Ready, Set, Go!

Ready
Topic: Even and odd functions

The graphs of even and odd functions make it easy to identify the type of function. **Even** functions have a line of symmetry along the *y*-axis, while **odd** functions have 180° rotational symmetry (symmetric about the origin).

**Label the following functions as even, odd, or neither.**

1. odd
2. even
3. odd
4. even
5. odd
6. neither
7. odd
8. even
9. odd
Set
Topic: Values of sine in the coordinate plane

Use the given point on the circle to find the value of sine. Recall that \( r = \sqrt{x^2 + y^2} \) and \( \sin \theta = \frac{y}{r} \).

10.\( \sin \theta = \frac{3}{5} \)

11.\( \sin \theta = \frac{\sqrt{3}}{2} \)

12. In each graph above, the angle of rotation is indicated by an arc and \( \theta \). Describe the angles of rotation that make the \( y \)-values of the points be positive and the angles of rotation that make the \( y \)-values be negative.

**Positive for** \( 0^\circ < \theta < 180^\circ \); **Negative for** \( 180^\circ < \theta < 360^\circ \)

13. In the graph at the right, the radius of the circle is 1. The intersections of the circle and the axes are labeled. Based on your observations from the task, what is the value of sine might be for:
\[ 90^\circ? \quad 180^\circ? \quad 270^\circ? \quad 360^\circ? \]

\[ 1 \quad 0 \quad -1 \quad 0 \]
14. Sketch the graph of a “Ferris wheel” that has a radius of 1 unit, with center at a height of 0, and makes one complete rotation in 360 seconds.

![Graph of a Ferris wheel]

Go

Topic: Solving problems using right triangle trigonometry

Make a sketch of the following problems and then solve.

15. A kite is flying, attached at the end of a string that is 1500 feet long. The string makes an angle of 43° with the ground. How far above the ground is the kite? Round your answer to the nearest foot.

1023 feet

16. A ladder leans against a building. The top of the ladder reaches a point on the building that is 12 feet above the ground. The foot of the ladder is 4 feet from the building. Find, to the nearest degree, the measure of the angle that the ladder makes with the level ground. What is the angle the ladder makes with the building?

71.6°

17. The shadow of a flagpole is 40.6 meters long when the angle of elevation of the sun is 34.6°. Find the height of the flagpole. Round your answer to the nearest tenth of a meter.

28 m
Ready, Set, Go!

**Ready**

*Topic: Comparing radius and arc length*

The wheels on the wagons that the pioneers used to cross the plains were smaller in the front than in the back. The front wheel had 12 spokes. The top of the front wheel measured 44 inches from the ground. The rear wheel had 16 spokes. The top of the rear wheel measured 59 inches from the ground. (For these problems disregard the hub at the center of the wheel. Assume the spokes meet in the center at a point.)

**Find the following values. Express answers in terms of \( \pi \).**

1. Find the area and the circumference of each wheel.
   - **Front:** \( A = 484\pi \text{ in}^2, C = 44\pi \text{ in} \)
   - **Rear:** \( A = 870.25\pi \text{ in}^2, C = 59\pi \text{ in} \)

2. Determine the central angle between the spokes on each wheel.
   - **Front:** \( 30^\circ \), **Rear:** \( 22.5^\circ \)

3. Find the distance on the circumference between two consecutive spokes for each wheel.
   - **Front:** \( 3.6\pi \text{ in} \), **Rear:** \( 3.6875\pi \text{ in} \)

4. The wagons could cover a distance of 15 miles per day. How many more times would the front wheels turn than the back wheels on an average day?
   - \((\text{Total “Big” wheel rotations}) - (\text{Total “Small” wheel rotations}) = 1748 \text{ more times}\)

5. A wheel rotates \( r \) times per minute. Describe how to find the number of degrees the wheel rotates in \( t \) seconds.
   - \( \frac{r \text{ revolutions}}{1 \text{ minute}} \cdot \frac{360^\circ}{1 \text{ revolution}} \cdot \frac{1 \text{ minute}}{60 \text{ seconds}} \cdot t \text{ seconds} \)
Set
Topic: Values of cosine in the coordinate plane

Use the given point on the circle to find the value of cosine. Recall that $r = \sqrt{x^2 + y^2}$ and $\cos \theta = \frac{x}{r}$.

6. $\cos \theta = \frac{4}{5}$

7. $\cos \theta = -\frac{1}{2}$

8. $\cos \theta = -\frac{\sqrt{2}}{2}$

9. $\cos \theta = \frac{\sqrt{3}}{2}$

10. In each graph, the angle of rotation is indicated by an arc and $\theta$. Describe the angles of rotation that make the $x$-values of the points be positive and the angles of rotation that make the $x$-values be negative.

**Positive:** $0^\circ < x < 90^\circ, 270^\circ < x < 360^\circ$; **Negative:** $90^\circ < x < 270^\circ$
11. In the graph at the right, the radius of the circle is 1. The intersections of the circle and the axes are labeled. Based on your observation in #10, what is the value of cosine might be for:

\[ 90^\circ \quad 180^\circ \quad 270^\circ \quad 360^\circ \]

\[
\begin{array}{cccc}
0 & -1 & 0 & 1
\end{array}
\]

**Go**

Topic: Measures in special triangles

\( \triangle ABC \) is a right triangle. Angle C is the right angle. Use the given information to find the missing sides and the missing angles.

12.  

13.  

14.  

15.
Find AD in the figures below:

16. \[ AD = 8\sqrt{3} \text{ ft} \]

17. \[ AD = 11\sqrt{2} \text{ m} \]

Topic: Graphing trigonometric functions

Graph the following functions (the graph of \( y = \sin x \) is given to assist you).

18. \( y = 2 \sin x \)

19. \( y = -3 \sin x - 1 \)

Graph the following functions (the graph of \( y = \cos x \) is given to assist you).

20. \( y = 4 \cos x \)

21. \( y = -\cos x + 1 \)
Ready, Set, Go!

Ready
Topic: Finding the length of an arc using proportions

Use the given degree measure of the central angle to set up a proportion to find the length of minor arc AB or the length of the semicircle. Leave your answers in terms of $\pi$.

Recall that $s = \frac{\theta}{360^\circ} (\pi d)$ where $s$ is the arc length.

1. 
2. 

\[ \frac{20\pi}{3} \text{ in} \hspace{2cm} 4\pi \text{ cm} \]

3. The circumference of circle A is 400 meters. The circumference of circle B is 800 meters. What is the relationship between the radius of circle A and the radius of circle B? Justify your answer.

2 times the radius of circle A equals the radius of circle B
Set
Topic: Measuring central angles

Veronica now thinks that the model of the archaeological site needs to have stakes that are equidistant from the vertical and horizontal axes. Therefore, she proposes using 8 evenly spaced stakes around each circle. Alyce also wants to make sure they record the distance around the circle to each new stake. As before, the central tower is located at the origin and the first of each set of 8 stakes for the inner and outer circles is placed at the points (12, 0) and (18, 0), respectively.

4. Your job is to determine the x- and y-coordinates for each of the remaining 8 stakes on each circle, as well as the arc lengths from the points (12, 0) or (18, 0), depending on which circle the stake is located. Keep track of the solutions in the table below.

