\lim_{x \to -1^+} f(x) = \mathbf{4}$

11. $\lim_{x \to -1^-} f(x) = \mathbf{8}$

12. $\lim_{x \to -1} f(x) =$ **DNE**

13. $\lim_{x \to 2^-} f(x) = \mathbf{1}$

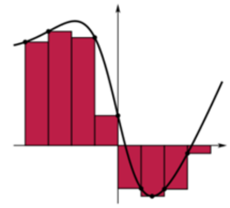
14. $\lim_{x \to 2^+} f(x) = -\infty$

15. $\lim_{x \to 3} f(x) = \mathbf{0}$

16. $\lim_{x \to 0} f(x) = \mathbf{3}$

17. $\lim_{x \to \infty} f(x) = \infty$

Ready, Set, Go!



Ready

Topic: Sigma notation

Recall:

$$\sum_{i=1}^{4} 3i^2 + 2 = [3(1)^2 + 2] + [3(2)^2 + 2] + [3(3)^2 + 2] + [3(4)^2 + 2] = 98$$

1. **Find the sum:**

$$\sum_{i=1}^{4} (2+i)^3 - 1 =$$

428

2. **Rewrite each in sigma (summation) notation:**

a. $2.4 + 2.8 + 3.2 + 3.6 + 4$

$$\sum_{i=1}^{5} 2.4 + 0.4(i-1)$$

or

$$\sum_{i=1}^{5} 2 + 0.4i$$

b. $[(2.4)^2 + 1] + [(2.8)^2 + 1] + [(3.2)^2 + 1] + [(3.6)^2 + 1] + [(4)^2 + 1]$

$$\sum_{i=1}^{5} [2.4 + 0.4(i-1)]^2 + 1$$

or

$$\sum_{i=1}^{5} (2 + 0.4i)^2 + 1$$

Topic: Infinite limits

3. **Find each limit as $n \to \infty$ (Reminder, the limit as $n \to \infty$ is similar to finding end behavior):**

a. $\frac{54-14n^2+1}{3n^2+5}$

$-\frac{14}{3}$

b. $\frac{n(2n+1)}{6n^2}$

$\frac{1}{3}$

c. $\frac{2n}{n} + \frac{n(n+1)}{6} \cdot \frac{1}{n^2}$

$\frac{13}{6}$

d. $10 - \frac{5n(2n+1)}{2n^2}$

5

e. $\left[3n - \frac{4n(n+1)}{2n} + \frac{4n(n+1)(2n+1)}{6n^2}\right] \cdot \frac{1}{n}$

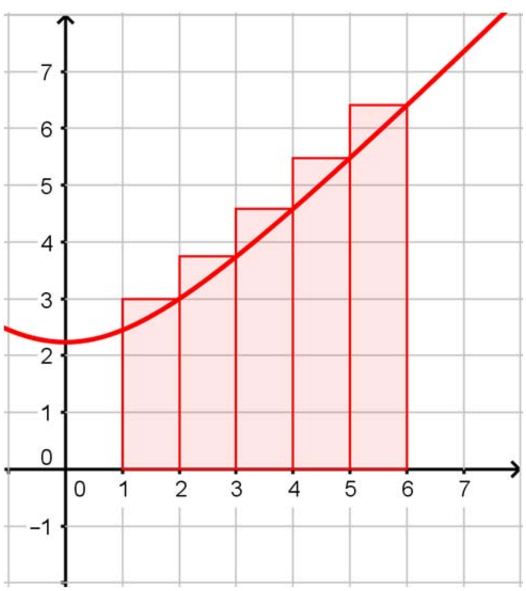
$\frac{7}{3}$

Set

Topic: Estimating areas using Riemann sums

4. Let $f(x) = \sqrt{x^2+5}$.

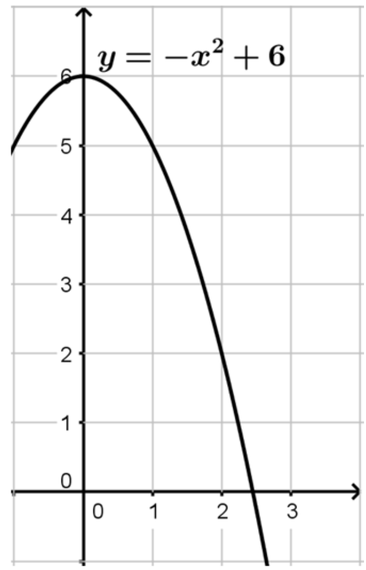
a. Graph the function over the domain $[1, 6]$ on the grid below.



b. Find an approximation of the area under the curve by using 5 right-endpoint rectangles of equal width.

