Name:

Ready, Set, Go!

Ready

Topic: Finding values for a pattern

1. Bob Cooper was born in 1900. By 1930 he had 3 sons, all with the Cooper last name. By 1960 each of Bob's 3 boys had exactly 3 sons of their own. By the end of each 30 year time period, the pattern of each Cooper boy having exactly 3 sons of their own continued. How many Cooper sons were born in the 30 year period between 1960 and 1990?
   27

2. Create a diagram that would show this pattern.

Year	1900	1930	1960	1990	2020
# of sons	1	3	9	27	81

3. Predict how many Cooper sons will be born between 1990 and 2020, if the pattern continues.
   81

4. Try to write an equation that would help you predict the number of Cooper sons that would be born between 2020 and 2050. If you can't find the equation, explain it in words.
   $f(n) = 3^{\frac{2050-1900}{30}} = 243$

5. How many Cooper sons were born in all from 1900 to 2020?
   121

Topic: Function Notation

For each of the following, find $f\,(1)$, $f\,(2)$ and $f\,(3)$

6. $f(x) = 2^x$

   $f(1) = 2, f(2) = 4, f(3) = 8$

7. $f(x) = 3(-2)^x$

   $f(1) = -6, f(2) = 12, f(3) = -24$

8. $f(x) = 2(x-1) + 3$

   $f(1) = 3, f(2) = 5, f(3) = 7$

Complete each table.

9.

Term	1st	2nd	3rd	4th	5th	6th	7th	8th
Value	66	50	34	18	**2**	**−14**	**−30**	**−46**

10.

Term	1st	2nd	3rd	4th	5th	6th	7th	8th
Value	−3	9	−27	81	**−243**	**729**	**−2187**	**6561**

Set

Topic: Completing a table

Fill in the table. Then write a sentence explaining how you figured out the values to put in each cell. Explain how to figure out what will be in cell #8.

11. You run a business making birdhouses. You spend $600 to start your business, and it costs you $5.00 to make each birdhouse.

# of birdhouses	0	1	2	3	4	5	6
Total cost to build	**600**	**605**	**610**	**615**	**620**	**625**	**630**

Explanation: **For the first birdhouse it costs $600 to start the business, plus $5 for the first birdhouse, and after that it is just $5 more for each additional birdhouse. The cost for making 8 birdhouses is then $635 + 5, or $640.**

12. You borrow $500 from a relative, and you agree to pay back the debt at a rate of $15 per month.

# of months	1	2	3	4	5	6	7
Amount of money owed	**500**	**485**	**470**	**455**	**440**	**425**	**410**

Explanation: **On month #1 you owe the total, $500, then every month after that you owe $15 less because you paid $15 toward the debt at the end of each month. On the 8th month, the amount of money owed would be 410 − 15, or $395**

Topic: Evaluating equations

Evaluate the following equations when $x = \{1, 2, 3, 4, 5\}$. Organize your inputs and outputs into a table of values for each equation. Let x be the input and y be the output.

13. $y = 4^x$

x	y
1	**4**
2	**16**
3	**4**
4	**256**
5	**1024**

14. $y = (-3)^x$

x	y
1	**−3**
2	**9**
3	**−27**
4	**81**
5	**−243**

15. $y = -3^x$

x	y
1	**−3**
2	**−9**
3	**−27**
4	**−81**
5	**−243**

Go

Topic: Good viewing window

When sketching a graph of a function, it is important that we see key points. For linear functions, we want a window that shows important information related to the story. Often, this means including both the x- and y-intercepts

Find an appropriate graphing window for each of the following linear functions. Fill in the blanks showing the lower and upper values and include the scale for each axis.

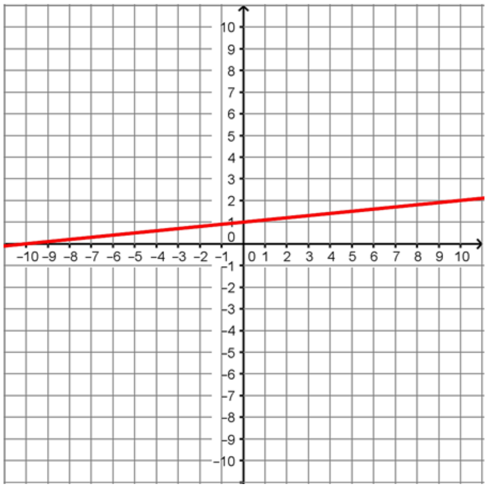
You may use an online graphing utility such as Desmos (https://www.desmos.com/calculator) or MATHPAPA (https://www.mathpapa.com/calc.html?q=)

Answers may vary. Sample answers provided below:

16. $f(x) = \frac{1}{10}x + 1$

x: [-20 , 20] by y: [-10 , 10]

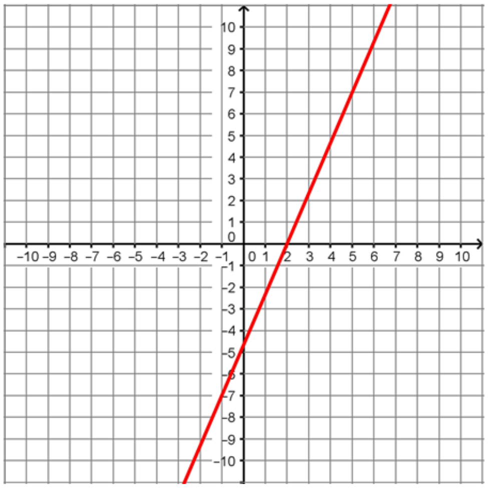
x-scale: 2 y-scale: 1



17. $7x - 3y = 14$

x: [-10 , 10] by y: [-10 , 10]

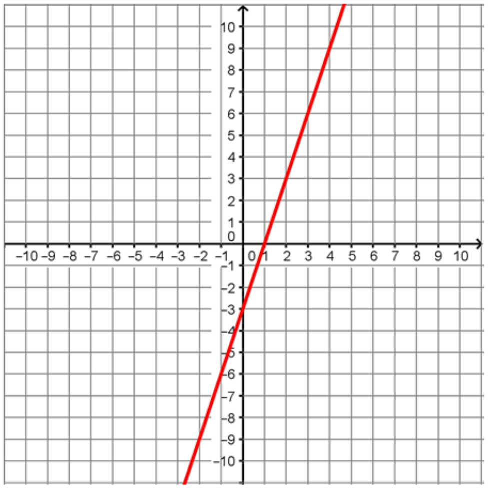
x-scale: 1 y-scale: 1



18. $y = 3(x - 5) + 12$

x: [-10 , 10] by y: [-10 , 10]