Javier suggests they record the location of each stake and its distance around the circle for the set of stakes on each circle. Veronica suggests it might also be interesting to record the ratio of the arc length to the radius for each circle.

5. Help Javier and Veronica complete this table.

<table>
<thead>
<tr>
<th>Stake 1</th>
<th>Location</th>
<th>Distance from (12, 0) along circular path</th>
<th>Ratio of arc length to radius</th>
<th>Location</th>
<th>Distance from (18, 0) along circular path</th>
<th>Ratio of arc length to radius</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stake 2</td>
<td>(12, 0)</td>
<td>0</td>
<td>0</td>
<td>(18, 0)</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Stake 3</td>
<td>(6√2, 6√2)</td>
<td>3π</td>
<td>4π/4</td>
<td>(9√2, 9√2)</td>
<td>4.5π</td>
<td>4π/4</td>
</tr>
<tr>
<td>Stake 4</td>
<td>(0, 12)</td>
<td>6π</td>
<td>π/2</td>
<td>(0, 18)</td>
<td>9π</td>
<td>π/2</td>
</tr>
<tr>
<td>Stake 5</td>
<td>(0, 12)</td>
<td>12π</td>
<td>π</td>
<td>(0, 18)</td>
<td>18π</td>
<td>π</td>
</tr>
<tr>
<td>Stake 6</td>
<td>(−6√2, 6√2)</td>
<td>9π</td>
<td>3π/4</td>
<td>(−9√2, 9√2)</td>
<td>13.5π</td>
<td>3π/4</td>
</tr>
<tr>
<td>Stake 7</td>
<td>(−12, 0)</td>
<td>15π</td>
<td>5π/4</td>
<td>(−18, 0)</td>
<td>18π</td>
<td>5π/4</td>
</tr>
<tr>
<td>Stake 8</td>
<td>(−6√2, −6√2)</td>
<td>21π</td>
<td>7π/4</td>
<td>(−9√2, −9√2)</td>
<td>22.5π</td>
<td>7π/4</td>
</tr>
</tbody>
</table>

6. What interesting patterns might Alyce, Javier, and Veronica notice in their work and their table?

Answers will vary
Topic: Locating points in terms of rectangular coordinates, arc length, reference angle, and radius

In the diagram, ΔABC is a right triangle. Point B lies on the circle and is described by the rectangular coordinates \((x, y)\), \(s\) is the length of the intercepted arc created by angle \(\theta\), \(r\) is the radius of circle A.

**Answer the following questions using the given information.**

7. B has rectangular coordinates \((5, 12)\).
   a. Find \(r\).
      \(13\)
   b. Find \(\theta\) to the nearest tenth of a degree.
      \(67.4^\circ\)
   c. Find \(s\) by using the formula \(s = \frac{\theta}{360^\circ} (d\pi)\).
      \(15.3\)
   d. Describe point B using the coordinates \((r, \theta)\).
      \((13, 67.4^\circ)\)
   e. Describe point B using the radius and arc length \((r, s)\).
      \((13, 15.3)\)

8. B is described by \((r, \theta)\) where \(\theta \approx 58.11^\circ\) and \(r = 53\).
   a. Find \((x, y)\) to the nearest whole number
      \((28, 45)\)
   b. Find \(s\) by using the formula \(s = \frac{\theta}{360^\circ} (d\pi)\).
      \(53.8\)
   c. Describe point B using the radius and arc length \((r, s)\).
      \((53, 53.8)\)

9. B is described by \((r, s)\) where \(s \approx 46\) and \(r = 37\).
   a. Find \(\theta\) by using the formula \(s = \frac{\theta}{360^\circ} (d\pi)\).
      \(71.2^\circ\)
   b. Find \((x, y)\) to the nearest whole number
      \((12, 35)\)
   c. Describe point B using the coordinates \((r, \theta)\).
      \((37, 71.2^\circ)\)
Go

Topic: Radian measurement

Label each point on the circle with the measure of the angle of rotation in standard position. Angle measures should be in radians. (Recall that a full rotation around the circle would be $2\pi$ radians.)

Example: The circle has been divided equally into 8 parts. Each part is equal to $\frac{2\pi}{8}$ or $\frac{\pi}{4}$ radians. Indicate how many parts of $\frac{\pi}{4}$ radians there are at each position around the circle.

10. Finish the example by writing the angle measures for points F, G, and H.

Label the figures below using a similar approach as in question 10.

11.

12.
Topic: Graphing sine and cosine functions

Graph the following functions (the graphs of \( y = \sin x \) and \( y = \cos x \) are given to assist you).

13. \( y = 3 \sin 2x \)  
14. \( y = \cos 3x \)

15. \( y = \cos(2x) + 2 \)  
16. \( y = -2 \sin \left( \frac{1}{2}x \right) - 1 \)
Ready, Set, Go!

**Ready**
Topic: Coterminal angles

State a negative angle of rotation that is *coterminal* with the given angle of rotation. *Coterminal* angles share the same terminal side of an angle of rotation. Sketch and label both angles.

**Example:** $\theta = 120^\circ$ is the given angle of rotation. The angle of rotation is indicated by the solid arc. The dotted angle of rotation is the coterminal angle with a rotation of $-240^\circ$.

1. Given $\theta = 20^\circ$
   Coterminal Angle $-340^\circ$

2. Given $\theta = 95^\circ$
   Coterminal Angle $-265^\circ$

3. Given $\theta = 225^\circ$
   Coterminal Angle $-135^\circ$
Set
Topic: Sine and cosine of radian measures

4. Label each point around the circle with the angle of rotation in radians starting from the point (1, 0).

5. Write the **exact** coordinates of the points on the circle below. Be mindful of the numbers that are negative.

6. Find the arc length, $s$, from the point (1, 0) to each point around the circle. Record your answers as decimal approximations to the nearest thousandth.
Use your calculator to find the following values.

7. \( \sin \frac{5\pi}{6} = 0.5 \)  
8. \( \sin \frac{\pi}{3} = 0.8660 \)  
9. Why are both of your answers to questions 7 & 8 positive?  
   Both angles are above the x-axis so the y-values are positive.

10. \( \cos \frac{2\pi}{3} = -0.5 \)  
11. \( \cos \frac{4\pi}{3} = -0.5 \)  
12. Why are both of your answers to questions 10 & 11 negative?  
   Both angles are left of the y-axis so the x-values are negative.

13. \( \sin \frac{\pi}{2} = 1 \)  
14. \( \cos \frac{\pi}{2} = 0 \)  
15. In which quadrants are sine and cosine both negative?  
   Quadrant III

16. Name an angle of rotation where sine is equal to -1.  
   \( \frac{3\pi}{2} = 270^\circ \)

17. Name an angle of rotation where cosine is equal to -1.  
   \( \pi = 180^\circ \)

Go

Topic: Inverse trigonometric functions

Use your calculator to find the value of \( \theta \) where \( 0 \leq \theta \leq 90^\circ \). Round your answers to the nearest degree.

18. \( \sin \theta = 0.82 \)  
   \( 55^\circ \)  
19. \( \cos \theta = 0.31 \)  
   \( 72^\circ \)  
20. \( \cos \theta = 0.98 \)  
   \( 11^\circ \)

21. \( \sin \theta = 0.39 \)  
   \( 23^\circ \)  
22. \( \sin \theta = 1 \)  
   \( 90^\circ \)  
23. \( \cos \theta = 0.71 \)  
   \( 45^\circ \)
Topic: Graphing sine and cosine functions

Graph the following functions (the graphs of $y = \sin x$ and $y = \cos x$ are given to assist you).

