# Linear and Exponential Functions 4.1

# Ready, Set, Go!

## Ready

Topic: Recognizing arithmetic and geometric sequences

Predict the next 2 terms in the sequence. State whether the sequence is arithmetic, geometric, or neither. Justify your answer.

1.	4, -20, 100, -500,	2.	3, 5, 8, 12, …
	25,000, —125, 000 Geometric, multiply by —5		17, 23 Neither

3. 64, 48, 36, 27, …

20.25, 15.1875 Geometric, multiply by  $\frac{3}{4}$ 

5. 40, 10,  $\frac{5}{2}$ ,  $\frac{5}{8}$ , ...

 $\frac{5}{32}, \frac{5}{128}$ Geometric, multiply by  $\frac{1}{4}$ 

- 7. -3.6, -5.4, -8.1, -12.15, ···
   8. -64, -47, -30, -13, ···

   -18.225, -27.228
   4, 21

   Geometric, multiply by 1.5
   Arithmetic, add 17
- 9. Create a predictable sequence of at least 4 numbers that is NOT arithmetic or geometric.

Answers will vary



4. 1.5, 0.75, 0, -0.75, ...

6. 1, 11, 111, 1111, …

**Neither** 

-1.5, -2.25 Arithmetic, subtract 0.75

# Set

Topic: Discrete and continuous relationships

## Identify whether the following statements represent a *discrete* or a *continuous* relationship.

10. The hair on your head grows  $\frac{1}{2}$  inch per month. **Continuous** 

11. For every ton of paper that is recycled, 17 trees are saved. Discrete

12. Approximately 3.24 billion gallons of water flow over Niagara Falls daily. Continuous

13. The average person laughs 15 times per day. Discrete

14. The city of Buenos Aires adds 6,000 tons of trash to its landfills every day. Continuous

15. During the Great Depression, stock market prices fell 75%. Continuous

## Go

Topic: Slopes of lines

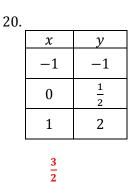
## Determine the slope of the line that passes through the following points.

16. (-15,9), (-10,4)	17. (0.5, 4), (3, 3.5)	18. (50,85), (60,80)
-1	$-\frac{1}{5}$	$-\frac{1}{2}$

1	q
T	7

x	у
-5	-20
-4	-17
-3	-14

3



x	у
-5	33
0	30
5	27
3	

5

21.

# Ready, Set, Go!

# Ready

Topic: Rates of change in linear models

## Say which situation has the greatest rate of change

1. The amount of stretch in a short bungee cord stretches 6 inches when stretched by a 3 pound weight. A slinky stretches 3 feet when stretched by a 1 pound weight.

## Slinky

2. A sunflower that grows 2 inches every day or an amaryllis that grows 18 inches in one week.

## Amaryllis

3. Pumping 25 gallons of gas into a truck in 3 minutes or filling a bathtub with 40 gallons of water in 5 minutes.

## **Pumping gas**

4. Riding a bike 10 miles in 1 hour or jogging 3 miles in 24 minutes.

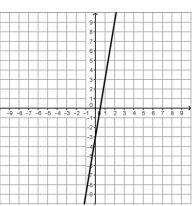
## **Riding a bike**

Topic: Recognizing the greater rate of change when comparing linear functions and exponential functions.

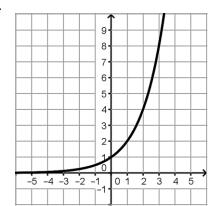
Identify whether situation "a" or situation "b" has the greater rate of change. b

5.	а

x	у
-10	-48
-9	-43
-8	-38
-7	-33

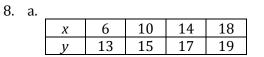


6. a.



7. a. Lee has \$25 withheld each week from his salary to pay for his subway pass.