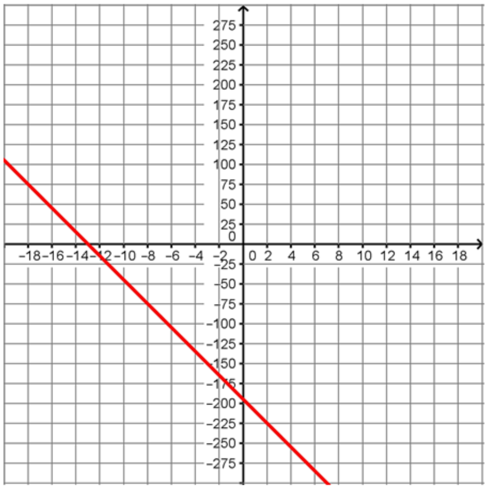
x-scale: 1 y-scale: 1



19. $f(x) = -15(x + 10) - 45$

x: [-20 , 20] by y: [-300 , 300]

x-scale: 2 y-scale: 25



Name:

Ready, Set, Go!

Ready

Topic: Write the equation of a line given two points.

Write an equation of the line that goes through the given two points.

1. $(5, 2)$ and $(7, 0)$

$y = -x + 7$

2. $(2, -4)$ and $(2, 6)$

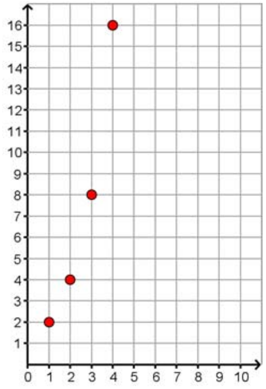
$x = 2$

Set

Topic: Recursive and explicit functions of arithmetic sequences

Below you are given various types of information. Write the recursive and explicit functions for each sequence. Finally, graph each sequence, making sure you clearly label your axes.

3. $2, 4, 8, 16, \ldots$



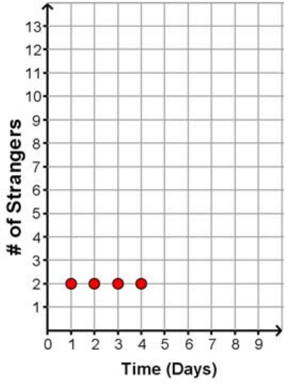
Recursive:

$f(1) = 2, f(n) = f(n-1) \times 2$

Explicit:

$f(n) = 2^n$

4. Each day Tania decides to do something nice for 2 strangers. Write recursive and explicit equations that represent the number of strangers Tania that does something nice for each day (not total number of strangers).



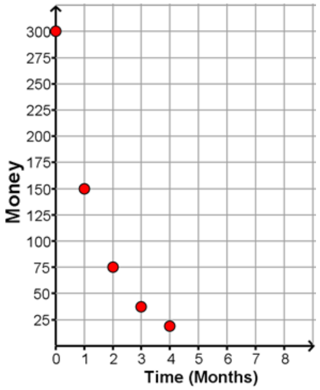
Recursive:

$f(1) = 2, f(n) = f(n-1)$

Explicit:

$f(n) = 2$

5. Claire has $300 in an account. She decides she is going to take out half of what's left in there at the end of each month.



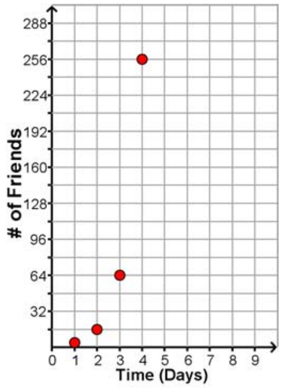
Recursive:

$\boldsymbol{f(0) = 300, f(n) = f(n-1) \times \frac{1}{2}}$

Explicit:

$\boldsymbol{f(n) = 300\left(\frac{1}{2}\right)^{n}}$

6. Tania creates a chain letter and sends it to four friends. Each day each friend is then instructed to send it to four friends and so forth.



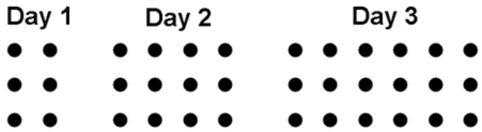
Recursive:

$\boldsymbol{f(1) = 4, f(n) = f(n-1) \times 4}$

Explicit:

$\boldsymbol{f(n) = 4^{n}}$

7.

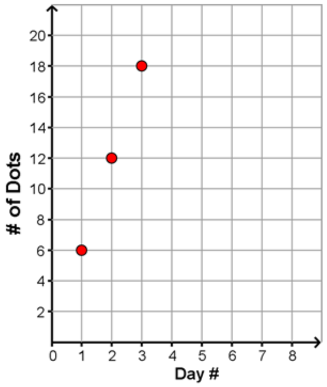


Recursive:

$\boldsymbol{f(1) = 6, f(n) = f(n-1) + 6}$

Explicit:

$\boldsymbol{f(n) = 6n}$



Topic: Summation notation for a series

8. Write out what is meant by:

a. $\sum_{n=1}^{5} 3^{n}$

$\boldsymbol{3 + 9 + 27 + 81 + 243 = 363}$

b. $\sum_{k=2}^{4} k$

$\boldsymbol{2 + 3 + 4 = 9}$

9. Write the following in summation notation:

a. $3 + 3 + 3 + 3$

$\boldsymbol{\sum_{x=1}^{4} 3}$

b. $2 + 4 + 6 + 8 + 10 + 12$

$\boldsymbol{\sum_{x=1}^{6} 2 + 2(x-1)}$

10. Are the following series equivalent? Explain your reasoning.

$\sum_{x=1}^{50} \frac{1}{x}$ and $\sum_{x=21}^{70} \frac{1}{x-20}$

Yes. Both sequences represent: $\boldsymbol{1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \cdots + \frac{1}{50}}$

Go

Topic: Arithmetic and geometric sequences

Determine if the following sequences are arithmetic, geometric, both, or neither.