24. $y = 2 \sin(x + 45^\circ)$  
   Hint: there will be a horizontal translation in this graph

25. $y = 1 + 2 \cos 3x$
Ready, Set, Go!

Ready
Topic: Functions and their inverses

Indicate which of the following functions have an inverse that is a function. If the function has an inverse, sketch it. Remember, the inverse will reflect across the line $y = x$. Finally, label each one as even, odd, or neither. Recall that an even function is symmetric with respect to the $y$-axis, while an odd function is symmetric with respect to the origin.

1. Inverse is not a function; Odd
2. Inverse is not a function; Even
3. Inverse is a function; Neither
4. Inverse is a function; Odd
**Set**

**Topic:** Graphs of the trigonometric functions

State the period, amplitude, vertical shift, and phase shift of the function shown in the graph. Then write the equation using the given trigonometric parent function.

5. \( y = \sin x \)

   - **Period:** \( 2\pi \)
   - **Amplitude:** \( 3 \)
   - **Vertical shift:** \( 0 \)
   - **Phase shift:** \( 0 \)
   - **Equation:** \( y = 3 \sin x \)

6. \( y = \sin x \)

   - **Period:** \( \frac{\pi}{2} \)
   - **Amplitude:** \( 2 \)
   - **Vertical shift:** \( -3 \)
   - **Phase shift:** \( 0 \)
   - **Equation:** \( y = 2 \sin(4x) - 3 \)

7. \( y = \cos x \)

   - **Period:** \( \pi \)
   - **Amplitude:** \( 1 \)
   - **Vertical shift:** \( 1 \)
   - **Phase shift:** \( 0 \)
   - **Equation:** \( y = 1 - \cos 2x \)

8. \( y = \cos x \)

   - **Period:** \( 2\pi \)
   - **Amplitude:** \( 1 \)
   - **Vertical shift:** \( 0 \)
   - **Phase shift:** \( \frac{\pi}{2} \)
   - **Equation:** \( y = \cos \left(x - \frac{\pi}{2}\right) \)
9. $y = \sin x$

- Period: $2\pi$
- Amplitude: 4
- Vertical shift: 0
- Phase shift: $-\frac{\pi}{2}$
- Equation: $y = 4\sin\left(x + \frac{\pi}{2}\right)$

10. The cofunction identity states that $\sin \theta = \cos(90^\circ - \theta)$ and $\sin(90^\circ - \theta) = \cos \theta$. How does this identity relate to the graph in #9?

$$4\sin\left(x + \frac{\pi}{2}\right) = 4\cos x$$

Explain where you would see this identity in a right triangle.

**Sine and cosine of complementary angles**

Describe the relationships between the graphs of $f(x)$ (solid) and $g(x)$ (dotted). Then write their equations.

11. $g(x)$ is a horizontal translation of $f(x)$.

$$g(x) = f\left(x - \frac{\pi}{4}\right)$$

$$f(x) = 3\cos x$$

$$g(x) = 3\cos\left(x - \frac{\pi}{4}\right)$$

12. $g(x)$ has an angular frequency that is 4 times that of $f(x)$.

$$f(x) = 2\cos\frac{x}{2}$$

$$g(x) = 2\cos(2x)$$
13. This graph could be interpreted as a shift or a reflection. Write the equations both ways.

\[ g(x) \text{ is reflected over the } x \text{ axis and has an increase in the amplitude when compared to } f(x). \]

\[ f(x) = -2 \sin x \quad \& \quad g(x) = 3 \sin x \]
\[ f(x) = 2 \sin(x - \pi) \quad \& \quad g(x) = -3 \sin(x - \pi) \]

Sketch the graph of the function. Include 2 full periods. Label the scale of your horizontal axis.

15. \[ y = 3 \sin \left( x - \frac{\pi}{2} \right) \]

14. \[ g(x) \text{ is a horizontal translation of } f(x). \]

\[ f(x) = -2 \sin x \quad \& \quad g(x) = -2 \sin \left( x - \frac{\pi}{4} \right) \]

16. \[ y = -2 \cos(x + \pi) \]
Name two values (if possible) for the angle of rotation, $0 < \theta \leq 2\pi$, that have the given trigonometric ratio.

17. $\sin \theta = \frac{\sqrt{2}}{2}$
\[ \theta = \frac{\pi}{4} \text{ and } \frac{3\pi}{4} \]

18. $\cos \theta = \frac{\sqrt{2}}{2}$
\[ \theta = \frac{\pi}{4} \text{ and } \frac{7\pi}{4} \]

19. $\cos \theta = -\frac{1}{2}$
\[ \theta = \frac{2\pi}{3} \text{ and } \frac{4\pi}{3} \]

20. $\sin \theta = 0$
\[ \theta = \pi, 2\pi \]

21. $\sin \theta = -\frac{\sqrt{3}}{2}$
\[ \theta = \frac{7\pi}{6} \text{ and } \frac{11\pi}{6} \]

22. $\cos \theta = -\frac{\sqrt{3}}{2}$
\[ \theta = \frac{5\pi}{6} \text{ and } \frac{7\pi}{6} \]

23. For which angles of rotation does $\sin \theta = \cos \theta$? Explain why.
\[ \frac{\pi}{4} \text{ and } \frac{5\pi}{4} \]

$\sin \theta$ and $\cos \theta$ have the same values at these angles since the triangles that can be formed with the $\theta$ axis is isosceles and sine and cosine have the same signs (positive/negative) in quadrants I and III.

Topic: Finding angle measures given trigonometric ratios in a right triangle.

Given the trigonometric values from a right triangle, find the indicated angle measures. Round angle measures to the nearest degree. Hint: draw a right triangle and label the lengths of the sides using the trigonometric ratio.

24. $\cot \theta = \frac{6}{7}$
\[ 49^\circ \]

25. $\sec \theta = 3$
\[ 71^\circ \]

26. $\csc \theta = \frac{8}{5}$
\[ 39^\circ \]
Ready, Set, Go!

Ready
Topic: Using the definition of tangent

Use what you know about the definition of tangent in a right triangle and your work with the new definitions of sine and cosine to find the exact value of tangent $\theta$ for each of the right triangles below.

1. $\tan \theta = \frac{3}{4}$
2. $\tan \theta = \frac{63}{-16}$
3. $\tan \theta = 1$
4. $\tan \theta = \frac{-2}{2\sqrt{3}} = -\frac{\sqrt{3}}{3}$

5. In each graph above, the angle of rotation is indicated by an arc and $\theta$. Describe the angles of rotation from 0 to $2\pi$ that make tangent be positive and the angles of rotation that make tangent be negative.

$\tan \theta > 0$ when $0 < \theta < \frac{\pi}{2}$ & $\pi < \theta < \frac{3\pi}{2}$

$\tan \theta < 0$ when $\frac{\pi}{2} < \theta < \pi$ & $\frac{3\pi}{2} < \theta < 2\pi$
**Set**

**Topic: Mathematical modeling using sine and cosine functions**

Many real-life situations such as sound waves, weather patterns, and electrical currents can be modeled by sine and cosine functions. The table below shows the depth of water (in feet) at the end of a wharf as it varies with the tides at various times during the morning.