## Depends (probably a)



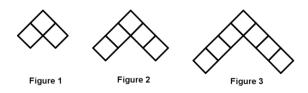
9. (a)  $y = 2(5)^x$ 

9 8. 7 6. 5. 4. 3. 2 -5 -4 -3 -2 01 2 3 4 5 -1

b

b. Jose owes his brother \$50. He has promised to pay half of what he owes each week until the debt is paid.

b. The number of rhombi in each shape.

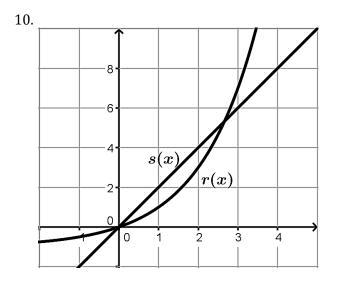


- b. In the children's book, *The Magic Pot*, every time you put one object into the pot, two of the same objects come out. Imagine that you have 5 magic pots.
- a. Examine the graph at the left from 0 to 1. Which graph do you think is growing faster?

**s**(**x**)

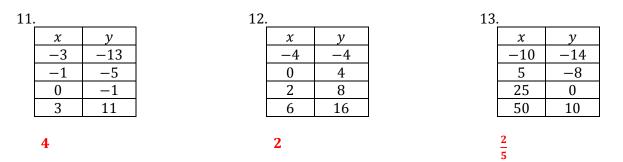
b. Now look at the graph from 2 to 3. Which graph is growing faster in this interval?

r(x)



## **Set** Topic: linear rates of change

## Determine the rate of change in each table below.



14. Complete the tables below to create the indicated type of function.

a.	Linear	Function
····		

Linear raneron		
x	f(x)	
-5		
-4		
-3		
-2		
-1		
0		

type of function.			
b.	Exponen	tial Funct	ion
	x	f(x)	
	-5		
	-4		
	-3		
	-2		
	-1		
	0		

Answers will vary. For the "Linear Function," check for equal differences over equal intervals. For the "Exponential Function," check for equal factors/ratios over equal intervals.

Topic: Recognizing linear and exponential functions.

#### For each representation of a function, decide if the function is linear, exponential, or neither.

15. The population of a town is decreasing at a rate of 1.5% per year.

16. Joan earns a salary of \$30,000 per year plus a 4.25% commission on sales

-			
Exr	on	en	tial

#### Linear

17. The number of gifts received each day of "The 12 Days of Vacation" as a function of the day. ("On the 4<sup>th</sup> day of Vacation my true love gave to me, 4 calling birds, 3 French hens, 2 turtle doves, and a partridge in a pear tree.")

## Neither

## Go

Topic: Recursive and explicit equations of geometric sequences.

#### Write the recursive and explicit equations for each geometric sequence.

18. Marissa has saved \$1000 in a jar. She plans to withdraw half of what's remaining in the jar at the end of each month.

Recursive equation:Explicit Equation: $f(0) = 1000, f(n) = f(n-1) \cdot \frac{1}{2}$  $f(n) = 1000 \cdot \left(\frac{1}{2}\right)^n$ 

19.	
Time	Number of
(Days)	Bacteria
1	10
2	100
3	1000
4	10000

Recursive Equation:  $f(1) = 0, f(n) = f(n-1) \cdot 10$ 

Explicit Equation:  $f(n) = 10^n$  or  $f(n) = 10 \cdot 10^{n-1}$ 

21. 1024, 256, 64, 16, ...

Recursive Equation:  $f(1) = 1024, f(n) = f(n-1) \cdot \frac{1}{4}$ 

Explicit Equation:  $f(n) = 4096 \cdot \left(\frac{1}{4}\right)^n$ 

10.		
	Folds in	Number of
	paper	rectangles
	0	1
	1	2
	2	4
	3	8

Recursive Equation:  $f(0) = 1, f(n) = f(n-1) \cdot 2$ 

Explicit Equation:  $f(n) = 2^n$ 

 $22. \ 3, 9, 27, 81, \dots$ 

Recursive Equation:  $f(1) = 3, f(n) = f(n-1) \cdot 3$ 

Explicit Equation:  $f(n) = 3^n$ 

# For each geometric sequence below, find the missing terms in the sequence. 23.

x	1	2	3	4	5
у	2	±6	18	±54	162

24.