$A \approx 23.2$

5. You are trying to find an approximation for $\int_1^2 6 - x^2\,dx$. The region is drawn for you at right.



a. Use 4 right-endpoint rectangles to find an approximation for the area.

$\boldsymbol{A \approx 3.28}$

b. Express the area approximation in sigma notation.

$$\boldsymbol{A \approx \sum_{i=1}^{4}[6 - (1 + 0.25\,i)^2](0.25)}$$

Go

Topic: Verifying trigonometric identities

Verify each trigonometric identity.

6. $\cot x + 1 = \csc x\,(\cos x + \sin x)$

Answers may vary

7. $\frac{\tan x - \cot x}{\sin x \cos x} = \sec^2 x - \csc^2 x$

Answers may vary

8. $\frac{\cos x}{1 - \sin x} = \frac{1 + \sin x}{\cos x}$

Answers may vary

9. $\frac{1 + \sin x}{\cos x} + \frac{\cos x}{1 + \sin x} = 2\sec x$

Answers may vary

10. $\tan^2 x \sin^2 x = \tan^2 x - \sin^2 x$

Answers may vary

11. $\csc^4 x - \cot^4 x = \csc^2 x + \cot^2 x$

Answers may vary

12. $\frac{\csc x + \cot x}{\tan x + \sin x} = \cot x \csc x$

Answers may vary

13. $\frac{\sin 2x}{\cos 2x + \sin^2 x} = 2 \tan x$

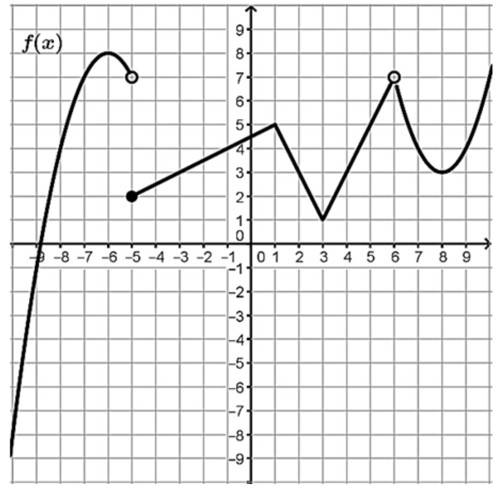
Answers may vary

Name **Limits & Introduction to Derivatives | 8.11H**

Set

Topic: Finding values of limits

Find the values of the limits using the graph at the right.



1. $\lim_{x \to -5^-} f(x) = \mathbf{7}$
2. $\lim_{x \to -5^+} f(x) = \mathbf{2}$
3. $\lim_{x \to 5} f(x) =$ **does not exist**
4. $\lim_{x \to 1^-} f(x) = \mathbf{5}$
5. $\lim_{x \to 1^+} f(x) = \mathbf{5}$
6. $\lim_{x \to 1} f(x) = \mathbf{5}$
7. $\lim_{x \to 6} f(x) = \mathbf{7}$
8. $\operatorname{Lim}_{x \to -\infty} f(x) = -\infty$
9. $\lim_{x \to \infty} f(x) = \infty$

Find the values of the following limits.

10. $\lim_{x \to -\frac{1}{2}} \frac{2x+1}{2x^2-x-1} = -\frac{2}{3}$
11. $\lim_{x \to 3} \frac{x^2-x-6}{x-3} = \mathbf{5}$
12. $\lim_{x \to -1} \frac{x^3+2x^2+x}{x^4+x^3+2x+2} = \mathbf{0}$
13. $\lim_{x \to 1} \frac{x^2+2x-3}{x^2-1} = \mathbf{2}$
14. $\lim_{x \to 0} \frac{\sqrt{4+x}-2}{x} = \frac{1}{4}$
15. $\lim_{x \to 9} \frac{\sqrt{x}-3}{x-9} = \frac{1}{6}$
16. $\lim_{x \to 2} \frac{\frac{1}{x}-\frac{1}{2}}{x-2} = -\frac{1}{4}$
17. $\lim_{x \to 0} \frac{\frac{1}{x+4}-\frac{1}{4}}{x} = -\frac{1}{16}$

18. Use the function $f(x) = \begin{cases} x^2+6, & x < 2 \\ x^3+2, & x \geq 2 \end{cases}$ to find the values of the following limits:

a. $\lim_{x \to 2^-} f(x) = \mathbf{10}$
b. $\lim_{x \to 2^+} f(x) = \mathbf{10}$
c. $\lim_{x \to 2} f(x) = \mathbf{10}$

19. Use the function $f(x) = \begin{cases} x+5, & x < 1 \\ x+7, & x \geq 1 \end{cases}$ to find the values of the following limits:

a. $\lim_{x \to 1^-} f(x) = \mathbf{6}$
b. $\lim_{x \to 1^+} f(x) = \mathbf{8}$
c. $\lim_{x \to 1} f(x) =$ **does not exist**