<table>
<thead>
<tr>
<th>$t$ (time)</th>
<th>midnight</th>
<th>2 A.M.</th>
<th>4 A.M.</th>
<th>6 A.M.</th>
<th>8 A.M.</th>
<th>10 A.M.</th>
<th>noon</th>
</tr>
</thead>
<tbody>
<tr>
<td>$d$ (depth)</td>
<td>8.16</td>
<td>12.16</td>
<td>14.08</td>
<td>12.16</td>
<td>8.16</td>
<td>5.76</td>
<td>7.26</td>
</tr>
</tbody>
</table>

We can use a trigonometric function to model the data. Suppose you choose cosine: $y = a \cos(b t - c) + d$, where $y$ is the depth at any time. The **amplitude** will be the distance from the average of the highest and lowest values. This will be the average depth, $d$.

6. Sketch the line that shows the average depth, $d$.

   See dashed line on the graph at the right. $y = 9.92$

7. Find the amplitude: $a = \frac{1}{2} (\text{high depth} - \text{low depth})$

   $4.16$

8. Find the period: $p = 2|\text{low time} - \text{high time}|$. Since a normal period for sine is $2\pi$, the new period for our model will be $\frac{2\pi}{p}$ so $b = \frac{2\pi}{p}$. Use the $p$ you calculated, divide and turn it into a decimal to find the value of $b$.

   $p = 12 \text{ hrs}, b \approx 0.5236$

9. High tide occurred 4 hours after midnight. The formula for the displacement is $4 = \frac{c}{b}$. Use $b$ and solve for $c$.

   $c = \frac{2\pi}{3}$

10. Now that you have your values for $a$, $b$, $c$, and $d$, you can put them into your equation:

    $y = a \cos(bt - c) + d$

    $y = 4.16 \cos\left(\frac{\pi}{6} t - \frac{2\pi}{3}\right) + 9.92$

11. Use your model to calculate the depth at 9 A.M. and 3 P.M.

    $6.31733 \text{ ft and 13.52267 ft}$

12. A boat needs at least 10 feet of water to dock at the wharf. During what interval of time in the afternoon can it safely dock?

    $(6.963, 13.037)$; from a little before 7:00am to a little after 1:00pm
Topic: Transformations of trigonometric graphs

Match each trigonometric representation on the left with an equivalent representation on the right. Then check your answers with a graphing utility. Record your answer in the space provided to the left of the question number.

13. \( y = -3 \sin \left( \theta + \frac{\pi}{2} \right) \)

A. \( y = -3 \sin \theta \)

14. \( y = 3 \cos \left( \theta + \frac{\pi}{2} \right) \)

B. \( y = -\sin \theta \)

F. \( y = 2 \cos \left( \theta + \frac{\pi}{2} \right) - 2 \)

15. \( y = \sin \left( 2 \left( \theta + \frac{\pi}{2} \right) \right) - 2 \)

C. \( y = \sin \theta \)

D. \( y = \sin \left( x + \pi \right) \)

E. \( y = \cos \left( x + \pi \right) + 3 \)

16. \( y = \sin \left( x + \pi \right) \)

B. \( y = -\sin \theta \)

F. \( y = \cos \left( x + \pi \right) + 3 \)
Use your calculator and what you know about where sine and cosine are positive and negative in the unit circle, to find the two angles that are solutions to each equation. Make sure $\theta$ is in the interval $0 < \theta \leq 2\pi$. Round your answers to 4 decimal places. (Your calculator should be set in radians.)

You will notice that your calculator will sometimes give you a negative angle. That is because the calculator is programmed to restrict the angle of rotation so that the inverse of the function is also a function. Since the requested answers have been restricted to positive rotations, if the calculator gives you a negative angle of rotation, you will need to figure out the positive coterminal angle for the angle that your calculator has given you.

19. $\sin \theta = \frac{4}{5}$

   |   |   |
   |---|---|---|
   | 0.9273 | 3.2418 | 4.3321 |
   | 2.2143 | 6.1830 | 5.0926 |

20. $\sin \theta = -\frac{1}{10}$

21. $\sin \theta = -\frac{13}{14}$

Note: When you ask your calculator for the angle, you are undoing the trigonometric function. Finding the angle is finding the inverse trigonometric function. When you see $\sin^{-1} \left(\frac{4}{5}\right)$, you are being asked to find the angle that makes $\sin \theta = \frac{4}{5}$ true. The answer would be the same as the answer your calculator gave you in #19. Another notation that represents the inverse sine function is $\arcsin \left(\frac{4}{5}\right)$.

Topic: Graphing sine and cosine

Sketch the graph of each function. Include two full periods. Label the scale of your horizontal axis.

22. $y = 3 \sin \left(x + \frac{\pi}{4}\right)$

23. $y = -2 \cos 3x$
Topic: Composite trigonometric functions

Recall that a composite function places one function such as \( g(x) \), inside the other, \( f(x) \), by replacing the \( x \) in \( f(x) \) with the entire function \( g(x) \). In general, the notation is \( f(g(x)) \). It is possible to do composition of the trigonometric functions. The answer to \( \sin^{-1} \left( \frac{1}{2} \right) \) is an angle of \( 30^\circ \). The composition of \( \sin \left( \sin^{-1} \left( \frac{1}{2} \right) \right) \) is simply asking “What is value of \( \sin 30^\circ \)?” The answer is \( \frac{1}{2} \).

Sine was just “undoing” what \( \sin^{-1} \theta \) was doing. Not all composite trigonometric functions are inverses such as problems 27 – 32.

Answer the following. For questions 28-32, it may help to draw a diagram.

24. \( \sin \left( \sin^{-1} \left( \frac{\sqrt{2}}{2} \right) \right) \)  
\( \frac{\sqrt{2}}{2} \)

25. \( \cos \left( \cos^{-1} \left( \frac{\sqrt{3}}{2} \right) \right) \)  
\( \frac{\sqrt{3}}{2} \)

26. \( \tan \left( \tan^{-1} \left( \frac{2}{3} \right) \right) \)  
\( \frac{2}{3} \)

27. \( \sin \left( \cos^{-1} \left( \frac{\sqrt{3}}{2} \right) \right) \)  
\( \frac{\sqrt{3}}{2} \)

28. \( \cos \left( \tan^{-1} \left( \frac{\sqrt{2}}{2} \right) \right) \)  
\( \frac{\sqrt{2}}{2} \)

29. \( \sin \left( \tan^{-1} \left( \frac{11}{4} \right) \right) \)  
\( \frac{11}{\sqrt{137}} \approx 0.9398 \)

30. \( \cos \left( \sin^{-1} \left( \frac{\sqrt{3}}{2} \right) \right) \)  
\( \frac{\sqrt{3}}{2} \)

31. \( \cos \left( \tan^{-1} \left( \frac{1}{1} \right) \right) \)  
\( \frac{1}{1} \)

32. \( \sin \left( \cot^{-1} (1) \right) \)  
\( 1 \)
Ready, Set, Go!

Ready
Topic: Rigid and non-rigid transformations of functions

The equation of a parent function is given. Write a new equation with the given transformations. Then sketch the new function on the same graph as the parent function. If the function has asymptotes, sketch them in.