11. 109, 94, 79, 64

    Arithmetic

12. Christine did 41 sit-ups on Tuesday, 44 sit-ups on Wednesday, 46 sit-ups on Thursday, 47 sit-ups on Friday.

    Neither

13. 1, 9, 81, 729, …

    Geometric

14. While sorting change into a piggy bank, Ruth put 14 coins in the first piggy bank, 14 coins in the second piggy bank, 14 coins in the third piggy bank, and 14 coins in the fourth piggy bank.

    Both

15. $6, -24, 144, -864$

    Geometric

16. A bookshelf has 7 shelves of different widths. Each shelf is narrower than the shelf below it. The bottom three shelves are 36 in., 31 in., and 26 in. wide. The shelf widths decrease by the same amount from bottom to top.

    a. What is the width of the top shelf?

       6 inches

    b. What is the total shelf space of all seven shelves?

       147 inches

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Topic: Arithmetic and geometric sequences

Find the missing values for each arithmetic or geometric sequence. Then say if the sequence has a constant difference or a constant ratio, and say what the constant difference/rate is.

1. 5, 10, 15, _____, 25, 30…

   Constant difference or a constant ratio?

   Constant Difference

   The constant difference/ratio is **5**

2. 20, 10, _____, 2.5, _____,…

   Constant difference or a constant ratio?

   Constant Ratio

   The constant difference/ratio is $\mathbf{\frac{1}{2}}$

3. 2, 5, 8, _____, 14, _____,…

   Constant difference or a constant ratio?

   Constant Difference

   The constant difference/ratio is **3**

4. 30, 24, ____, 12, 6, …

   Constant difference or a constant ratio?

   Constant Difference

   The constant difference/ratio is $\mathbf{-6}$

Set

Topic: Determine recursive equations

Two consecutive terms in an arithmetic sequence are given. Find the constant difference and the recursive equation.

5. If $f(3) = 5$ and $f(4) = 8$.

   $f(5) = \mathbf{11}$, $f(6) = \mathbf{14}$ Recursive Function: $\boldsymbol{f(1) = -1,\ f(n) = f(n-1) + 3}$

6. If $f(2) = 20$ and $f(3) = 12$.

   $f(4) = \mathbf{4}$, $f(5) = \mathbf{-4}$ Recursive Function: $\boldsymbol{f(1) = 28,\ f(n) = f(n-1) - 8}$

Topic: Recursive and explicit equations

Determine whether each situation represents an arithmetic or geometric sequence and then find the recursive and explicit equation for each.

7. 2 , 4 , 6 , 8 , …

Arithmetic or Geometric? **Arithmetic**

Recursive: $\boldsymbol{f(1) = 2,\ f(n) = f(n-1) + 2}$

Explicit: $\boldsymbol{f(n) = 2n}$

8.

Time (days)	Number of Cells
1	5
2	8
3	12.8
4	20.48

Arithmetic or Geometric? **Geometric**

Recursive: $\boldsymbol{f(n) = f(n-1) \times 1.6, f(1) = 5}$

Explicit: $\boldsymbol{f(n) = 5(1.6)^{n-1}}$

9. Cami invested $6,000 dollars into an account that earns 10% interest each year.

Arithmetic or Geometric? **Geometric**

Recursive: $\boldsymbol{f(0) = 6000, f(n) = f(n-1) \times 1.1}$

Explicit: $\boldsymbol{f(n) = 6000(1.1)^{n}}$

10. Scott decides to add running to his exercise routine and runs a total of one mile his first week. He plans to double the number of miles he runs each week.

Arithmetic or Geometric? **Geometric**

Recursive: $\boldsymbol{f(1) = 1, f(n) = f(n-1) \times 2}$

Explicit: $\boldsymbol{f(n) = 2^{n-1}}$

11. Vanessa has $60 to spend on rides at the State Fair. Each ride cost $4.

Arithmetic or Geometric? **Arithmetic**

Recursive: $\boldsymbol{f(1) = 4, f(n) = f(n-1) + 4}$

Explicit: $\boldsymbol{f(n) = 4n}$

12. Michelle likes chocolate so much that she eats it every day and it always 3 more pieces than the previous day. She ate 3 pieces on day 1.

Arithmetic or Geometric? **Arithmetic**

Recursive: $\boldsymbol{f(1) = 3, f(n) = f(n-1) + 3}$

Explicit: $\boldsymbol{f(n) = 3n}$

Go

Topic: Evaluate using function notation

Find each value.

13. $f(n) = 5^n$. Find $f(2)$.

$\mathbf{25}$

14. $f(n) = (-2)^n$. Find $f(3)$.

$\mathbf{-8}$

15. $f(n) = 3 + 4(n-1)$. Find $f(5)$ and $f(6)$.

$\boldsymbol{f(5) = 19, f(6) = 23}$

Topic: Solving systems of linear equations

Solve the system of equations using a matrix.

16. $\begin{cases} y = 2x - 10 \\ x - 4y = 5 \end{cases}$

$\mathbf{(5, 0)}$

17. $\begin{cases} x - 7y = 6 \\ -3x + 21y = -18 \end{cases}$

Infinitely Many Solutions

18. $\begin{cases} 5x - 4y = 3 \\ 6x + 4y = 30 \end{cases}$

$\mathbf{(3, 3)}$

19. $\begin{cases} 2x - 3y = -12 \\ -x + 2y = 4 \end{cases}$

$\mathbf{(-12, -4)}$

Name: **Sequences and Series** | **3.4H**

Ready, Set, Go!

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Topic: Constant Ratios

Find the constant ratio for each geometric sequence.

1. $2, 4, 8, 16, \ldots$
   $\mathbf{2}$

2. $\frac{1}{2}, 1, 2, 4, 8, \ldots$
   $\mathbf{2}$

3. $-5, 10, -20, 40, \ldots$
   $\mathbf{-2}$

4. $10, 5, 2.5, 1.25, \ldots$
   $\mathbf{\frac{1}{2}}$

Set

Topic: Recursive and explicit equations

Fill in the blanks for each table and then write the recursive and explicit equation for each sequence.