 1.						
x	1	2	3	4	5	
у	$\frac{1}{9}$	$-\frac{1}{3}$	1	-3	9	

# Ready, Set, Go!

# Ready

Topic: Comparing arithmetic and geometric sequences.

The 1<sup>st</sup> and 5<sup>th</sup> terms of a sequence are given. Fill in the missing numbers for an arithmetic sequence. Then fill in the numbers for a geometric sequence.

1.						
	Arithmetic	3	14.25	25.5	36.75	48
	Geometric	3	<b>±6</b>	12	<u>+</u> 24	48

2.

Arithmetic	-12	-9.1875	-6.375	-3.5625	-0.75
Geometric	-12	<b>±6</b>	-3	±1.5	-0.75

Topic: Comparing linear and exponential models

In questions 3-8, compare different characteristics of each type of function by filling in the cells of each table as completely as possible.

	y = 4 + 3x	$y = 4(3^x)$	
3. Type of growth	Linear	Exponential	
4. What kind of sequence corresponds to each model?	Arithmetic	Geometric	
5. Make a table of values	x         y           0         4           1         7           2         10           3         13	x         y           0         4           1         12           2         36           3         108	
6. Find the rate of change	Add 3	Not Constant	
<ul><li>7. Graph each equation.</li><li>Compare the graphs.</li><li>What is the same?</li><li>What is different?</li></ul>	-9 -8 -7 -6 -5 -4 -3 -2 -1 -2 -3 -5 -4 -5 -5 -6 -7 -9 -9 -9 -9 -9 -9 -9 -9 -9 -9	$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	
8. Find the <i>y</i> -intercept for each function.	(0,4)	(0,4)	



- 9. Find the *y*-intercepts (where the graph crosses the *y*-axis and when the value of *x* is 0) for the following equations:
  - a. y = 3x(0, 0)

b. 
$$y = 3^x$$
  
(0, 1)

10. Explain how you can find the *y*-intercept of a linear equation and how that is different from finding the *y*-intercept of a geometric equation.

y = mx + b, *b* is the *y*-intercept  $y = a \cdot b^x$ , a is the *y*-intercept

# Set

Topic: Comparing linear and exponential functions.

- 11. *Calcu-rama* had a net income of 5 million dollars in 2010, while a small competing company, *Computafest*, had a net income of 2 million dollars. The management of *Calcu-rama* develops a business plan for future growth that projects an increase in net income of 0.5 million per year, while the management of *Computafest* develops a plan aimed at increasing its net income by 15% each year.
  - a. Express the projected net incomes in these two business plans as recursive formulas.

Calcu-rama: f(n + 1) = f(n) + 0.5, f(0) = 5 where n = 0 corresponds to the year 2010 Computation computations for f(n + 1) = f(n)(1, 15), f(0) = 2 where n = 0 corresponds to the year 2010

b. Write an explicit equation for the net income as a function of time for each company's business plan.

Calcu-rama: f(n) = 5 + 0.5nComputafest:  $f(n) = 2(1.15)^n$ 

c. Compare your answers in *a* and *b*. How are the two representations similar? How do they differ? What relationships are highlighted in each representation?

**Answers vary** 

d. Explain why if both companies are able to meet their net income growth goals, the net income of *Computafest* will eventually be larger than that of *Calcu-rama*. In what year will the net income of *Computafest* be larger than that of *Calcu-rama*?

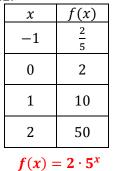
#### Exponential growth functions increase at a greater rate than linear functions

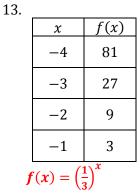
# Go

Topic: Writing explicit equations for linear and exponential models.