1. \( y = x^2 \)
   Vertical Shift: down 8
   Horizontal Shift: left 6
   Dilation: \( \frac{1}{4} \)
   Equation: \( y = \frac{1}{4}(x + 6)^2 - 8 \)
   Domain: \( (-\infty, \infty) \)
   Range: \([8, \infty)\)

2. \( y = \frac{1}{x} \)
   Vertical Shift: up 3
   Horizontal Shift: left 4
   Dilation: \(-1\)
   Equation: \( y = -\frac{1}{x+4} + 3 \)
   Domain: \( (-\infty, -4) \cup (-4, \infty) \)
   Range: \((-\infty, 3) \cup (3, \infty)\)
3. \( y = \sqrt{x} \)
   Vertical Shift: none
   Horizontal Shift: left 5
   Dilation: 3
   Equation: \( y = 3\sqrt{x} + 5 \)
   Domain: \([5, \infty)\)
   Range: \([0, \infty)\)

4. \( y = \sin x \)
   Vertical Shift: up 1
   Horizontal Shift: left \( \frac{\pi}{2} \)
   Dilation (amplitude): 3
   Equation: \( y = 3 \sin \left(x + \frac{\pi}{2}\right) + 1 \)
   Domain: \((-\infty, \infty)\)
   Range: \([-2, 4]\)

Set
Topic: Features of the graphs of the trigonometric functions

5. \( \triangle ABC \) is a right triangle with \( AB = 1 \).
   Use the information in the figure to label the length of the sides and measure of
   the angles.

6. \( \triangle RST \) is an equilateral triangle.
   \( RS = 1 \) and \( SA \) is an altitude
   Use the information in the figure to label the length of the sides, \( RA \), and the
   exact length of \( SA \).
   Label the measure of \( \angle RSA \) and \( \angle SRA \).
7. Use the information from the figures in questions 6 and 7 to fill in the table.

<table>
<thead>
<tr>
<th>Function</th>
<th>$\theta = 30^\circ$</th>
<th>$\theta = \frac{\pi}{6}$</th>
<th>$\theta = 45^\circ$</th>
<th>$\theta = \frac{\pi}{4}$</th>
<th>$\theta = 60^\circ$</th>
<th>$\theta = \frac{\pi}{3}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sin \theta$</td>
<td>$\frac{1}{2}$</td>
<td>$\frac{1}{2}$</td>
<td>$\frac{\sqrt{2}}{2}$</td>
<td>$\frac{\sqrt{2}}{2}$</td>
<td>$\frac{\sqrt{3}}{2}$</td>
<td>$\frac{\sqrt{3}}{2}$</td>
</tr>
<tr>
<td>$\cos \theta$</td>
<td>$\frac{\sqrt{3}}{2}$</td>
<td>$\frac{\sqrt{3}}{2}$</td>
<td>$\frac{\sqrt{2}}{2}$</td>
<td>$\frac{\sqrt{2}}{2}$</td>
<td>$\frac{1}{2}$</td>
<td>$\frac{1}{2}$</td>
</tr>
<tr>
<td>$\tan \theta$</td>
<td>$\frac{\sqrt{3}}{3}$</td>
<td>$\frac{\sqrt{3}}{3}$</td>
<td>$1$</td>
<td>$1$</td>
<td>$\sqrt{3}$</td>
<td>$\sqrt{3}$</td>
</tr>
</tbody>
</table>

8. Name the angles of rotation, in radians, where sine equals 0 in the domain $[0, 2\pi]$.
   $0, \pi, 2\pi$

9. Name the angles of rotation, in radians, where cosine equals 0 in the domain $[0, 2\pi]$.
   $\frac{\pi}{2}, \frac{3\pi}{2}, \frac{\pi}{2}$

10. Name the angles of rotation, in radians, where tangent equals 0 in the domain $[0, 2\pi]$.
    $0, \pi, 2\pi$

11. Name the angles of rotation, in radians, where tangent is undefined in the domain $[0, 2\pi]$.
    $\frac{\pi}{2}, \frac{3\pi}{2}$

**Topic: Graphing tangent functions.**

**Graph each tangent function. Be sure to indicate the location of the asymptotes.**

12. $y = 3 \tan(2\theta) - 4$

13. $y = 2 \tan \left(\frac{1}{2} \theta\right) + 2$
Go
Topic: Trigonometric facts

Answer the questions below. Be sure you can justify your thinking.

14. Given triangle $ABC$ with $\angle C$ being the right angle, what is $m\angle A + m\angle B$?
   $90^\circ$

15. Identify the quadrants in which $\sin \theta$ is positive.
   Quadrants I & II

16. Identify the quadrants in which $\cos \theta$ is negative.
   Quadrants II & III

17. Identify the quadrants in which $\tan \theta$ is positive.
   Quadrants I & III

18. Explain why it is impossible for $\sin \theta > 1$.
   Answers may vary. Sample answer: because the leg of a right triangle can never be longer than the hypotenuse.

19. Name the angles of rotation, in radians, for when $\sin \theta = \cos \theta$.
   $\frac{\pi}{4}$ & $\frac{5\pi}{4}$

20. Which trigonometric function has the same value when the angle of rotation is positive or negative?
   cosine

21. Explain why $\sin \theta = \cos(90^\circ - \theta)$
   Answers may vary. Sample answer: In a right triangle, $A = 90^\circ - B$ and $B = 90^\circ - A$ and $\sin A = \cos B$, then $\sin A = \cos(90^\circ - A)$.

22. Write the Pythagorean identity and then prove it.
   $\sin^2 \theta + \cos^2 \theta = 1$
   Proof:
   $a^2 + b^2 = c^2$
   $\sin \theta = \frac{a}{c}$
   $\cos \theta = \frac{b}{c}$
   $\frac{a^2 + b^2}{c^2} = 1$
   $\left(\frac{a}{c}\right)^2 + \left(\frac{b}{c}\right)^2 = 1$
   $\sin^2 \theta + \cos^2 \theta = 1$

23. Explain why, in the unit circle, $\tan \theta = \frac{y}{x}$
   Answers may vary. Sample answer: $\tan \theta = \frac{\text{opposite}}{\text{adjacent}}$ on the unit circle opposite = $y$-values and adjacent = $x$-values.

24. Which function represents the slope of the hypotenuse in a right triangle?
   tangent
25. Name the trigonometric function(s) that are odd functions.
\[ \sin \theta \text{ & } \tan \theta \]

Topic: Graphing sine and cosine

**Sketch the graph of each function. Include two full periods. Label the scale of your horizontal axis.**

26. \( y = 3 \sin \left( \frac{\pi}{4} x + \frac{\pi}{2} \right) \)

![Graph of 26](image)

27. \( y = \cos \left( x - \frac{\pi}{2} \right) + 3 \)

![Graph of 27](image)

28. \( y = 2\sin 3(x + \pi) - 1 \)

![Graph of 28](image)

29. \( y = 4\cos \left( \frac{\pi}{8} x \right) - 5 \)

![Graph of 29](image)
Ready, Set, Go!

Ready
Topic: Function combinations

The functions $f(x)$, $g(x)$, and $h(x)$ are given in the graphs below. Graph the indicated combination on the same axes.