Topic: Finding derivatives using the limit process

Find the derivative of each function. That is, find:

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

20. $f(x) = x^2 + 3x$

    $f'(x) = 2x + 3$

21. $f(x) = 2x^3 - x$

    $f'(x) = 6x^2 - 1$

22. $f(x) = \frac{3}{x}$

    $f'(x) = -\frac{3}{x^2}$

23. $f(x) = \sqrt{x-4}$

    $f'(x) = \frac{1}{2\sqrt{x-4}}$

Topic: Average rate of change and instantaneous rate of change

24. The function $V(x) = x^3$ describes the volume of a cube. $V(x)$ is measured in cubic inches. The length, width, and height are each measured in inches.

    a. Find the average rate of range of the volume with respect to x as x changes from 5 inches to 5.1.

    76.51 cubic inches per inch

    b. Find the average rate of range of the volume with respect to x as x changes from 5 inches to 5.01 inches.

    75.1501 cubic inches per inch

    c. Find the instantaneous rate of change of the volume with respect to x at the moment when $x = 5$ inches.

    75 cubic inches per inch

25. A ball is thrown straight up from a rooftop 160 feet high with an initial speed of 48 feet per second. The function $s(t) = -16t^2 + 48t + 160$ describes the ball's height above the ground, $s(t)$, in feet, t seconds after it is thrown. The ball misses the rooftop on its way down and eventually strikes the ground.

a. What is the instantaneous speed of the ball 2 seconds after it is thrown?

−16 feet per second

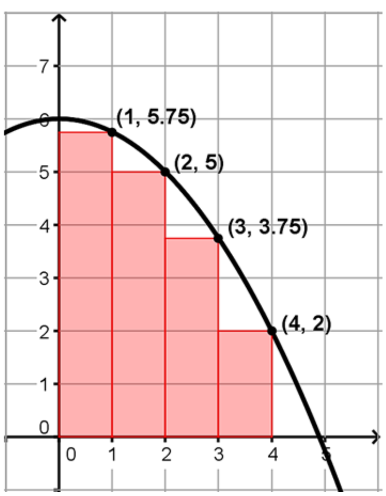
b. What is the instantaneous speed of the ball when it hits the ground?

−112 feet per second

Topic: Estimating areas using Riemann sums

26. Consider the function, $g(x) = -\frac{1}{4}x^2 + 6$ for $0 \le x \le 4$. Estimate $\int_0^4 g(x)\,dx$ using right-endpoint rectangles of width 1 unit. Follow the steps below as necessary to complete the problem:

a. Sketch a neat graph showing the curve over the indicated domain. Draw in right-endpoint rectangles from the x-axis to the curve showing a width of 1 unit for each rectangle. The rectangles should be below the curve.



b. Find the height of each rectangle by using the y-values of $g(x)$.

See points labeled on graph above

c. Write an expression for the sum of the areas of the rectangles.

$\mathbf{1 \cdot f(1) + 1 \cdot f(2) + 1 \cdot f(3) + 1 \cdot f(4) = (1 \cdot 5.75) + (1 \cdot 5) + (1 \cdot 3.75) + (1 \cdot 2)}$

d. Estimate $\int_0^4 g(x)\,dx$.

$\mathbf{16.5\ un^2}$

27. Consider the function, $g(x) = -\frac{1}{4}x^2 + 6$ for $0 \le x \le 4$. Estimate $\int_0^4 g(x)\,dx$ using right-endpoint rectangles of width 0.5 units. Follow the steps below as necessary to complete the problem:

a. Sketch a neat graph showing the curve over the indicated domain. Draw in right-endpoint rectangles from the x-axis to the curve showing a width of 0.5 units for each rectangle. The rectangles should be below the curve.

b. Find the height of each rectangle by using the y-values of $g(x)$.

See points labeled on graph above

c. Write an expression for the sum of the areas of the rectangles.

$\mathbf{0.5 \cdot f(0.5) + 0.5 \cdot f(1) + 0.5 \cdot f(1.5) + 0.5 \cdot f(2) + 0.5 \cdot f(2.5) + 0.5 \cdot f(3) + 0.5 \cdot f(3.5) + 0.5 \cdot f(4)}$

$\mathbf{(0.5 \cdot 5.94) + (0.5 \cdot 5.75) + (0.5 \cdot 5.44) + (0.5 \cdot 5) + (0.5 \cdot 4.44) + (0.5 \cdot 3.75) + (0.5 \cdot 2.94) + (0.5 \cdot 2)}$

d. Estimate $\int_0^4 g(x)\,dx$.
$\mathbf{17.63\ un^2}$

28. Why is the approximation in question 27 better than the one in question 26?
More rectangles gives a better approximation