5. Table 1

n	1	2	3	4	5
$f(n)$	5	7	9	**11**	**13**

Recursive: $\mathbf{f(1) = 5, f(n) = f(n-1) + 2}$

Explicit: $\mathbf{f(n) = 5 + 2(n-1)}$

6. Table 2

n	$f(n)$
1	-2
2	-4
3	-6
4	$\mathbf{-8}$
5	$\mathbf{-10}$

Recursive:
$\mathbf{f(1) = -2,\ f(n) = f(n-1) - 2}$

Explicit:
$\mathbf{f(n) = -2(n-1) - 2}$
or $\mathbf{f(n) = -2n}$

7. Table 3

n	$f(n)$
1	3
2	9
3	27
4	**81**
5	**243**

Recursive:
$\mathbf{f(1) = 3,\ f(n) = f(n-1) \times 3}$

Explicit:
$\mathbf{f(n) = 3^n}$
or $\mathbf{f(n) = 3 \cdot 3^{n-1}}$

8. Table 4

n	$f(n)$
1	27
2	9
3	3
4	**1**
5	$\mathbf{\frac{1}{3}}$

Recursive:
$\mathbf{f(1) = 27, f(n) = f(n-1) \times \frac{1}{3}}$

Explicit:
$\mathbf{f(n) = 27\left(\frac{1}{3}\right)^{n-1}}$

Topic: Subscript notation for sequences

Other textbooks may use subscript notation to write rules for sequences. Use the examples below to write the recursive and explicit rules for the following sequences.

Example Sequence	**Function Notation**	**Subscript Notation**
$3, 5, 7, 9, \dots$	Recursive: $f(1) = 3, f(n) = f(n-1) + 2$ Explicit: $f(n) = 2n + 1$	Recursive: $a_1 = 3, a_n = a_{n-1} + 2$ Explicit: $a_n = 2n + 1$
$3, 9, 27, 81, \dots$	Recursive: $f(1) = 3, f(n) = f(n-1) \cdot 3$ Explicit: $f(n) = 3^n$	Recursive: $a_1 = 3, a_n = a_{n-1} \cdot 3$ Explicit: $a_n = 3^n$

9. $22, 19, 16, \dots$

Recursive: $\boldsymbol{a_1 = 22, a_n = a_{n-1} - 3}$

Explicit: $\boldsymbol{a_n = -3n + 25}$

10. $1, 5, 25, \dots$

Recursive: $\boldsymbol{a_1 = 1, a_n = a_{n-1} \cdot 5}$

Explicit: $\boldsymbol{a_n = 5^{n-1}}$

Topic: Arithmetic series

11. Find the sum of the first 12 terms of the sequence $f(n) = -2n + 10$

-36

12. Find the sum: $\sum_{x=1}^{21} 3x - 1$

672

13. Find the sum of the first 150 terms of the sequence 20, 15, 10, 5, …

$-52,875$

14. Find the sum of the first 200 even numbers.

402,000

15. The Agnesi High School auditorium has exactly 26 rows of seats. The rows are labeled, in order, from the front of the auditorium to the back from A through Z. There are 8 seats in the row A. Each row after the first row has two more seats than the previous row. There are 10 seats in row B, 12 seats in row C and so on.
    a. How many seats are there in row Z?

    There are 58 seats in Row Z.

    b. What is the total number of seats in the Agnesi High School auditorium?

    858 seats

16. The first figure contains one segment. For each successive figure, six segments replace each segment. This is an example of a fractal.
    a. How many segments are in each of the first four figures of the sequence?

    1, 6, 36, 216

    b. Write a recursive definition for the sequence.

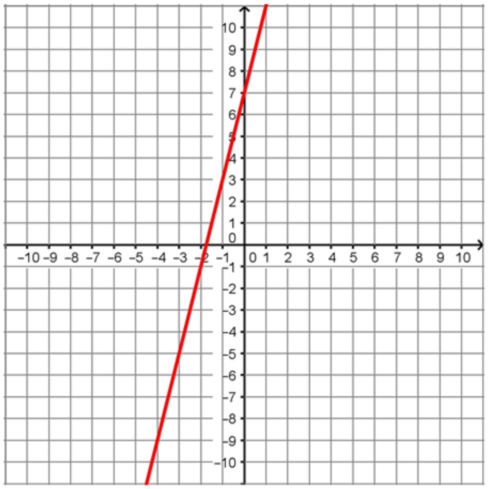
    $\boldsymbol{f(1) = 1, f(n) = f(n-1) \cdot 6}$

Go

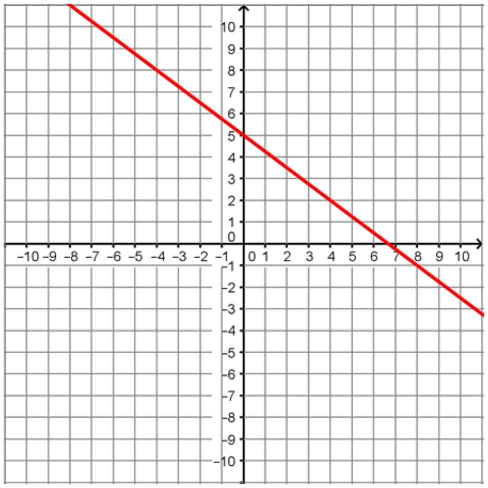
Topic: Graphing linear equations and labeling your axes.

Graph the following linear equations. Label your axes.

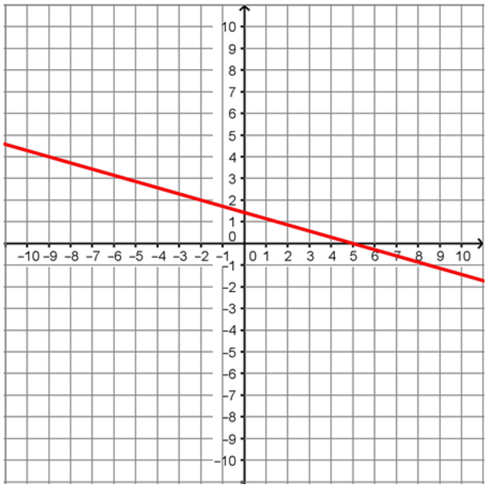
17. $y = 4x + 7$



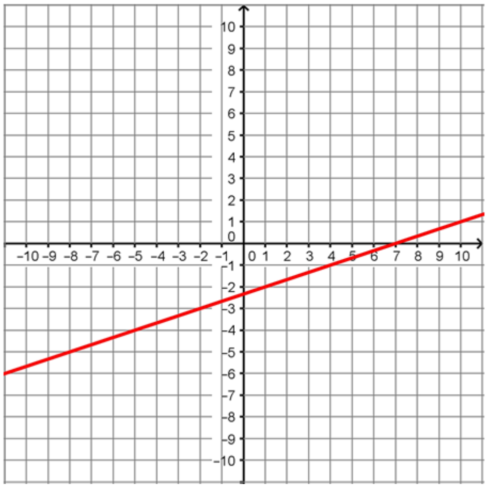
18. $y = -\frac{3}{4}x + 5$



19. $2x + 7y = 10$



20. $x - 3y = 7$



Name: **Sequences and Series | 3.5H**

Ready, Set, Go!