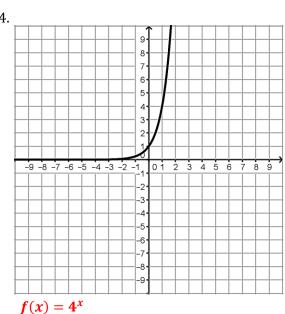
# Write the explicit equation for the tables and graphs below.



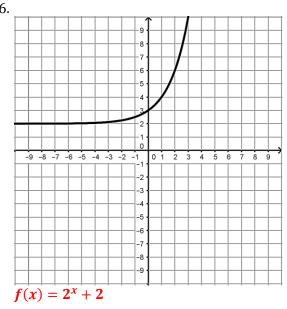




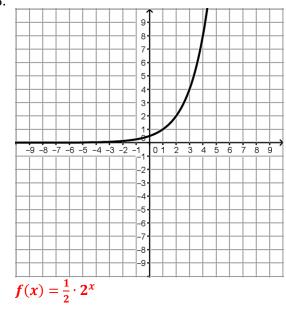
14.



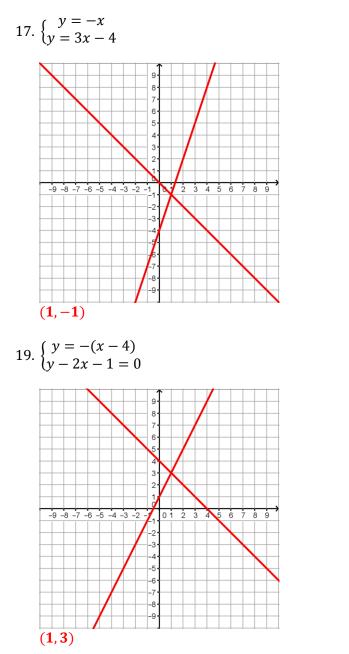


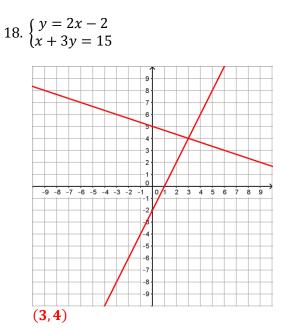






# Find the solution to the systems of equations by graphing.





# Ready, Set, Go!

## Ready

Topic: Writing equations of lines.

## Write the equation of a line in slope-intercept form: y = mx + b, using the given information.

1. 
$$m = -7, b = 4$$
  
 $y = -7x + 4$   
2.  $m = \frac{3}{8}, b = -3$   
 $y = \frac{3}{8}x - 3$   
3.  $m = 16, b = -\frac{1}{5}$   
 $y = 16x - \frac{1}{5}$ 

Write the equation of the line in point-slope form:  $y = m(x - x_1) + y_1$ , using the given information.4. m = 9, (0, -7)5.  $m = \frac{2}{3}, (-6, 1)$ 6. m = -5, (4, 11)y = 9x - 7 $y = \frac{2}{3}(x + 6) + 1$ y = -5(x - 4) + 11