1. $f(x) + h(x)$
2. $\frac{1}{h(x)}$
3. $f(x)g(x)$

Set
Topic: Graphing reciprocal trigonometric functions

Graph each function. Remember, it may be helpful to sketch sine, cosine, or tangent first. Use the space below the function to construct a table of values to use for your graph.

4. $y = 3 \csc(2x) - 1$

<table>
<thead>
<tr>
<th>$x$</th>
<th>$3 \sin(2x) - 1$</th>
<th>$3 \csc(2x) - 1$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
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<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
5. \( y = 1 - \sec(3x) \)

\[
\begin{array}{|c|c|c|}
\hline
x & 1 - \cos(3x) & 1 - \sec(3x) \\
\hline
\end{array}
\]

6. \( y = 2 + 3 \csc(2x) \)

\[
\begin{array}{|c|c|c|}
\hline
x & 2 + 3 \sin(2x) & 2 + 3 \csc(2x) \\
\hline
\end{array}
\]

7. \( y = -3 + 2 \cot(2x) \)

\[
\begin{array}{|c|c|c|}
\hline
x & -3 + 2 \tan(2x) & -3 + 2 \cot(2x) \\
\hline
\end{array}
\]
Write the equation of the given graphs. Check the accuracy of your equations using a graphing utility.

8. Use $\csc x$
   \[ y = -\csc x \]

9. Use $\sec x$
   \[ y = 2\sec(2x) \]

---

Go

**Topic: Finding all six trigonometric ratios**

**Use the given information to find the exact value of all remaining trigonometric ratios of $\theta$. Hint: It may be helpful to draw a diagram.**

10. $\sin \theta = \frac{3}{5}, \sec \theta < 0$

11. $\sec \theta = \frac{7}{2}, \tan \theta > 0$

12. $\cot \theta = \frac{4}{3}, \pi < \theta < \frac{\pi}{2}$

\[
\begin{align*}
\cos \theta &= -\frac{4}{5} \\
\tan \theta &= -\frac{3}{4} \\
csc \theta &= \frac{5}{3} \\
sec \theta &= -\frac{5}{4} \\
cot \theta &= -\frac{4}{3}
\end{align*}
\]

\[
\begin{align*}
\sin \theta &= \frac{2\sqrt{11}}{7} \\
\cos \theta &= \frac{2}{7} \\
\tan \theta &= \sqrt{11} \\
csc \theta &= \frac{7\sqrt{11}}{22} \\
cot \theta &= \frac{\sqrt{11}}{11}
\end{align*}
\]

13.

\[
\begin{align*}
\sin \theta &= \frac{\sqrt{3}}{2} \\
\cos \theta &= -\frac{1}{2} \\
\tan \theta &= -\sqrt{3} \\
csc \theta &= \frac{2\sqrt{3}}{3} \\
sec \theta &= -2 \\
cot \theta &= -\frac{\sqrt{3}}{3}
\end{align*}
\]
Ready, Set, Go!

Ready
Topic: Vertical and horizontal shifts on a graph

A school building is kept warm only during school hours. The graph below shows the temperature, $G$, in °F, as a function of time, $t$, in hours after midnight. At midnight ($t = 0$), the building’s temperature is 50°F. This temperature remains the same until 4 am. Then the heater begins to warm the building so that by 8 am the temperature is 70°F. That temperature is maintained until 4 pm, when the building begins to cool. By 8 pm, the temperature has returned to 50°F and will remain at that temperature until 4 am.

1. In January, many students are sick with the flu. The custodian decides to keep the building 5°F warmer. Sketch the graph of the new schedule on the original axes.

2. If $f(t)$ is the function that describes the original temperature setting, what would be the function for the January setting?
   
   January = $f(t) + 5$

3. In the spring, the drill team begins early morning practice. The custodian then changes the original setting to start 2 hours earlier. The building now begins to warm at 2 am instead of 4 am and reaches 70°F at 6 am. It begins cooling off at 2 pm instead of 4 pm and returns to 50°F at 6 pm instead of 8 pm. Sketch the graph of the new schedule on the graph above.

4. If $f(t)$ is the function that describes the original temperature setting, what would be the function for the spring setting?
   
   Spring = $f(t + 2)$
**Set**

Topic: Fundamental trigonometric identities

The **cofunction identities** state: \( \sin \theta = \cos \left( \frac{\pi}{2} - \theta \right) \) and \( \cos \theta = \sin \left( \frac{\pi}{2} - \theta \right) \)

**Complete the statements, using the Cofunction identities.**

5. \( \sin 70^\circ = \cos _{20}^\circ \)  
6. \( \cos \frac{\pi}{8} = \sin \frac{3\pi}{8} \)  
7. \( \cos 54^\circ = \sin _{36}^\circ \)

8. \( \sin 9^\circ = \cos _{81}^\circ \)  
9. \( \cos 72^\circ = \sin _{8}^\circ \)  
10. \( \sin \frac{5\pi}{12} = \cos \frac{\pi}{12} \)

Use the sum identities for sine and cosine to find the exact value of each.

11. \( \cos 15^\circ = \frac{\sqrt{6}+\sqrt{2}}{4} \)

12. \( \sin 165^\circ = \frac{\sqrt{6}-\sqrt{2}}{4} \)

13. Let \( \sin \theta = \frac{3}{4} \) and \( \theta \) lies in quadrant II.
   a. Use the Pythagorean identity, \( \sin^2 \theta + \cos^2 \theta = 1 \), to find the value of \( \cos \theta \).
   \[ \cos \theta = -\frac{\sqrt{7}}{4} \]
   b. Use the Quotient identity, \( \tan \theta = \frac{\sin \theta}{\cos \theta} \), the given information, and your answer in part a to calculate the value of \( \tan \theta \).
   \[ \tan \theta = -\frac{3\sqrt{7}}{7} \]

14. Let \( \cos \beta = \frac{12}{13} \) and \( \beta \) lies in quadrant IV.
   a. Find \( \sin \beta \) using the Pythagorean identity \( \sin^2 \theta + \cos^2 \theta = 1 \)
   \[ \sin \beta = \frac{5}{13} \]
   b. Find \( \tan \beta \) using the quotient identity \( \tan \theta = \frac{\sin \theta}{\cos \theta} \)
   \[ \tan \beta = -\frac{5}{12} \]
   c. Find \( \cos \left( \frac{\pi}{2} - \beta \right) \) using a cofunction identity.
   \[ \cos \left( \frac{\pi}{2} - \beta \right) = \frac{5}{13} \]
Use trigonometric identities to show that the two sides are equivalent. Show all of your steps.

Answers may vary. Sample answers provided below.