Ready

Topic: Arithmetic and geometric sequences

For each set of sequences, find the first five terms. Compare arithmetic sequences and geometric sequences. Which grows faster? When?

1. Arithmetic sequence: $f(1) = 2$, common difference, $d = 3$
   Geometric sequence: $g(1) = 2$, common ratio, $r = 3$

Arithmetic:	Geometric:
$f(1) = \mathbf{2}$	$g(1) = \mathbf{2}$
$f(2) = \mathbf{5}$	$g(2) = \mathbf{6}$
$f(3) = \mathbf{8}$	$g(3) = \mathbf{18}$
$f(4) = \mathbf{11}$	$g(4) = \mathbf{54}$
$f(5) = \mathbf{14}$	$g(5) = \mathbf{162}$

Which value do you think will be more, $f(100)$ or $g(100)$? Why?

$g(100)$ because the value increases much faster when multiplying the previous term by the same value as opposed to adding the same value to the previous term.

2. Arithmetic sequence: $f(1) = 2$, common difference, $d = 10$
   Geometric sequence: $g(1) = 2$, common ratio, $r = 3$

Arithmetic:	Geometric:
$f(1) = \mathbf{2}$	$g(1) = \mathbf{2}$
$f(2) = \mathbf{12}$	$g(2) = \mathbf{6}$
$f(3) = \mathbf{22}$	$g(3) = \mathbf{18}$
$f(4) = \mathbf{32}$	$g(4) = \mathbf{54}$
$f(5) = \mathbf{42}$	$g(5) = \mathbf{162}$

Which value do you think will be more, $f(100)$ or $g(100)$? Why?

$g(100)$ because the value increases much faster when multiplying the previous term by the same value as opposed to adding the same value to the previous term, even if the value added is much larger than the value multiplied by as seen in this example.

Set

Topic: Arithmetic sequences

Each of the tables below represents an arithmetic sequence. Find the missing terms in the sequence, showing your method.

3. Table 1

n	1	2	3
$f(n)$	3	**7.5**	12

$\frac{12-3}{2} = 4.5$

4. Table 2

n	$f(n)$
1	2
2	**10**
3	**18**
4	26

$\frac{26-2}{3} = 8$

5. Table 3

n	$f(n)$
1	24
2	**15**
3	6
4	**−3**

$\frac{6-24}{2} = -9$

6. Table 4

n	$f(n)$
1	16
2	**12**
3	**8**
4	4
5	**0**

$\frac{4-16}{3} = -4$

7. Table 5

n	2	3	4	5	6
$f(n)$	32	**27**	**22**	**17**	12

$\frac{12-32}{4} = -5$

Topic: Geometric sequences

Each of the tables below represents a *geometric* sequence. Find the missing terms in the sequence, showing your method.

8. Table 1

n	1	2	3
$f(n)$	3	**6**	12

$3r^2 = 12, \qquad r = 2$

9. Table 2

n	$f(n)$
1	2
2	6
3	18
4	54

$2r^3 = 54,\ r = 3$

10. Table 3

n	$f(n)$
1	5
2	±10
3	20
4	±40

$5r^2 = 20,\ r = \pm2$

11. Table 4

n	$f(n)$
1	4
2	±12
3	36
4	±108
5	324

$4r^4 = 324,\ r = \pm3$

12. Table 5

n	3	4	5	6
$f(n)$	18	*54*	*162*	486

$18r^3 = 486,\quad r = 3$

Go

Topic: Sequences

Determine the recursive and explicit equations for each (if the sequence is not arithmetic or geometric, try your best). Express answers in both function subscript notation.

13. 5, 9, 13, 17, ... This sequence is: [✓ Arithmetic], Geometric , Neither

Recursive Equation: $f(1) = 5,\ f(n) = f(n-1) + 4$
$a_1 = 5, a_n = a_{n-1} + 4$

Explicit Equation: $f(n) = 5 + 4(n-1)$
$a_n = 4n + 1$

14. 60, 30, 0, −30 , ... This sequence is: [✓ Arithmetic], Geometric , Neither

Recursive Equation: $f(1) = 60,\ f(n) = f(n-1) - 30$
$a_1 = 60, a_n = a_{n-1} - 30$

Explicit Equation: $f(n) = 60 - 30(n-1)$
$a_n = -30n + 90$

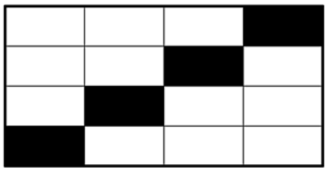
15. $60, 30, 15, \frac{15}{2}, \ldots$ This sequence is: Arithmetic, [✓ Geometric], Neither

Recursive Equation: $f(1) = 60,\ f(n) = f(n-1) \times \frac{1}{2}$
$a_1 = 60, a_n = a_{n-1} \cdot \frac{1}{2}$

Explicit Equation: $f(n) = 60\left(\frac{1}{2}\right)^{n-1}$
$a_n = 60\left(\frac{1}{2}\right)^{n-1}$

16.