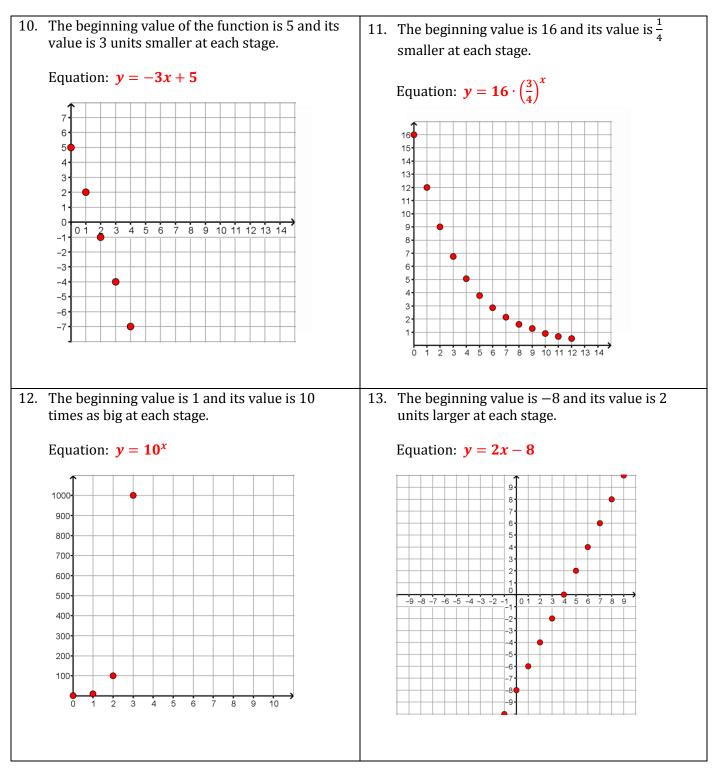
7. 
$$(2,-5) (-3,10)$$
8.  $(0,-9) (3,0)$ 9.  $(-4,8) (3,1)$  $y = -3(x-2) - 5$  $y = 3x - 9$  $y = -1(x+4) + 8$ orororor $y = -3(x+3) + 10$  $y = 3(x-3)$  $y = -1(x-3) + 1$ 



# Set

Topic: Graphing linear and exponential functions

# Make a graph of the function based on the following information. Add your axes. Choose an appropriate scale and label your graph. Then write the equation of the function.



# Go

Topic: Recognizing linear and exponential functions.

## For each representation of a function, decide if the function is linear, exponential, or neither.

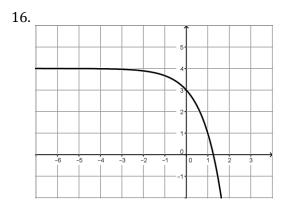
15.

14. 3x + 4y = -3

#### Linear

Side of a square	Area of a square
1 inch	1 in <sup>2</sup>
2 inches	4 in <sup>2</sup>
3 inches	9 in <sup>2</sup>
4 inches	16 in <sup>2</sup>

Neither



**Exponential (Students most likely won't see this, but the equation is**  $f(x) = -(3)^x + 4$ **)** 

Topic: Slope-intercept form

**Rewrite the equations in slope-intercept form.** 17. 2y + 10 = 6x + 12

y = 3x + 1

18.  $(y - 13) = \frac{1}{2}(8x - 14)$ 

$$y = 4x + 6$$

19. 
$$(y + 11) = -7(x - 2)$$
  
 $y = -7x + 3$ 
20.  $3(2x - y) = 9x + 12$   
 $y = -x - 4$ 

21. 
$$y - 2 = \frac{1}{5}(10x - 25)$$
  
 $y = 2x - 3$   
22.  $y + 1 = \frac{3}{4}(x + 3)$   
 $y = \frac{3}{4}x + \frac{5}{4}$ 

## Name:

# Ready, Go!

# Ready

Topic: Simple interest

When a person borrows money, the lender usually charges "rent" on the money. This "rent" is called interest. Simple interest is a percent "**r**" of the original amount borrowed "**P**" multiplied by the time "**t**", usually in years. The formula for calculating the interest is i = Prt.

#### Calculate the simple interest owed on the following loans.

1. $P = $1000$	r = 11%	t = 2 years	i = <b>\$200</b>
2. $P = $6500$	r = 12.5%	t = 5 years	i = <b>\$4062.50</b>
3. $P = $20,000$	r = 8.5%	t = 6 years	i = <b>\$10,200</b>
4. $P = $700$	<i>r</i> = 20%	t = 6 months	i = <b>_\$70</b>

## Go

Topic: Solving multi-step equations

#### Solve the following equations

5. $12 + 6x - 4 = 5 + 2(3x - 1)$	6. $5(2x+4) = 3(x+5) - 19$
No Solution	$x = -\frac{24}{7}$
	,
7. $7 - 3(4x + 2) = 6(2x + 3) - 17$	8. $2(x+1) = 6(x-3)$
x = 0	<i>x</i> = 5

9. What does it mean when you have solved an equation?

Solving an equation means you have found the value(s) of *x* that make the equation true.