15. \( \tan \theta \cos \theta = \sin \theta \)

\[
\frac{\sin \theta}{\cos \theta} \cdot \cos \theta = \sin \theta \\
\sin \theta = \sin \theta
\]

16. \((1 + \cos \beta)(1 - \cos \beta) = \sin^2 \beta\)

\[
1 - \cos^2 \beta = \sin^2 \beta \\
\sin^2 \beta = \sin^2 \beta
\]

17. \(\frac{2(\tan x - \cot x)}{\tan^2 x - \cot^2 x} = \sin 2x\)

\[
\frac{2(\tan x - \cot x)}{(\tan x - \cot x)(\tan x + \cot x)} = \sin 2x \\
\frac{2}{\tan x + \cot x} = \sin 2x \\
\frac{2}{\sin x \cos x} = \sin 2x \\
\frac{2}{\sin x \cos x} = \sin 2x \\
2 \cdot \frac{\sin x \cos x}{1} = \sin 2x \\
\sin 2x = \sin 2x
\]

18. \(\sin^2 W - \cos^2 W = 2 \sin^2 W - 1\)

\[
\sin^2 W - (1 - \sin^2 W) = 2 \sin^2 W - 1 \\
\sin^2 W - 1 + \sin^2 W = 2 \sin^2 W - 1 \\
2 \sin^2 W - 1 = 2 \sin^2 W - 1
\]

19. \((\cos x + \sin x)(\cos x - \sin x) = \cos 2x\)

\[
\cos^2 x - \sin^2 x = \cos 2x \\
\cos 2x = \cos 2x
\]

20. \(\frac{1}{\sec x - \tan x} = \tan x + \sec x\)

\[
\frac{1}{\sec x - \tan x} \cdot \frac{\sec x + \tan x}{\sec x + \tan x} = \tan x + \sec x \\
\frac{\sec x + \tan x}{\sec x - \tan x} = \tan x + \sec x \\
\frac{\sec x + \tan x}{\sec^2 x - \tan^2 x} = \tan x + \sec x \\
\frac{\sec x + \tan x}{\tan^2 x + 1 - \tan^2 x} = \tan x + \sec x \\
\frac{\sec x + \tan x}{1} = \tan x + \sec x \\
\tan x + \sec x = \tan x + \sec x
\]
Go
Topic: Trigonometric values of the special angles

Find two solutions of the equation. Give both answers in degrees \((0 \leq \theta \leq 360^\circ)\) and radians \((0 \leq \theta \leq 2\pi)\). Do not use a calculator.

21. \(\sin \theta = \frac{1}{2}\)
   
   degrees: \(30^\circ, 150^\circ\)
   
   radians: \(\frac{\pi}{6}, \frac{5\pi}{6}\)

22. \(\sin \theta = -\frac{1}{2}\)
   
   degrees: \(210^\circ, 330^\circ\)
   
   radians: \(\frac{7\pi}{6}, \frac{11\pi}{6}\)

23. \(\cos \theta = \frac{\sqrt{2}}{2}\)
   
   degrees: \(45^\circ, 315^\circ\)
   
   radians: \(\frac{\pi}{4}, \frac{7\pi}{4}\)

24. \(\cos \theta = -\frac{\sqrt{3}}{2}\)
   
   degrees: \(210^\circ, 330^\circ\)
   
   radians: \(\frac{7\pi}{6}, \frac{11\pi}{6}\)

25. \(\tan \theta = -1\)
   
   degrees: \(135^\circ, 315^\circ\)
   
   radians: \(\frac{3\pi}{4}, \frac{7\pi}{4}\)

26. \(\tan \theta = \sqrt{3}\)
   
   degrees: \(60^\circ, 240^\circ\)
   
   radians: \(\frac{\pi}{3}, \frac{4\pi}{3}\)
Set, Go!

Set
Topic: Using trigonometric ratios to solve problems

Perhaps you have seen *The London Eye* in the background of a recent James Bond movie or on a television show. When it opened in March of 2000, it was the tallest Ferris wheel in the world. The passenger capsule at the very top is 135 meters above the ground. The diameter is 120 meters.

1. How high off the ground is the center of the Ferris wheel?
   
   \[75 \text{ m}\]

2. How far from the ground is the very bottom passenger capsule?
   
   \[15 \text{ m}\]

3. Assume there are 32 passenger capsules, evenly spaced around the circumference. Find the height from the ground of each of the even numbered passenger capsules shown in the figure. Use the figure at the right to help you think about the problem.

   **Height:** \(75 + 60 \sin \theta\)

   \[2: 97.96 \text{ m} \quad 4: 117.43 \text{ m} \quad 6: 130.43 \text{ m} \quad 8: 135 \text{ m}\]

4. Choose the equation(s) at the right that has the same graph as \(y = \cos \theta\).

   Use the unit circle to explain why they are the same.

   **The x-coordinate is the same for positive and negative angles of rotation.**

   a. \(y = \cos(\theta - \pi)\)
   
   b. \(y = \cos(\theta + \pi)\)

5. Choose the equation(s) at the right that has the same graph as \(y = -\sin \theta\).

   Use the unit circle to explain why they are the same.

   **With a phase shift of \(\pi\), the y-coordinates are on the opposite side of the unit circle.**

   a. \(y = \sin(\theta + \pi)\)
   
   b. \(y = \sin(\theta - \pi)\)
In the diagram, \( \triangle ABC \) is a right triangle. Point B lies on the circle and is described by the rectangular coordinates \((x, y)\), \(s\) is the length of the intercepted arc created by angle \(\theta\), \(r\) is the radius of circle A.

**Answer the following questions using the given information.**

6. B has rectangular coordinates \((33, 56)\).
   a. Find \(r\).
      \[65\]
   b. Find \(\theta\) to the nearest tenth of a degree.
      \[59.5^\circ\]
   c. Find \(s\) by using the formula \(s = \frac{\theta}{360^\circ}(d\pi)\).
      \[67.5\]
   d. Describe point B using the coordinates \((r, \theta)\).
      \[(65, 59.5^\circ)\]
   e. Describe point B using the radius and arc length \((r, s)\).
      \[(65, 67.5)\]

7. B is described by \((r, \theta)\) where \(\theta \approx 25.01^\circ\) and \(r = 85\).
   a. Find \((x, y)\) to the nearest whole number.
      \[(77, 36)\]
   b. Find \(s\) by using the formula \(s = \frac{\theta}{360^\circ}(d\pi)\).
      \[37.1\]
   c. Describe point B using the radius and arc length \((r, s)\).
      \[(85, 37.1)\]
Topic: Using the calculator to find angles of rotation

Use your calculator and what you know about where sine and cosine are positive and negative in the unit circle, to find the two angles that are solutions to each equation. Make sure \( \theta \) is in the interval \( 0 < \theta \leq 2\pi \). Round your answers to 4 decimal places. (Your calculator should be set in radians.)

You will notice that your calculator will sometimes give you a negative angle. That is because the calculator is programmed to restrict the angle of rotation so that the inverse of the function is also a function. Since the requested answers have been restricted to positive rotations, if the calculator gives you a negative angle of rotation, you will need to figure out the positive coterminal angle for the angle that your calculator has given you.

8. \( \cos \theta = \frac{11}{12} \)

\[ 0.4111 \text{ & } 5.8720 \]

9. \( \cos \theta = -\frac{7}{8} \)

\[ 2.6362 \text{ & } 3.6470 \]

10. \( \cos \theta = -\frac{2}{5} \)

\[ 1.9823 \text{ & } 4.3009 \]

For each function, identify the amplitude, period, horizontal shift, and vertical shift. Then sketch a graph.

11. \( f(t) = 14 \cos \left( \frac{\pi}{6} (t - 8) \right) + 8 \)

amplitude: \( 14 \)

period: \( 12 \)

horizontal shift: \( 8 \) right

vertical shift: \( 8 \) up

12. \( f(t) = 4.5 \sin \left( \frac{\pi}{4} t + \frac{3}{4} \right) + 8 \)

amplitude: \( 4.5 \)

period: \( 8 \)

horizontal shift: \( \frac{3}{\pi} \) left

vertical shift: \( 8 \) up
13. The angle of depression from the top of a building to a car parked in the parking lot is 32.5°. How far from the top of the building is the car on the ground, if the car is 330 meters from the bottom of the building? Round your answer to the nearest tenth of a meter.