(The percentage of tiles shaded black)

This sequence is: Arithmetic, [✓ Geometric], Neither

Recursive Equation: $f(1) = 100, f(n) = f(n-1) \times \frac{1}{2}$ Explicit Equation: $f(n) = 100\left(\frac{1}{2}\right)^{n-1}$

$a_1 = 100, a_n = a_{n-1} \cdot \frac{1}{2}$ $a_n = 100\left(\frac{1}{2}\right)^{n-1}$

17. 4, 7, 12, 19, … This sequence is: Arithmetic, Geometric, [✓ Neither]

Recursive Equation: $f(1) = 4,\ f(n) = f(n-1) + 2n - 1$ Explicit Equation: $f(n) = n^2 + 2n + 1$

$a_1 = 4, a_n = a_{n-1} + 2n - 1$ $a_n = n^2 + 2n + 1$

****Note: Students are not expected to be able to write the recursive or explicit equations for question 15 at this point****

18. Write the following series in summation notation: $20 + 14 + 8 + 2 - 4 - 10$

$\sum_{n=1}^{6} 20 - 6(n-1)$

19. Find the sum of $\sum_{n=3}^{6} 2^{n-1}$

60

Ready, Set, Go!

Ready

Topic: Comparing linear equations and arithmetic sequences

1. Describe similarities and differences between linear equations and arithmetic sequences.

Similarities	Differences
• **Both have a consistent change for every interval.** • **Both can be represented as functions of a variable.** • **Both have points lie on a line.**	• **Linear equations represent all solutions to all *x*-values, whereas arithmetic sequences choose only specific values.**

Set

Topic: representations of arithmetic sequences

Use the given information to complete the other representations for each arithmetic sequence.

2. Recursive Equation:

$\boldsymbol{f(1) = 8, f(n) = f(n-1) + 8}$

Explicit Equation:

$\boldsymbol{f(n) = 8n}$

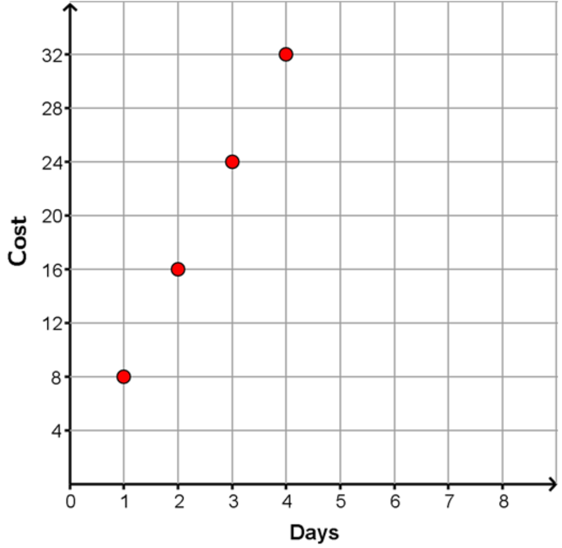
Table:

Days	Cost
1	8
2	16
3	24
4	32

Create a Context:

It costs $8 per day to rent a kayak.

Graph:



3. Recursive Equation:

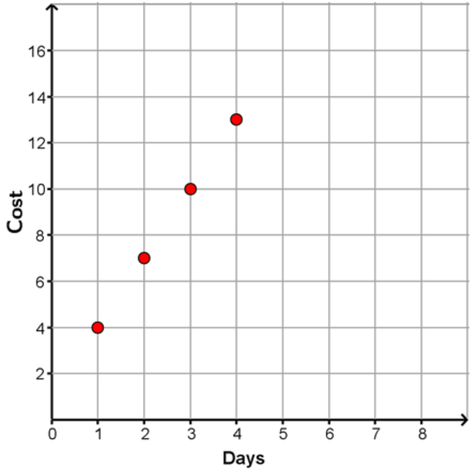
$f(1) = 4, f(n) = f(n-1) + 3$

Explicit Equation: $\mathbf{f(n) = 4 + 3(n-1)}$

Table:

Hour	**Cost**
1	**4**
2	**7**
3	**10**
4	**13**

Graph:



Create a Context:

It costs a flat fee of $1 to check out skates, and then $3 per hour for the rental.

4. Recursive Equation:

$\mathbf{f(1) = 4,}$

$\mathbf{f(n) = f(n-1) + 5}$

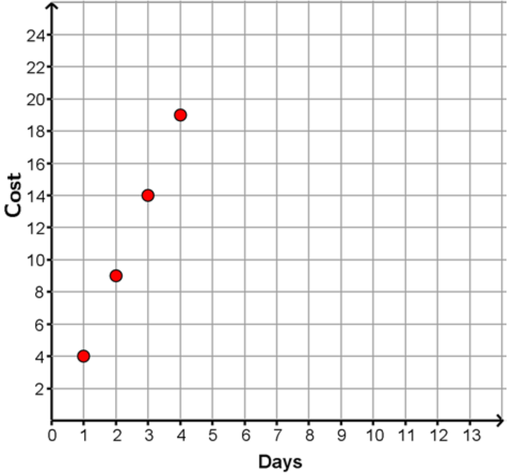
Explicit Equation:

$f(n) = 4 + 5(n-1)$

Table:

Days	**Cost**
1	**4**
2	**9**
3	**14**
4	**19**

Graph:



Create a Context:

It Costs $4 to rent snorkel gear on the first day, and then $5 every day after that.

5. Recursive Equation:

$f(1) = 14, f(n) = f(n-1) + 2$

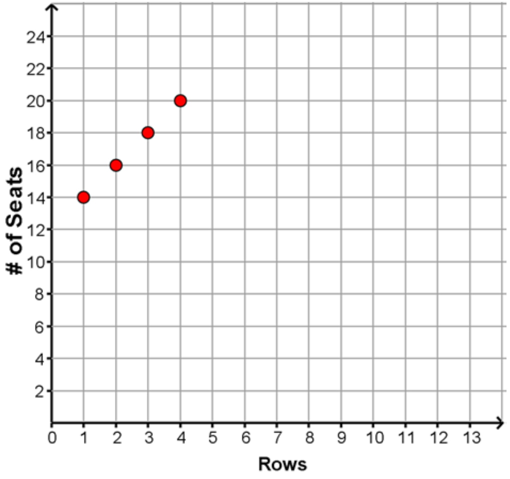
Explicit Equation:

$f(n) = 14 + 2(n-1)$

Table:

Row	# of seats
1	14
2	16
3	18
4	20

Graph:



Create a Context:

Janet wants to know how many seats are in each row of the theater. Jamal lets her know that each row has 2 seats more than the row in front of it. The first row has 14 seats.

Topic: Application of arithmetic and geometric series

Write a series representation using summation notation for each scenario and then find the sum.

6. Logs are stacked in a pile with 24 logs on the bottom row and 15 on the top row. There are 10 rows in all with each row having one more log than the one above it. How many logs are in the stack?