10. Explain how a linear equation can have more than one solution.

There are an infinite number of pairs of *x* and *y* values that make any linear equation true.



**TE-59** 

# For each geometric sequence below, find the missing terms in the sequence. 11.

x	1	2	3	4	5
у	10	±5	2.5	±1.25	0.625

12

X	1	2	3	4	5
у	g	gz	$gz^2$	$gz^3$	$gz^4$

13.

13.						
	X	1	2	3	4	5
	у	-3	±9	-27	±81	-243

## Name:

## Set, Go!

## Set

Topic: Evaluate using the formulas for simple interest or compound interest.

**Given the formula for simple interest:** *i* = *Prt*, calculate the simple interest paid. (*Remember*, *i* = *interest*, *P* = *the principal*, *r* = *the interest rate per year as a decimal*, *t* = *time in years*)

1. Find the simple interest you will pay on a 5 year loan of \$7,000 at 11% per year.

## \$3850

2. How much interest will you pay in 2 years on a loan of \$1500 at 4.5% per year?

#### **\$135**

	i =	P P	× r	× t
3.	\$4,059	\$11,275	12%	3 years
4.	\$1,428	\$5,100	4%	7 years
5.	\$93.75	\$1,250	15%	6 months
6.	\$54	\$900	8%	9 months

#### Use i = Prt to complete the table. All interest rates are annual.

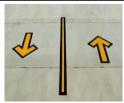
Given the formula for compound interest:  $A = P(1 + r)^t$ , write a compound interest function to model each situation. Then calculate the balance after the given number of years. *(Remember: A = the balance after t years, P = the principal, t =the time in years, r = the annual interest rate expressed as a decimal)* 

7. \$22,000 invested at a rate of 3.5% compounded annually for 6 years.

## \$27,043.62

8. \$4300 invested at a rate of 2.8% compounded annually for 15 years.

## \$6,506.77



9. Suppose that when you are 15 years old, a magic genie gives you the choice of investing \$10,000 at a rate of 7% or \$5,000 at a rate of 12%. Either choice will be compounded annually. The money will be yours when you are 65 years old. Which investment would be the best? Justify your answer.

**\$10,000 at 7% ≈ \$294,570** 

\$5,000 at 12% ≈ \$1,445,010

### Go

Topic: Using order of operations when evaluating expressions

Evaluate the expressions for the given values of the variables.

 10.  $pq \div 6 + 10$ ; when p = 7 and q = -3 11. m + n(m - n); when m = 2, and n = 6 

 6.5
 -22

 12.  $(b - 1)^2 + ba^2$ ; when a = 5, and b = 3 13. y(x - (9 - 4y)); when x = 4, and y = -5 

 79
 125

14. 
$$x - (x - (x - y^3))$$
; when  $x = 7$ , and  $y = 2$   
-1
-522
-522

# Ready, Set, Go!

# Ready

Topic: Discrete vs. continuous

1. Give a contextual example that can be modeled by a discrete function.

## Answers will vary

2. Give a contextual example that can be modeled by a continuous function.

## **Answers will vary**

Topic: Arithmetic and geometric means

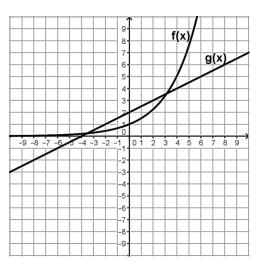
3. The first and 5th terms of a sequence are given. Fill in the missing numbers for an arithmetic sequence. Then fill in the numbers for a geometric sequence.

Arithmetic	-6250	-4690	-3130	-1570	-10
Geometric	-6250	±1250	<b>-250</b>	<b>±50</b>	-10

Topic: Comparing rates of change

4. Compare the rate of change in the pair of functions in the graph by identifying the interval where it appears that f(x) is changing faster and the interval where it appears that g(x) is changing faster. Verify your conclusions by making a table of values for each function and exploring the rates of change in your tables.