391.3 m

Graph each function.

14. \( y = \sin 4x + 2 \)  

15. \( y = \cos \left( x - \frac{\pi}{2} \right) - 4 \)

![Graph of sin 4x + 2](image1)

![Graph of cos(x - pi/2) - 4](image2)

Go

Topic: Trigonometric ratios in a right triangle

Find the other two trigonometric ratios based on the one that is given. Hint: draw and label a right triangle using the given trigonometric ratio.

<table>
<thead>
<tr>
<th></th>
<th>sin ( \theta )</th>
<th>cos ( \theta )</th>
<th>tan ( \theta )</th>
</tr>
</thead>
<tbody>
<tr>
<td>16.</td>
<td>( \frac{4}{5} )</td>
<td>( \frac{3}{5} )</td>
<td>( \frac{4}{3} )</td>
</tr>
<tr>
<td>17.</td>
<td>( \frac{12}{13} )</td>
<td>( \frac{5}{13} )</td>
<td>( \frac{12}{5} )</td>
</tr>
<tr>
<td>18.</td>
<td>( \sqrt{2} )</td>
<td>( \sqrt{2} )</td>
<td>( 1 )</td>
</tr>
<tr>
<td>19.</td>
<td>( \frac{1}{2} )</td>
<td>( \frac{2\sqrt{3}}{3} )</td>
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<td>20.</td>
<td>( \frac{41}{40} )</td>
<td>( \frac{9}{41} )</td>
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<td>21.</td>
<td>( \frac{\sqrt{3}}{2} )</td>
<td>( \frac{1}{2} )</td>
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Name: Trigonometric Functions 4.12H

Ready, Set, Go!

Ready
Topic: Solving equations

Solve each equation for the indicated variable.
1. \( x^2 - 13x + 36 = 0 \)
   \[ x = 9, x = 4 \]

2. \( \frac{2}{x+1} - \frac{1}{x-1} = \frac{2x}{x^2-1} \)
   \[ x = -3 \]

3. \( 3x^2 = 12x \)
   \[ x = 0, x = 4 \]

4. \( 5x^2 + 2 = 11x \)
   \[ x = \frac{1}{5}, x = 2 \]

Set
Topic: Inverse trigonometric functions

Use the given information to find the missing angle \((0 \leq \theta \leq 2\pi)\). Round answers to the thousandths place.

5. \( \cos \theta = 0.9848; \sin \theta > 0 \)
   \[ 0.175 \]

6. \( \sin \theta = 0.9925; \tan \theta < 0 \)
   \[ 1.693 \]

7. \( \cos \theta = 0.0872; \theta \) is in Quadrant IV
   \[ 4.8 \]

8. \( \tan \theta = 0.3839; \cos \theta < 0 \)
   \[ 3.508 \]

9. \( \cos \theta = 0; \sin \theta > 0 \)
   \[ 1.570 \]

10. \( \sin \theta = -0.1908; \tan \theta > 0 \)
    \[ 3.334 \]

11. \( \tan \theta = -0.4663; \sin \theta > 0 \)
    \[ 2.705 \]

12. \( \tan \theta = -0.4663; \cos \theta > 0 \)
    \[ 5.847 \]

13. \( \tan \theta = -1; \sin \theta > 0 \)
    \[ 2.356 \]

14. \( \sin \theta = -1 \)
    \[ 4.712 \]

15. Explain why #14 needed only 1 clue to determine a unique value for \( \theta \), and the others required at least 2 clues.

    Both sine and cosine equal 1 and \(-1\) in only one location
Go

Topic: Verifying trigonometric identities

Half of an identity and the graph of this half are given. Use the graph to make a conjecture as to what the right side of the identity should be. Then prove your conjecture.

16. \((\sin \frac{x}{2} + \cos \frac{x}{2})^2 = ~ \)

17. \(\sin 2x \sec x = ~ \)

\[
\begin{align*}
= \sin x + 1 \\
\text{proofs will vary}
\end{align*}
\]

\[
\begin{align*}
= 2 \sin x \\
\text{proofs will vary}
\end{align*}
\]
Name: 

Trigonometric Functions 4.13H

Ready, Set, Go!

Ready
Topic: Using graphs to evaluate function expressions

Use the graph of \( f(x) \) and \( g(x) \) below to find the indicated values.

1. \( f(-4)g(-4) \)
   \( 0 \)
2. \( f(-2)g(-2) \)
   \( 4 \)
3. \( 2f(4) + 4g(2) \)
   \( 32 \)
4. \( g(-5) - f(-4) + g(5) + f(4) \)
   \( 0 \)
5. \( \frac{f(2)}{g(2)} \)
   \( \frac{1}{2} \)

Set
Topic: Solving trigonometric equations

Find all solutions of the equations in the interval \([0, 2\pi)\).

6. \( 2\cos^2 x - \cos x = 1 \)
   \( x = 0, \frac{2\pi}{3}, \frac{4\pi}{3} \)
7. \( 2\sin^2 x - 3\sin x = -1 \)
   \( x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{\pi}{2} \)
8. \( \cos^2 x + \sin x - 1 = 0 \)

\[ x = 0, \pi, \frac{\pi}{2} \]

9. \( 2 \sin 2x - \sqrt{2} = 0 \)

\[ x = \frac{\pi}{8}, \frac{3\pi}{8}, \frac{9\pi}{8}, \frac{11\pi}{8} \]

10. \( \sqrt{3} \tan 3x = 0 \)

\[ x = \frac{\pi}{3}, \frac{2\pi}{3}, \pi, \frac{4\pi}{3}, \frac{5\pi}{3} \]

11. \( 3 \csc^2 5x = -4 \)

no solution

12. \( \cos (x + \frac{\pi}{6}) - \cos (x - \frac{\pi}{6}) = 1 \)

\[ x = \frac{3\pi}{2} \]
Go
Topic: Graphing trigonometric functions

Match the equation with the correct graph.
A. \( y = \sin 2x \)  
B. \( y = (\sin x) + 2 \)  
C. \( y = 3 \sin x \)
D. \( y = -(\sin x) - 2 \)  
E. \( y = -2 \sin x \)  
F. \( y = 3 \sin 2x \)

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SDUHSD Math 3 Honors
Topic: Locating points in terms of rectangular coordinates, arc length, reference angle, and radius.

In the diagram, $\Delta ABC$ is a right triangle. Point B lies on the circle and is described by the rectangular coordinates $(x, y)$, $s$ is the length of the intercepted arc created by angle $\theta$, $r$ is the radius of circle A.

**Answer the following questions using the given information.**

19. B is described by $(r, s)$ where $s \approx 62.26$ and $r = 73$.
   a. Find $\theta$ by using the formula $s = \frac{\theta}{360^\circ} (d\pi)$.
      \[48.9^\circ\]
   b. Find $(x, y)$ to the nearest whole number
      \[(48, 55)\]
   c. Describe point B using the coordinates $(r, \theta)$.
      \[(73, 48.9^\circ)\]