$\sum_{n=1}^{10} 24 - (n-1) = 195$

7. Each hour, a grandfather clock chimes the number of times that corresponds to the time of day. For example, at 3:00, it will chime 3 times. How many times does the clock chime in a day?

$2 \cdot \sum_{n=1}^{12} n = 156$

8. A company is offering a job with a salary of $30,000 for the first year and a 5% raise each year after that. If that 5% raise continues every year, find the total amount of money you will have earned by the end of your 5th year.

$\sum_{n=1}^{5} 30,000(1.05)^{n-1} = \$166,162.96$

Go

Topic: Writing explicit equations

Given the recursive equation for each arithmetic sequence, write the explicit equation.

9. $f(n) = f(n-1) - 2;\ \ f(1) = 8$

$\boldsymbol{f(n) = 8 - 2(n-1)}$

10. $f(n) = 5 + f(n-1);\ \ f(1) = 0$

$\boldsymbol{f(n) = 5(n-1)}$

11. $f(n) = f(n-1) + 1;\ \ f(1) = \frac{5}{3}$

$\boldsymbol{f(n) = \frac{5}{3} + (n-1)}$

Name: Sequences and Series | Review

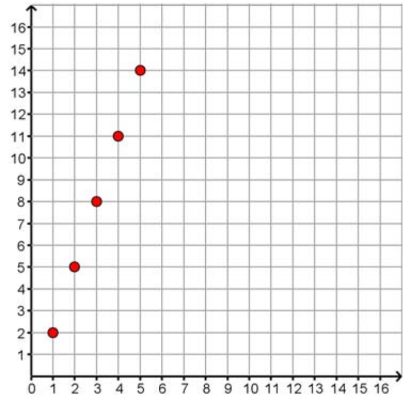
Use the given information to state as much as possible about each sequence. Your answer should include: type of sequence, the common difference or common ration, a table of at least 5 terms, a graph, the recursive rule, and the explicit rule.

1. Type: **Arithmetic**

n	$f(n)$
1	**2**
2	**5**
3	**8**
4	**11**
5	**14**

Common difference/ratio: **3**

Recursive rule:
$f(1) = 2,\ f(n) = f(n-1) + 3$



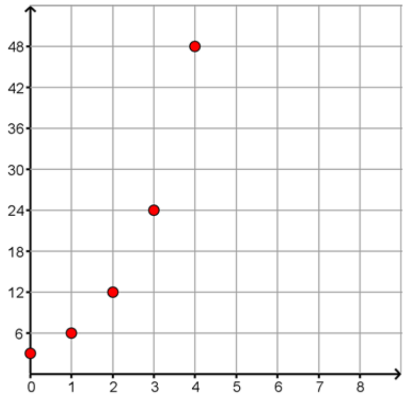
Explicit rule:
$\boldsymbol{f(n) = 2 + 3(n-1)}$

2. Type: **Geometric**

n	$f(n)$
0	**3**
1	**6**
2	**12**
3	**24**
4	**48**

Common difference/ratio: **2**

Recursive rule:
$\boldsymbol{f(1) = 6, f(n) = f(n-1) \times 2}$



Explicit rule:
$f(n) = 3 \cdot 2^n$

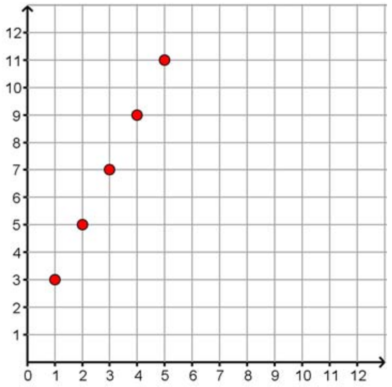
3. Type: Arithmetic

n	$f(n)$
1	3
2	**5**
3	**7**
4	**9**
5	**11**

Common difference/ratio: 2

Recursive rule:

$f(1) = 3, f(n) = f(n-1) + 2$



Explicit rule:

$f(n) = 3 + 2(n-1)$

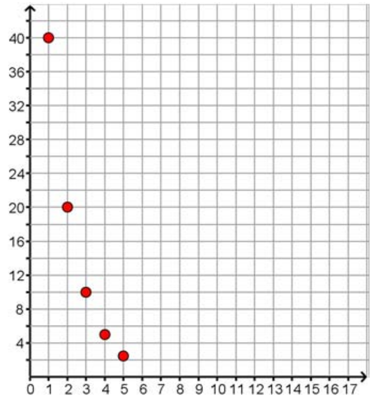
4. Type: ***Geometric***

n	$f(n)$
1	40
2	**20**
3	**10**
4	**5**
5	**2.5**

Common Ratio = $\frac{1}{2}$

Recursive rule:

$f(1) = 40, f(n) = f(n-1) \times \frac{1}{2}$



Explicit rule:

$f(n) = 40\left(\frac{1}{2}\right)^{n-1}$

5. Explain how you tell if a sequence is arithmetic and if a sequence is geometric.

If a sequence is arithmetic there is a common difference that is added or subtracted from the previous value to get the next value, whereas, if a sequence is geometric there is a common ratio that is multiplied by the previous value to get the next value. In addition, an arithmetic sequence creates a linear function, whereas a geometric sequence creates an exponential function.

For #6-8, determine if each sequence is arithmetic or geometric. Find the values of the next two terms. Then write the explicit and recursive formulas for each sequence.