 $f(x) \approx (-\infty, 0.5)$  $g(x) \approx (0.5, \infty)$ 



Topic: Determining if a sequences is linear, exponential, or neither.

## 5. Identify the following sequences as linear, exponential, or neither.

a.	–23, –6, 11, 28, <mark>Linear</mark>	b.	49, 36, 25, 16, Neither	C.	5125, 1025, 205, 41, Exponential
d.	2, 6, 24, 120, <mark>Neither</mark>	e.	0.12, 0.36, 1.08, 3.24, <mark>Exponential</mark>	f.	21, 24.5, 28, 31.5, <mark>Linear</mark>



# Set

Topic: Comparing linear and exponential functions using multiple representations

# Describe the defining characteristics of each type of function by filling in the cells of each table as completely as possible.

	y = 6 + 5x	$y = 6(5)^x$	
6. Type of growth	Linear	Exponential	
7. What kind of sequence corresponds to each model?	Arithmetic	Geometric	
8. Make a table of values	x       y         0       6         1       11         2       16         3       21         4       26	xy061302150375043750	
9. Find the rate of change	5	Not Constant	
10. Graph each equation. Compare the graphs. What is the same? What is different?	9 7 7 6 5 7 6 5 7 6 7 6 7 6 7 6 7 6 7 6 7 7 6 7 7 6 7 7 6 7 7 6 7 7 7 7 6 7 7 7 7 7 7 7 7 7 7 7 7 7	2000 1800 1600 1400 1000 1000 00 10	
11. Find the <i>y</i> -intercept for each function.	(0,6)	(0,6)	
12. Write the recursive form of each equation.	f(0) = 6 f(x) = f(x-1) + 5	$f(0) = 6$ $f(x) = f(x-1) \times 5$	

# There were 2 girls in my grandmother's family, my mother and my aunt. They each had 3 daughters. My two sisters, 3 cousins, and I each had 3 daughters. Each one of our 3 daughters have had 3 daughters.

13. If the pattern of each girl having 3 daughters continues for 2 more generations (my mom and aunt being the 1<sup>st</sup> generation, I want to know about the 5<sup>th</sup> generation), how many daughters will be born then?

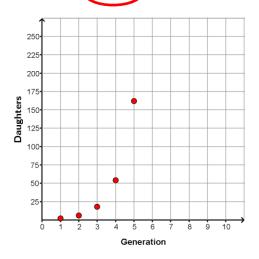
**162** 

14. Write the explicit equation for this pattern.

 $y = \frac{2}{3} \cdot 3^x$ 

15. Create a table and a graph describing this pattern. Is this situation discrete or continuous?

S



## Go

Topic: Solving multi-step equations

#### Solve the following equations.

x = -5	No Solution	Infinitely Many Solutions
16. $5x + 3 = 2(x - 6)$	17. $6x - 12x + 10 = 2(-3x - 6)$	18. $13x - 12x + \frac{1}{2} = x + \frac{3}{6}$

Recall the following formulas: S

# Using the formulas for simple interest or compound interest, calculate the following.

19. The simple interest on a loan of \$12,000 at an interest rate of 17% for 6 years.

## \$12,240

20. The simple interest on a loan of \$20,000 at an interest rate of 11% for 5 years.

# \$11,000

21. The amount owed on a loan of \$20,000, at 11%, compounded annually for 5 years.

# \$33,701.16

22. Compare the interest paid in #26 to the interest paid in #27. Which kind of interest do you want if you have to take out a loan?

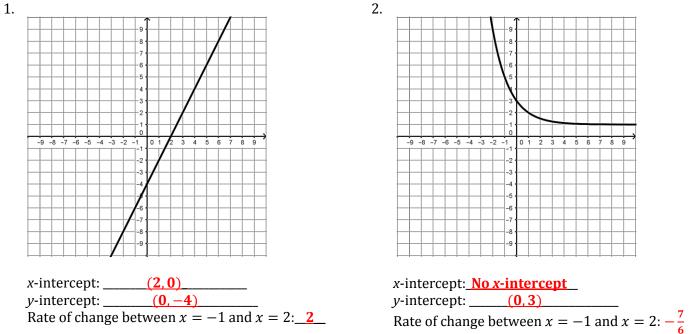
There is a \$22,701.16 difference between the simple and compound interests described above. You would want the simple interest.

23. The amount in your savings account at the end of 30 years, if you began with \$2500 and earned an interest rate of 7% compounded annually.

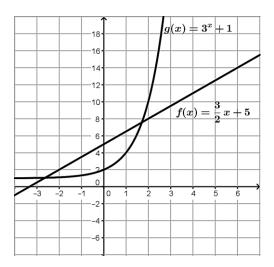
## \$19,030.64

## Module 4 Review Homework Characteristics of Functions:

## For each of the functions find the following information.



3. Discuss and compare the functions by analyzing the rates of change, intercepts, and where one function is greater or less than the other.



- $f(x): \text{ rate of change: } \frac{3}{2}$ x-intercept:  $\left(\frac{10}{3}, 0\right)$ y-intercept: (0, 5)
- g(x): rate of change: not constant x-intercept: none y-intercept: (0, 2)

f(x) is greater than g(x) from about x = -2.75 to 1.75.

#### Linear and Exponential Models:

4. Write an explicit formula to model the number of dots per day.

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Day 1

Day 3

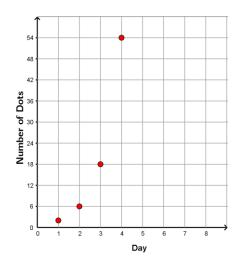
$$f(x) = \frac{2}{3} \cdot 3^{x}$$
 or  $2 \cdot 3^{x-1}$ 

#### Model the function using the table provided:

Day 2

Day	Number of Dots
1	2
2	6
3	18
4	54

Use your table to create a graph of the function:



5. Sherry has a huge doll collection of 80 dolls. Her mom tells her that she needs to get rid of 5 per year to get it down to a decent number before leaving for college. Write an explicit formula to model the number of dolls per year. If she is 12, how many will she have left when she is 18?

f(x) = 80-5xWhen she is 18, x = 6 so f(6) = 50. Sherry will have 50 dolls when she is 18 years ols.

6. You bought a Toyota Corolla in 2004 for \$12,500. The car's value depreciates by 7% a year. How much is the car worth now? How much is it worth in 2020?

 $f(x) = 12500 \cdot (1 - 0.07)^x$ In 2020, x = 16 so  $f(16) \approx $3914.15$ . The car will be worth approximately \$3,914.15 in 2020.

- 7. The population of a large city increases by a rate of 3% a year. When the 2000 census was taken, the population was 1.2 million.
  - a. Write a model for this population growth.

 $f(x) = 1.2 \cdot (1 + 0.03)^x$ 

b. What should the population be now? What is the projected population for 2020?

In 2014, x = 14 so f(14) = 1.815 million people. In 2020, x = 20 so f(20) = 2.167 million people.

8. Bank Plans:

Suppose you worked mowing lawns all summer and earned \$50. Two savings institutions, Linear Luck and Exponential Experiment want you to let them "hold onto your money" for a while.

**Linear Luck**: This savings plan will add \$100 to your balance for every month that you leave your money in the account.

**Exponential Experiment**: This savings plan will multiply your balance by 2 every month that you leave your money in their account.

Analyze the plans: Write the explicit function for each account, and decide which account is best at what times.

Linear Luck: f(x) = 100x + 50

**Exponential Experiment:**  $f(x) = 50 \cdot 2^x$ 

Linear Luck is better for only the first 2 months. After about 2 and a half months, both accounts will be about \$315. After that, Exponential Experiment is much better.