6. $90, 30, 10, \frac{10}{3}, \ldots$

Arithmetic or [✓ Geometric]

Next 2 terms: $\frac{10}{9}, \frac{10}{27}$

Recursive Formula: $f(1) = 90, f(n) = f(n-1) \times \frac{1}{3}$ Explicit Formula: $f(n) = 90\left(\frac{1}{3}\right)^{n-1}$

7. $42, 34, 26, 18, \ldots$

[✓ Arithmetic] or Geometric

Next 2 terms: **10, 2**

Recursive Formula: $f(1) = 42, f(n) = f(n-1) - 8$ Explicit Formula: $f(n) = 42 - 8(n-1)$

8. $6, 13, 20, 27, \ldots$

[✓ Arithmetic] or Geometric

Next 2 terms: **34, 41**

Recursive Formula: $f(1) = 6, f(n) = f(n-1) + 7$ Explicit Formula: $f(n) = 6 + 7(n-1)$

9. Find the missing terms of the arithmetic sequence below. Be sure to show all work.

n	1	2	3	4	5	6
$f(n)$	7	**2**	**−3**	**−8**	**−13**	−18

$\frac{-18-7}{5} = -5$

10. Find the missing terms of the geometric sequence below. Be sure to show all work.

n	1	2	3	4	5	6
$f(n)$	**3.5**	7	**14**	**28**	56	**112**

$7r^3 = 56, \ r = 2$

11. Find the missing terms of the geometric sequence below. Be sure to show all work.

n	1	2	3	4	5	6
$f(n)$	972	**±324**	**108**	**±36**	12	**±4**

$972r^4 = 12, \ r = \pm\frac{1}{3}$

12. Find the sum of the first 50 multiples of 6.

$\left(\frac{6+300}{2}\right)(50) = 7,650$

13. A child building a tower with blocks uses 15 for the bottom row. Each row has 2 fewer blocks than the previous row. Suppose that there are 8 rows in the tower.

a. How many blocks are used for the top row?

1 block

b. What is the total number of blocks in the tower?

64 blocks

14. What is the common ratio of the series modeled by $\sum_{n=1}^{6} 4(-3)^{n-1}$?

-3

15. How many terms are in the geometric sequence having a first term of 2, a last term of 32, and a common ratio of -2?

5 terms

16. A snail is crawling straight up a wall. The first hour it climbs 16 inches, the second hour it climbs 12 inches, and each succeeding hour, it climbs only three-fourths the distance it climbed the previous hour. Assume the pattern continues.

a. How far does the snail climb during the fifth hour?

$\frac{81}{16} = 5.0625$ **inches**

b. What is the total distance the snail has climbed in five hours?

$\frac{781}{16} = 48.8125$ **inches**

c. Express the total distance with summation notation.

$\sum_{n=1}^{5} 16\left(\frac{3}{4}\right)^{n-1}$

Module 4H Introduction Homework

The following problems are intended for students to work on after the Module 3H Test. The first four problems introduce the beginning tasks of Module 4H. The problems are meant to be done on their own and will be discussed during the warm up the next class. The following page is blank for the teacher to copy and give to each student after the test. Below are the solutions for the challenge problems.

For #1-4, create a mathematical model for that includes a table, graph, and equation:

1. My little sister, Savannah, is three years old. She has a piggy bank that she wants to fill. She started with five pennies and each day when I come home from school, she is excited when I give her three pennies that are left over from my lunch money. Create a mathematical model for the number of pennies in the piggy bank on day n.

   $p = 3n + 5$ **where** p **represents the number of pennies Savannah has and** n **represents the number of days that have passed.**

   Check for table & graph.

2. Our family has a small pool for relaxing in the summer that holds 1500 gallons of water. I decided to fill the pool for the summer. When I had 5 gallons of water in the pool, I decided that I didn't want to stand outside and watch the pool fill, so I had to figure out how long it would take so that I could leave, but come back to turn off the water at the right time. I checked the flow on the hose and found that it was filling the pool at a rate of 2 gallons every five minutes. Create a mathematical model for the number of gallons of water in the pool at t minutes.

   $f(t) = 0.4t + 5$
   Check for table & graph.

3. I'm more sophisticated than my little sister so I save my money in a bank account that pays me 3% interest on the money in the account at the end of each month. (If I take my money out before the end of the month, I don't earn any interest for the month.) I started the account with $50 that I got for my birthday. Create a mathematical model of the amount of money I will have in the account after m months.

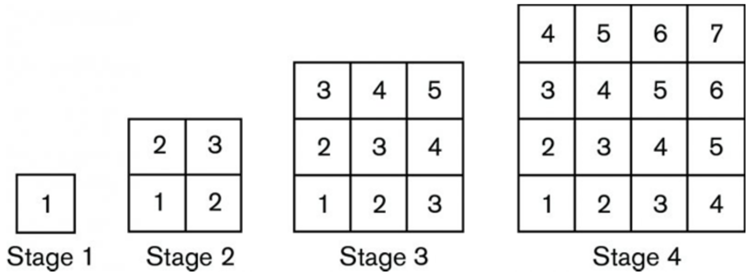
   $f(m) = 50(1.03)^x$
   Check for table & graph.

4. At the end of the summer, I decide to drain the swimming pool that holds 1500 gallons of water. I noticed that it drains faster when there is more water in the pool. That was interesting to me, so I decided to measure the rate at which it drains. I found that it was draining at a rate of 3% every minute. Create a mathematical model of the gallons of water in the pool at *t* minutes.

   $f(t) = 1500(0.97)^t$
   Check for table & graph.

Module 3 Challenge Problems

5. Consider the pattern of square grids shown. The sum of the numbers in the square grid at Stage 3 is 27. If the pattern continues, what will be the sum of the numbers in the square grid at Stage 7?



   The sum of the numbers at the Stages 1 through 4 are 1, 8, 27 and 64, respectively. Notice that these numbers are all perfect cubes: $1 = 1^3, 8 = 2^3, 27 = 3^3, 64 = 4^3$. These sums form a sequence where the nth term of the sequence is n^3. Therefore, the sum of the numbers at Stage 7 will be $7^3 = 343$.

6. The sum of a list of seven positive integers is 42. The mean, median and mode are consecutive integers, in some order. What is the largest possible integer in the list?

   We are told that the sum of the seven integers is 42, so it follows that these numbers have a mean of $\frac{42}{7} = 6$. Thus, there are three possibilities for the consecutive measures of central tendency 4, 5 and 6; 5, 6 and 7; 6, 7 and 8. But the values 4, 5 and 6 will yield the largest possible integer in such a list. Since the mean is 6, the mode and median could be 4 and 5, respectively, or vice versa.

   Suppose, 4 is the mode and 5 is the median, to get the largest integer we would have the series $1 + 4 + 4 + 5 + 6 + 7 + x = 42$. This gives a value of $x = 15$.

   Suppose 4 is the median and 5 is the mode. Since the mean does not need to be one of the integers in the list, to get the largest integer we would have the series $1 + 2 + 3 + 4 + 5 + 5 + x = 42$. This gives a value of $x = 22$.

   Therefore, the largest possible integer in the list is 22.